



LI. On multiple reflexion

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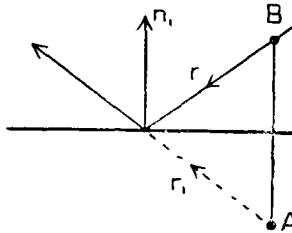
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LI. *On Multiple Reflexion.* By L. SILBERSTEIN, Ph.D.,
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THE purpose of the present paper is to give a very simple method of dealing with reflexions from any number of plane mirrors. The subject has been taken up in connexion with some technical problems concerning the construction of the kind of triple mirrors known as central mirrors.

Consider first a single plane mirror. Let the unit vector n_1 represent its normal, drawn away from the reflecting side. Let the direction of the incident ray be given by the vector r , and that of the reflected ray by r_1 . The tensors r , r_1 of these vectors are irrelevant. It will be convenient, however, to make them equal. If both are taken as unit vectors, then their scalar product rr_1 will give at once the cosine of the angle included between the incident and the reflected rays.

Now, by the fundamental law of reflexion, $r_1 - r = \vec{AB}$ has



the direction n_1 and the size $-2n_1r$, that is,

$$r_1 = r - 2n_1(n_1r), \quad . \quad . \quad . \quad . \quad . \quad (A)$$

or, using the dot as separator,

$$r_1 = [1 - 2n_1 \cdot n_1]r = \Omega_1 r.$$

Thus, the linear vector operator or the dyadic,

$$\Omega_1 = 1 - 2n_1 \cdot n_1,$$

when applied \dagger to the incident ray r , gives the reflected ray r_1 . In view of this property the operator Ω_1 can be called *the reflector* belonging to the mirror in question. It is a pure *versor*, i. e. it leaves intact the tensor of the operand.

* Communicated by the Author.

\dagger It may be applied to the operand either as prefactor, $r_1 = \Omega_1 r$, or as a postfactor, $r_1 = r \Omega_1$, the operator for a simple mirror being self-conjugate or symmetrical. The operator for a multiple mirror, however, is not symmetrical, and to avoid confusion we shall use it always as a *prefactor*.

In fact, squaring (A) and remembering that $n_1^2=1$, we have

$$r_1^2=r^2+4(n_1r)^2-4(n_1r)^2=r^2,$$

identically. Thus, r being a unit vector, so is also r_1 . Notice in passing that

$$\Omega_1\Omega_1=\Omega_1^2=1-4n_1\cdot n_1+4(n_1\cdot n_1)^2=1-4n_1\cdot n_1+4n_1\cdot n_1,$$

i. e. $\Omega_1^2=1$, which is an obvious property.

Let, now, the ray r_1 , reflected from the first mirror, impinge upon a second mirror whose normal is n_2 , again a unit vector. Then the ray r_2 , reflected from the second mirror, will be

$$r_2=\Omega_2r_1=\Omega_2\Omega_1r,$$

where $\Omega_2=1-2n_2\cdot n_2$; and since Ω_2 applied to the vector Ω_1r gives the same result as $\Omega_2\Omega_1$ applied to r (associative property), no separating signs are needed. Thus the reflector of the double mirror 1, 2 is simply $\Omega=\Omega_2\Omega_1$. Similarly, if r_2 impinges upon a third mirror, the reflected ray is $r_3=\Omega r$, where $\Omega=\Omega_3\Omega_2\Omega_1$, and so on.

Thus the reflector of a multiple mirror consisting of any number κ of plane mirrors, taken in the prescribed succession 1, 2, 3, ... κ , is

$$\Omega=\Omega_\kappa\ldots\Omega_3\Omega_2\Omega_1=\prod_{i=\kappa}^{i=1}\Omega_i, \quad . \quad . \quad . \quad (1)$$

where

$$\Omega_i=1-2n_i\cdot n_i, \quad . \quad . \quad . \quad . \quad (2)$$

n_i being the unit normal of the i th mirror*. If r is the incident and r' the finally reflected ray, we have

$$r'=\Omega r, \quad . \quad . \quad . \quad . \quad (1a)$$

and, by what has been said before,

$$r'^2=r^2. \quad . \quad . \quad . \quad . \quad (3)$$

If, therefore, r is a unit vector, so is r' , and if θ be the angle between the incident and the finally reflected ray, we have simply

$$\cos \theta=r\Omega r. \quad . \quad . \quad . \quad . \quad (4)$$

* Notice that, for a given multiple mirror, Ω will, in general, be different according to the order of succession of the component reflexions, that is, of the suffixes i . In short, the "products" $\Omega_3\Omega_2\Omega_1$, etc., are associative, but, in general, *not commutative*. They become so only for certain multiple mirrors, *i. e.* for some particular arrangements of the component mirrors. If a finite beam of parallel rays r impinges upon, say, a triple mirror, some pencils may be reflected as $\Omega_3\Omega_2\Omega_1r$, others as $\Omega_3\Omega_1\Omega_2$, and so on. Thus the several reflected pencils will, in general, diverge from one another.

In particular, when the arrangement of the component mirrors is such that

$$\Omega = -1,$$

then every incident ray will be sent back parallel to its own path. Such multiple mirrors are called *central* mirrors.

By a well-known theorem of vector algebra, the general reflector (1), being a pure versor, could always be expressed by $\Omega = \mathbf{a} \cdot \mathbf{i} + \mathbf{b} \cdot \mathbf{j} + \mathbf{c} \cdot \mathbf{k}$, where both \mathbf{a} , \mathbf{b} , \mathbf{c} , and \mathbf{i} , \mathbf{j} , \mathbf{k} are some normal systems of unit vectors, both right-handed, or both left-handed. No use, however, will be made here of this fundamental property, since obviously the most natural entities to represent the properties of any multiple mirror are the normals $\mathbf{n}_1, \mathbf{n}_2$, etc. themselves. These appear in Ω as dyads, such as $\mathbf{n}_1 \cdot \mathbf{n}_1$ or $\mathbf{n}_2 \cdot \mathbf{n}_1$, etc., or as scalar products $\mathbf{n}_1^2 = 1$, $\mathbf{n}_1 \mathbf{n}_2 = \cos(\mathbf{n}_1, \mathbf{n}_2)$, and so on. In certain cases it may be advantageous to employ the unit edges of consecutive mirrors, *i. e.* apart from the scalar factors $\sin(\mathbf{n}_1, \mathbf{n}_2)$, etc., the vector products $\mathbf{V} \mathbf{n}_1 \mathbf{n}_2$, and so on.

The utility of the above method of treatment, in which the clumsy and often unmanageable formulæ of spherical trigonometry are replaced by the simple operator (1), *needing no drawings whatever*, will best be exhibited on a number of examples. We shall begin with the simplest case of a double mirror and then proceed to more complicated ones. In each case the procedure will consist in simply "multiplying" out the dyads $\mathbf{n} \cdot \mathbf{n}$ contained in the component reflectors. And in doing so we have only to remember that juxtaposed vectors, not separated by dots, are fused into ordinary scalar products. Thus, $\mathbf{n}_1 \cdot \mathbf{n}_1 \mathbf{n}_2 \cdot \mathbf{n}_2 = \mathbf{n}_1 (\mathbf{n}_1 \mathbf{n}_2) \cdot \mathbf{n}_2 = a_{12} \mathbf{n}_1 \cdot \mathbf{n}_2$, where $a_{12} = \mathbf{n}_1 \mathbf{n}_2 = \cos(\mathbf{n}_1, \mathbf{n}_2)$. In short, the "product" of any number of dyads is always a dyad, including an ordinary scalar factor. In what follows we shall employ the general notation

$$\mathbf{n}_i \mathbf{n}_j = \cos(\mathbf{n}_i, \mathbf{n}_j) = a_{ij} = a_{ji}. \quad . \quad . \quad . \quad (5)$$

The incident ray or operand \mathbf{r} need not be written out in each case; it is enough to deal with the operators Ω_i and with their resultants, *i. e.* with the reflectors themselves. All of the operations involved being associative as well as distributive, the multiplication will be done as in ordinary algebra, the only precaution (owing to non-commutativity) being the preservation of order.

Double mirror.—The unit normals of the component mirrors being $\mathbf{n}_1, \mathbf{n}_2$, and $\mathbf{n}_1 \mathbf{n}_2 = a_{12} = a$ the cosine of their

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included angle, we have, by (1), (2),

$$\Omega = \Omega_2 \Omega_1 = 1 - 2[\mathbf{n}_1 \cdot \mathbf{n}_1 + \mathbf{n}_2 \cdot \mathbf{n}_2] + 4\mathbf{a} \cdot \mathbf{n}_2 \cdot \mathbf{n}_1, \quad (6)$$

or, introducing the vector $\mathbf{p} = \mathbf{n}_1 - 2\mathbf{a}\mathbf{n}_2$,

$$\Omega = 1 - 2\mathbf{p} \cdot \mathbf{n}_1 - 2\mathbf{n}_2 \cdot \mathbf{n}_2. \quad (7)$$

The meaning of this operational equation is seen at once by remembering that $\mathbf{r}' = \Omega \mathbf{r}$; thus

$$\mathbf{r} - \mathbf{r}' = 2(\mathbf{r}\mathbf{n}_1)\mathbf{p} + 2(\mathbf{r}\mathbf{n}_2)\mathbf{n}_2, \quad (7a)$$

i. e. whatever the incident ray \mathbf{r} , the vector $\mathbf{r}' - \mathbf{r}$ is normal to the common edge of the two mirrors. In other words, the projections of \mathbf{r} and \mathbf{r}' upon the edge are equal to one another.

From (6) we see that, in general, $\Omega_2 \Omega_1$ differs from $\Omega_1 \Omega_2$, since the last term $4\mathbf{a}\mathbf{n}_2 \cdot \mathbf{n}_1$ is not symmetrical. Thus, a beam of parallel rays \mathbf{r} (broad enough to impinge upon both mirrors) is split by the double mirror into two beams \mathbf{r}' , \mathbf{r}'' oblique to one another, e. g. such that

$$\mathbf{r}' - \mathbf{r}'' = 4\mathbf{a}[\mathbf{n}_2 \cdot \mathbf{n}_1 - \mathbf{n}_1 \cdot \mathbf{n}_2]\mathbf{r}.$$

In particular, if the double mirror is *orthogonal*, we have $\mathbf{a} = \mathbf{n}_1 \mathbf{n}_2 = 0$, and, independently of the order of reflexions,

$$\Omega = \Omega_2 \Omega_1 = \Omega_1 \Omega_2 = 1 - 2\mathbf{n}_1 \cdot \mathbf{n}_1 - 2\mathbf{n}_2 \cdot \mathbf{n}_2.$$

But since $\mathbf{n}_1 \perp \mathbf{n}_2$, we have

$$1 = \mathbf{n}_1 \cdot \mathbf{n}_1 + \mathbf{n}_2 \cdot \mathbf{n}_2 + \mathbf{e} \cdot \mathbf{e},$$

where \mathbf{e} is a unit vector along the common edge of the two mirrors*. Therefore, for an *orthogonal* double mirror,

$$\Omega = -[1 - 2\mathbf{e} \cdot \mathbf{e}], \quad (8)$$

that is to say, the reflexion from such a double mirror is equivalent to the reflexion from a simple mirror whose normal is \mathbf{e} , followed by a simple reversal (-1). This is valid for *any* incident ray \mathbf{r} . More especially, if the incident ray is normal to the edge, or $\mathbf{r}\mathbf{e} = 0$, we have $\mathbf{r}' = \Omega \mathbf{r} = -\mathbf{r}$, that is to say, the ray is sent back parallel to itself. The latter property is familiar from ordinary geometrical constructions.

* If $\mathbf{i}, \mathbf{j}, \mathbf{k}$ be any normal system of unit vectors, the dyadic $\mathbf{i} \cdot \mathbf{i} + \mathbf{j} \cdot \mathbf{j} + \mathbf{k} \cdot \mathbf{k}$ is equivalent to 1, or is, in Gibbs's nomenclature, an *idemfactor*: $[\mathbf{i} \cdot \mathbf{i} + \mathbf{j} \cdot \mathbf{j} + \mathbf{k} \cdot \mathbf{k}]\mathbf{r} = \mathbf{r}$, for any \mathbf{r} . Notice that, since \mathbf{e} appears only through the dyad $\mathbf{e} \cdot \mathbf{e}$, the sense of \mathbf{e} is, obviously, a matter of indifference.

Returning to the general formula (6) we have for the angle θ between the rays \mathbf{r} and \mathbf{r}' , $\cos \theta = \mathbf{r} \Omega \mathbf{r}$, i. e.

$$\frac{1}{2}(1 - \cos \theta) = r_1^2 + r_2^2 - 2ar_1r_2,$$

where $r_1 = \mathbf{r} \mathbf{n}_1$, $r_2 = \mathbf{r} \mathbf{n}_2$ are the projections of the incident ray upon the mirror normals. On the other hand, since $\mathbf{r} = r_1 \mathbf{n}_1 + r_2 \mathbf{n}_2 + (\mathbf{r} \mathbf{e}) \mathbf{e}$ and \mathbf{r} is a unit vector, we have

$$r_1^2 + r_2^2 + 2ar_1r_2 = 1 - (\mathbf{r} \mathbf{e})^2,$$

so that the last equation can be written

$$\frac{1}{2}(1 - \cos \theta) = 1 - 4ar_1r_2 - (\mathbf{r} \mathbf{e})^2. \quad . \quad . \quad . \quad (9)$$

This gives the angle θ for any double mirror and for any \mathbf{r} .

From this general formula we see at once that there is no such double mirror which would send back (parallel to its own path) *every* incident ray, in short, that there are no central double mirrors. In fact, $\cos \theta = -1$ would mean $4ar_1r_2 + (\mathbf{r} \mathbf{e})^2 = 0$, and this cannot be fulfilled for all directions of \mathbf{r} .

Triple mirror.—The reflector in this case is $\Omega = \Omega_3 \Omega_2 \Omega_1$, that is, the product of $\Omega_3 = 1 - 2\mathbf{n}_3 \cdot \mathbf{n}_3$ into the operator (6) of the preceding section. Writing, therefore, $\mathbf{n}_1 \mathbf{n}_2 = a_{12}$, etc., we have, for any triple mirror (the order of reflexions being 1, 2, 3),

$$\begin{aligned} \Omega = \Omega_{123} = 1 - 2[\mathbf{n}_1 \cdot \mathbf{n}_1 + \mathbf{n}_2 \cdot \mathbf{n}_2 + \mathbf{n}_3 \cdot \mathbf{n}_3] \\ + 4[a_{12}\mathbf{n}_2 \cdot \mathbf{n}_1 + a_{23}\mathbf{n}_3 \cdot \mathbf{n}_2 + a_{31}\mathbf{n}_3 \cdot \mathbf{n}_1] - 8a_{12}a_{23}\mathbf{n}_3 \cdot \mathbf{n}_1. \end{aligned} \quad (10)$$

Here again, the third and fourth terms being non-symmetrical, an incident beam of parallel rays will give rise to six reflected beams $\Omega_{123}\mathbf{r}$, $\Omega_{132}\mathbf{r}$, etc., which will, in general, not be parallel to one another. These reflected beams become parallel to one another, i. e. Ω becomes independent of the order of reflexions 1, 2, 3, when, and only when,

$$a_{12} = a_{23} = a_{31} = 0,$$

i. e. when the three component mirrors are *perpendicular* to one another. In that case, $\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$ being a triad of normal unit vectors, the dyadic $\mathbf{n}_1 \cdot \mathbf{n}_1 + \mathbf{n}_2 \cdot \mathbf{n}_2 + \mathbf{n}_3 \cdot \mathbf{n}_3$ becomes an idemfactor or 1, and therefore,

$$\Omega = -1; \mathbf{r}' = -\mathbf{r}. \quad . \quad . \quad . \quad . \quad (11)$$

That is to say, every incident ray is sent back parallel to itself. The orthogonal triple mirror is a *central* mirror.

Returning to the general triple mirror, let r_1, r_2, r_3 be the

direction cosines of the incident ray, *i. e.* the projections of \mathbf{r} upon $\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$. Then the direction cosines r_1', r_2', r_3' of the reflected ray \mathbf{r}' will be, by (10),

$$\left. \begin{aligned} r_1' &= r_1(1-2r_1), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ r_2' &= r_2(1-2r_2+4a_{12}r_1), \quad . \quad . \quad . \quad . \quad . \quad . \\ r_3' &= r_3(1-2r_3+4a_{23}r_2+4a_{31}r_1-8a_{12}a_{23}r_1). \end{aligned} \right\} \dots \quad (12)$$

These scalar formulæ can at once be used for numerical calculation.

The angle $\theta = (\mathbf{r}, \mathbf{r}')$ is given by $\cos \theta = \mathbf{r} \Omega \mathbf{r}$, *i. e.* by (10),

$$\frac{1 - \cos \theta}{2} = r_1^2 + r_2^2 + r_3^2 - 2[a_{12}r_1r_2 + a_{23}r_2r_3 + a_{31}r_3r_1] + 4a_{12}a_{23}r_3r_1.$$

On the other hand, we have, by squaring

$$\mathbf{r} = \mathbf{r}_1\mathbf{n}_1 + \mathbf{r}_2\mathbf{n}_2 + \mathbf{r}_3\mathbf{n}_3^*,$$

$$r^2 = 1 = r_1^2 + r_2^2 + r_3^2 + 2[a_{12}r_1r_2 + a_{23}r_2r_3 + a_{31}r_3r_1],$$

and therefore, for any incident ray, whose order of reflexions is 123,

$$\frac{3 - \cos \theta}{4} = r_1^2 + r_2^2 + r_3^2 + 2a_{12}a_{23}r_3r_1. \quad . \quad . \quad (13)$$

From this general formula, which enables us to calculate at once the angle θ for any incident ray, we can see also that the orthogonal mirror is the only possible central mirror. In fact, the right-hand member of (13) becomes equal to 1 (*i. e.* $\cos \theta = -1$) for *any* direction of \mathbf{r} , when, and only when, $\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$ are normal to one another.

If the three mirrors constitute a *regular pyramid*, *i. e.* if

$$a_{12} = a_{23} = a_{31} = \cos \omega, \text{ say, } . \quad . \quad . \quad . \quad (14)$$

then the reflector (10) becomes

$$\begin{aligned} \Omega &= 1 - 2[\mathbf{n}_1 \cdot \mathbf{n}_1 + \mathbf{n}_2 \cdot \mathbf{n}_2 + \mathbf{n}_3 \cdot \mathbf{n}_3] \\ &\quad + 4a[\mathbf{n}_2 \cdot \mathbf{n}_1 + \mathbf{n}_3 \cdot \mathbf{n}_2 + \mathbf{n}_3 \cdot \mathbf{n}_1] - 8a^2\mathbf{n}_3 \cdot \mathbf{n}_1, \end{aligned} \quad (10a)$$

and the formula (13) for the angle $\theta = (\mathbf{r}, \mathbf{r}')$,

$$\frac{3 - \cos \theta}{4} = r_1^2 + r_2^2 + r_3^2 + 2 \cos^2 \omega r_1r_3, \quad . \quad (13a)$$

for any incident ray, the order of succession of the reflexions being 123, for (10a), and either 123 or 321 for (13a). If the order is 231 (or 132) or 312 (or 213), we have only to write, in the last term of (13a), r_2r_1 and r_3r_2 respectively.

* We assume, of course, that $\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$ are not coplanar, *i. e.* that the three reflecting planes constitute a pyramid, not a prism.

More especially, if the incident ray is *equally inclined* to the three reflecting planes of the regular pyramid, we have

$$r_1 = r_2 = r_3 = \frac{1}{\sqrt{3}} \cot \frac{\omega}{2},$$

and (13 a) becomes

$$\frac{3 - \cos \theta}{4} = \left(1 + \frac{2}{3} \cos^2 \omega\right) \cot^2 \frac{\omega}{2}. \quad \dots (13 b)$$

In this case the angle θ is independent of the order of reflexions, as was to be expected. The reflected beams, although not parallel to one another, are equally inclined to, and symmetrically disposed around, the direction of the incident beam \mathbf{r} . These reflected beams coincide in direction when, and only when, $\omega = 90^\circ$, *i. e.* when the mirror becomes an orthogonal and, therefore, a central mirror.

Further discussion of the above formulæ and the construction of similar ones for quadruple and more complicated mirrors are left to the reader. Here but two further remarks on the general reflector Ω :—

Reversal of the order of reflexions.—Let \mathbf{r} be the incident ray, \mathbf{r}' the finally reflected ray when the order of reflexions is $123\dots\kappa$, and \mathbf{s}' the finally reflected ray when the order of reflexions is $\kappa\dots321$. Then, if $\Omega = \Omega_\kappa\dots\Omega_3\Omega_2\Omega_1$, as in (1),

$$\mathbf{r}' = \Omega \mathbf{r}, \quad \mathbf{s}' = \mathbf{r} \Omega,$$

whence

$$\mathbf{r} \mathbf{r}' = \mathbf{r} \Omega \mathbf{r}, \quad \mathbf{s}' \mathbf{r} = \mathbf{r} \Omega \mathbf{r},$$

and therefore, for any multiple mirror,

$$\mathbf{r}' \mathbf{r} = \mathbf{s}' \mathbf{r}, \quad \dots \dots \dots (14)$$

while $\mathbf{r}' \mathbf{s}' = \mathbf{r} \Omega^2 \mathbf{r}$. That is, the reflected rays \mathbf{r}' , \mathbf{s}' , although not parallel to one another, are always *equally inclined to the incident ray* \mathbf{r} . The equality $\theta_{123} = \theta_{321}$, exhibited by (13 a) is but a special instance of this general property.

Images of given objects.—Hitherto we have considered \mathbf{r} and \mathbf{r}' as determining the directions of the incident and the reflected rays. In order to obtain the image of a given point-object, let the end-point of the vector \mathbf{r} , drawn from a fixed origin O , determine the position of the object, and the end-point of \mathbf{r}' , drawn from the same origin, the position of the image. Then, in the case of a simple mirror, we have again

$$\mathbf{r}' = \Omega_1 \mathbf{r} = [1 - 2\mathbf{n}_1 \cdot \mathbf{n}_1] \mathbf{r},$$

provided that O is a point of the reflecting plane itself.

Similarly, in the case of a double mirror,

$$\mathbf{r}' = \Omega_2 \Omega_1 \mathbf{r},$$

if O is taken on the common edge of the component mirrors. Thus, in the case of any multiple mirror, we have, as before,

$$\mathbf{r}' = \Omega \mathbf{r}, \quad \Omega = \prod_{i=\kappa}^{i=1} \Omega_i,$$

where $\Omega_i = 1 - 2\mathbf{n}_i \cdot \mathbf{n}_i$, provided that all the component mirrors have a *common* point of intersection, and that this point is taken as the *origin* of the vectors \mathbf{r}, \mathbf{r}' .

Under these circumstances, therefore, the treatment of point-objects and their images is formally the same as that of incident and reflected rays.

September 5, 1916.

LII. *On an X-Ray Vacuum Spectrograph.* By MANNE SIEGBAHN, *Dr. phil.*, and EINAR FRIMAN, *Dr. phil.**

IN order to examine the high frequency spectra of the elements by long wave-lengths the authors have had a vacuum spectrograph built. Hereby, as our former measurements † have shown, the following conditions must be satisfied. First, the crystal must be movable, as otherwise irregularities in the structure of the crystal may be of great influence (comp. Rutherford and Andrade and E. Wagner). Secondly, in order to get a good resolving power, besides using a fine slit it is of great importance to focus the rays.

The apparatus built on this principle is shown in fig. 1, in both horizontal and vertical section. The spectrograph consists of a round metallic box of 6 mm. thickness, a height of 8 cm. and an inner diameter of 30 cm. The upper part BB, 3.5 cm. broad, is carefully plane-ground, as well as the corresponding part CC of the cover D. This as well as the bottom is furnished with radial reinforcements in order to resist the pressure better. The cover has a handle and a screw, the latter being used to lift it after the air has been admitted. The box is supported by three set-screws. In the middle of the bottom there is a conical hole with a metallic cone E, fitting well in it. This cone, being kept in the hole by a ring screwed into the box, after lubricating can be turned without the air passing through. The crystal table, placed

* Communicated by the Authors.

† Phil. Mag. vol. xxxi. p. 403 (1916); vol. xxxii. p. 39 (1916).