are the best chapters in a book full of good things—a book which, while it is hardly suitable for a beginner working privately, is one that, if used under the guidance of an experienced teacher, can hardly be matched.

With regard to notation, the author uses i for steady current, which apparently forces him on p. 282 to utilise j for the imaginary unit. It seems a pity that he is not content to use the Greek letter iota, which is now fairly standardised. Also, instead of the large L for limit, one would prefer to see the usual lim.

I noticed only one misprint, in l. 5 up, p. 165. J. M. CHILD.

Leçons sur l'approximation des Fonctions d'une Variable Réelle. By C. DE LA VALLÉE POUSSIN. Pp. 150+vi. 12 fr. 1919. (Gauthier-Villars.)

M. S. Bernstein, in a paper, "Sur les Recherches Récentes relatives à la meilleure approximation des fonctions continues par des polynômes," read at the Fifth International Congress of Mathematicians in 1912, very clearly stated the problem dealt with in this book. Since Weierstrass, in 1885, enunciated his classic theorem "that any function, continuous in an interval (a, b), can be developed in a series of polynomials, uniformly convergent in that interval," various mathematicians have given different proofs of the theorem and have constructed different polynomials of degree n, $R_n(x)$, such that the maximum value of the difference $|f(x)-R_n(x)|$ in the interval tends to zero, as n increases indefinitely. The approximations obtained by different methods, for the same function and for the same value of n, are not always the same, and this fact has led to the search for those polynomials $P_n(x)$ for which the maximum value of the difference $|f(x)-P_n(x)|$ tends most rapidly towards zero. The polynomials, $P_n(x)$, which have this property, are called "Polynomials of Approximation," and the minimum value, $E_n[(x)]$, of the modulus $|f(x)-P_n(x)|$ is called "the best approximation" of the given function in the interval. Polynomials of approximation had been used even before the discovery of Weierstrass, but it is only during the last twelve years that the systematic study of the magnitude of the best approximation, $E_n[f(x)]$, has been undertaken. Yet a consideration of this magnitude is necessary to complete Weierstrass' Theorem, which really implies that $\underset{n\to\infty}{\text{Lt}}\, E_n[f(x)]=0$, whatever continuous function of x, $f(x)$ may be.

The author of this book was (in 1908) among the first to consider the problem systematically and he was followed by Dunham Jackson (1911) and S. Bernstein (1911 and 1912). Their results are summarised by M. Bernstein in the paper mentioned at the beginning of this review. Jackson and Bernstein published further papers in 1913. It is probable that the war delayed further research on the subject by our author, who is Professor of Mathematics at ill-fated Louvain. However, during the dark days, he received hospitality from Paris, and the early months of 1918 brought new and important results from his pen. The present volume is, in the main, the reproduction of a series of lectures delivered at the Sorbonne during May and June of that year.

The main fact that emerges from the various researches is the existence of a close connection between the differential properties of the function $f(x)$ and the asymptotic law of the decrement of $E_n[f(x)]$. This book treats of this reciprocal relation. It shows that, whether the development is by polynomials or by trigonometric expressions, the order of the best possible approximation depends on the continuity and differential properties of the function or, in the case of an analytic function, on the nature and situation of the singular points. Conversely, if it is given that a function can be represented with an approximation of a certain order, it is shown that the differential properties of the function can be deduced. The author has, of course, profited largely by the work of Bernstein and Jackson, but he has made the subject peculiarly his own and he has welded the various results into one synthetic whole. In many cases, he has extended Bernstein's results and, at times, has replaced Bernstein's proofs by others, more accurate or more elegant. Jackson obtained results first for the representation by polynomials and deduced from these corresponding theorems for the trigonometric representation. M. de la Vallée Poussin very skilfully reverses this order; that his is the more natural order is evident from the greater power and the more general results obtained by his treatment.

The last two chapters, on "Fonctions analytiques présentant des certaines singularités," well illustrate the power and elegance of his method.

Even if M. de la Vallée Poussin's name were not on the title page, the inclusion of this volume in the remarkable "Collection de Monographies sur la Théorie des Fonctions," edited by M. Émile Borel, would be a sufficient guarantee of its authority and general excellence. A. DAKIN.

Elements of Vector Algebra. By L. SILBERSTEIN. Pp. iv + 42. 5s. net. 1919. (Longmans.)

This little book was written on the suggestion of Messrs. Adam Hilger, and contains the work required for reading the *Simplified Method of Tracing Rays*, by the same author. It gives an account of the addition of vectors, of their scalar multiplication (indicated by juxtaposition) and of their vector multiplication (indicated by a prefixed V). The fundamental formulae are deduced and some geometrical applications given. This fills just more than half the book. Then follows a short account of linear vector operators, of dyads and dyadics, and some hints on differentiation. A few illustrations would have been helpful here. With this book alone, the reader who is ignorant of, say, Hydrodynamics or Elasticity, would be at a loss to understand what applications could be made of the latter part. H. G. F.

Fermat's Last Theorem. (Revised Edition.) By M. CASHMORE. Pp. 55. 2s. 1918. (G. Bell & Sons.)

Three attempted proofs are given of this well-known puzzle. Unfortunately all are fallacious. H. G. F.

Les spectres Numériques. By M. PETROVITCH. With preface by M. ÉMILE BOREL. 1919. (Gauthier-Villars.)

The defect of this book is that there is nothing in it. It may seem very improbable that a book published by Gauthier-Villars, and introduced by M. Borel, should contain no proposition of interest. All that I can say is that I can find none, and that, reading between the lines of M. Borel's preface, I am inclined to suspect that that very eminent mathematician is of approximately the same opinion.

Given a series of integers, say

$$31, \ 17, \ 3, \ 169, \ 24, \ldots,$$

we can, in an infinity of ways, embody them, interspersed with zeros, in a sequence

$$31017000300169000024\ldots;$$

and such a sequence may be called a "numerical spectrum". Again, given a function $f(z)$, there may be an operation Δ such that

$$\Delta f(z) = a_0 + a_1 z + a_2 z^2 + \ldots$$

is a power series with integral coefficients. From these coefficients we may form a "spectrum", and we may, if we please, call this a "spectrum of f relative to Δ".

On these foundations M. Petrovitch builds an elaborate structure of definitions. These would be justified if some application of interest could be found for them. All that appears from the book is that M. Petrovitch has found none. G. H. HARDY.

The Theory of the Imaginary in Geometry. By J. L. S. HATTON. Pp. vi + 216. 18s. net. 1920. (Camb. Univ. Press.)

In writing this book Prof. Hatton had a great opportunity; for the subject is one of extreme importance both logically and didactically, and there is no English treatise* in which it is dealt with adequately. The book, moreover, shows abundant evidence of thought and ingenuity. If then I am disappointed with the result, it is because I differ fundamentally from the author in my judgment as to what methods of developing the theory are, in the light of modern knowledge, the best.

* No reader of Prof. Veblen's admirable *Projective Geometry* will suspect me of using "English" to include "American". I ought perhaps to except Mr. Mathews' *Projective Geometry*; but Mr. Mathews passes over these particular matters rather lightly.