



CIII. On the forces acting upon the poles of the electric arc

Professor A.M. Tyndall

To cite this article: Professor A.M. Tyndall (1921) CIII. On the forces acting upon the poles of the electric arc , Philosophical Magazine Series 6, 42:252, 972-981, DOI: [10.1080/14786442108633837](https://doi.org/10.1080/14786442108633837)

To link to this article: <http://dx.doi.org/10.1080/14786442108633837>



Published online: 08 Apr 2009.



Submit your article to this journal [↗](#)



Article views: 2



View related articles [↗](#)



Citing articles: 3 View citing articles [↗](#)

CIII. *On the Forces acting upon the Poles of the Electric Arc.*
By PROFESSOR A. M. TYNDALL*.

IN the preceding paper by Mr. H. E. G. Beer and the author it is shown that the hydrostatic pressure at the poles of the electric arc, first observed by Dewar, may be regarded as arising from an electric wind set up by the motion of space charges in the vicinity of the electrodes; and, further, that as the main body of the arc contributes practically nothing to the wind, it must contain practically equal quantities of each sign of ion per cubic centimetre at all points.

Reference is also made to the work of Duffield, Burnham, and Davis, who observed the existence of a *mechanical* pressure on each pole of the arc tending to thrust them apart. In looking for the cause of this pressure, Duffield rules out all motion of ions which arises from the electrostatic forces within the arc; and adopting Pollock's view of the projection of high-speed electrons from the cathode, he regards the pressure on the cathode as representing the recoil of this projection, while that on the anode is due to the transference of the momentum of the electrons to it either directly by the electrons themselves or indirectly by its communication to the gas as drag.

It seems to the author it is not legitimate to rule out entirely the effects of the electrostatic forces within the arc, and that it is desirable to re-examine the theoretical basis of the Duffield effect in the light of the further information which the study of the Dewar hydrostatic pressure has provided.

When an ion moves viscously anywhere in a gas between two electrodes, it receives momentum from the electric field and simultaneously hands it on to the gas, while the electrodes experience reaction equal and opposite to the force acting on the ion.

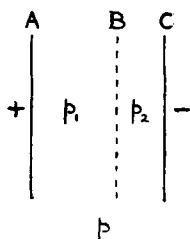
If the electrodes are so large and so near one another that there is no appreciable loss of momentum from the gas between them to that outside, the momentum in the gas will be passed back to the electrodes and will be exactly equal and opposite to that which they receive from the electrical reaction. This is, however, only general when the forces on both electrodes are taken together.

Before a similar general statement is possible for the electrodes taken separately, one special condition must obtain—namely that of symmetry.

* Communicated by the Author.

Take the case of discharge between two large parallel electrodes A and C close together (that is virtually between two infinite parallel plates). Let the dotted line B represent a thin uniform sheet of negative ions which has started at the cathode C and is on its way to the anode A. Let the initial pressure of the gas between A and C be " p ," and let the conditions be such that the field between A and C is sensibly uniform; also assume the motion of B to be viscous, *i. e.* steady on the average. The drag of B on the gas is constant and calls into existence a constant difference of pressure $p_1 - p_2$ between AB and BC, due to the transference of a small amount of gas from right to left through B.

Fig. 1.



Since the electrodes are large and the distance AC short, there is no eddying, and the momentum of the drag is passed straight on to that of static pressure without any appreciable amount taking the form of motion of gas in the process.

At the beginning of B's journey close to C, p_1 will be equal to p , and the whole change in static pressure will occur in p_2 . Hence C will feel the whole of B's drag as a suction, while A will be unaffected.

As B travels across, p_1 rises above p while p_2 approaches p until, when B is close to A, $p_2 = p$ and the whole change of static pressure occurs in p_1 . The momentum imparted to the gas is thus returned wholly to C at the start and wholly to A at the finish, and divided between A and C in varying proportions on the journey. But the process is symmetrical, and, if the whole journey is taken into account, it will be seen that half the drag momentum is received by A and half by C, the resultant force on each being directed towards the left. As, moreover, half the drag is also felt by A as electrical reaction and the other half by C, these being forces directed towards the right, it follows that A and C will be separately in equilibrium when the whole journey of the ions is considered. This argument holds equally when the discharge takes the form of a constant current between A and C.

If the diameters of A and C are not infinite compared with the distance between them, it will still be true that the whole drag is taken by C at the start and by A at the finish during those stages of the motion when the distance from B to either electrode is still very small. But in the main region between them the direct effect of the drag will be to set the gas in motion; and it is only after this motion has been transformed into static pressures and suction that the electrodes receive back their shares of the drag momentum. None the less, so long as the motion of the gas is not excessive, that is to say so long as it is viscous, the process will be symmetrical and the electrodes will each end by receiving half the total momentum. In this case also, therefore, the electrodes will be separately in equilibrium when the whole journey is taken into account.

But now let us consider the effect of a dissymmetry in the form of the electrodes.

If C is of wire gauze and A is continuous, the electrical reactions are unaltered, but the plate now receives nearly the whole momentum from the gas, because gas can flow in through C behind the layer B as it moves forward. The electrodes are therefore in this case thrust apart by equal and opposite momenta. If C is continuous and A of wire gauze, the electrodes will be thrust towards one another. Or if the electrodes are of very different sizes, the balance of forces at each separately will be upset. The case of the electric windmill is well known. Also when a fine point discharges against a large plate delicately suspended, the plate moves towards or away from the point according as it is or is not perforated to allow the wind to pass through it. It appears, therefore, that only when complete symmetry in all the discharge conditions is present can it be said with certainty that each electrode separately will be in equilibrium in respect of the viscous motion of ions between them.

But when the ions are accelerating under the force of the field, symmetry is generally lost and the electrodes are no longer separately in equilibrium. Two cases of this type which bear on the present discussion are referred to below—the starting from rest of an ion which later reaches the steady state of viscous motion, and the projection of ions from one electrode to the other without perceptible drag on the gas.

Given the correctness of Pollock's view of high-speed electrons projected from the cathode with a velocity of 1.4×10^8 cm. per sec., Duffield deduces that they must carry

one-half of the current in the arc. The absence of wind-pressure in the body of the arc makes it impossible to suppose that the remainder is carried by positive ions. A further supply of relatively low-speed ions must therefore be available for the purpose. For the same reason, these low-speed ions must be associated with a positive space-charge equal and opposite to their own; otherwise the drag on the gas would be enormous—some thousands of dynes per sq. cm. for an arc-length of 1 cm.

This positive charge is doubtless brought into position and held there by its attraction for the negative ions, from which it follows that the high-speed electrons in shooting across the arc will also be associated with an equal and opposite space-charge.

Now, in order to produce a Duffield effect, it would appear that these electrons must retain their original momentum in travelling across the arc; because, if they exert any drag on the gas, they will very rapidly be pulled up quite close to the cathode, thereby setting up a suction there which will balance the recoil of the electrode and so reduce the Duffield effect to zero.

On the projected electron view, the arc will thus be traversed by four sets of ions—the slow-moving negatives N_1 with their compensating positives M_1 , and the high-speed electrons N_2 with their positives M_2 . Of these, N_2 alone do not drag the gas—a condition which is very difficult to reconcile with the absence of wind in the body of the arc; for absence of wind implies that, while the proportion of N_1 to N_2 is purely accidental, the space-charges of M_1 and M_2 together are equal to the space-charge of N_1 alone, though the value of M_1 is determined only by N_1 and that of M_2 by N_2 .

It is, of course, easy to understand how the drags of N_1 and M_1 may neutralize one another, since their mutual attractions tend to equalize their space-charges; but on that view we should expect to find a wind due to M_2 blowing from anode to cathode. The drag of this wind for an arc 1 cm. long is given by the product of the field in the body of the arc and the ratio of the current carried by the projected electrons to their velocity. Taking a field of 25 volts per cm., a current of 10 amperes, and a velocity of 1.4×10^8 cm. per sec., this gives a slope of pressure of 8.9 dynes per cm. of arc-length directed towards the cathode. One half of this, or 4.4 dynes, would be observed on the gauge attached as it was in the wind-pressure measurements, viz. with one limb to the electrode

and the other to the open air; and curves giving the variation of hydrostatic pressure at, say, the anode with length of arc should therefore dip by that amount as the arc-length increases.

But this is a pressure which would have been readily observed if it had been present; whereas the pressure-distance curves given in curve III. of the previous paper show no sign of it, and in fact slope, if anything, in the opposite direction.

This result is, moreover, consistent with other evidence militating against the view that electrons of speed of the order of 10^8 cm. per second would shoot across the arc without exerting an appreciable drag. This velocity is considerably below that at which an electron is usually regarded as acquiring an abnormal free path. Also, Campbell Swinton failed to detect in the arc electrons sufficient energy to penetrate an aluminium sheet of thickness 0.0026 mm., which, taking Whiddington's constants for β -rays, is equivalent to 0.094 mm. of air. The author is indebted to Mr. E. G. Hill, M.Sc., for carrying out a somewhat allied experiment on ions from a very fine point and from a white-hot loop in point-plane discharge. Any projected electrons must under these conditions be subjected at the point to the action of a very high accelerating field, which would presumably increase their range. He failed, however, with these sources to find ions of sufficient energy to penetrate foil of this thickness, even when the potential of the point or loop was 10,000 volts and the discharge-gap only a few millimetres. Nor was any effect obtained down to a pressure of 5 mm. of mercury.

Lastly, Pollock's argument that the forward E.M.F. of 6 volts which, according to Duddell*, exists at the cathode face must lead to the projection of electrons, is not above question.

Pollock neglected the opposing potential step which this E.M.F. would set up; yet, if the two were equal, the electrons would leave the cathode with no excess momentum. It is true that after leaving the cathode the electrons have to pass through what is presumably the cathode dark space, where they encounter a further step of 11.7 volts†, in which, if they have a free fall, they will acquire a speed of about 2×10^8 or 1.4 times that calculated by Pollock. The following argument shows that this might also give rise to a Duffield effect.

* Duddell, Phil. Trans. vol. 203. A. p. 332.

† Duddell, *loc. cit.*

Take the above figure, and let B now represent a layer of positive electricity giving rise to the cathode dark space. All electrons passing from C to B will be urged forward with a force P, half of which is produced by the charge on B and half by that on C. C therefore feels a direct backward thrust of $\frac{1}{2}P$, and as the attraction of the electrons for B urges B towards C, a hydrostatic pressure of $\frac{1}{2}P$ will also be felt by C. The net effect is that C feels the full recoil of the momentum imparted to the electron with the same result, though not by the same mechanism, as if the electron had been projected from C by forces with C's surface. As before, this recoil will be balanced by a suction if the electrons rapidly lose their momentum on leaving the dark space, with the result that no Duffield pressure will be thereby produced. But it will be unbalanced if they shoot across the arc without drag. Taking Duffield's calculation but substituting the drop of 11.6 volts in the dark space for the 6.1 volts at the cathode face, one obtains for the current carried by these high-speed electrons one-third of the total current instead of a half. The corresponding pressure-slope set up by the compensating positive ions will be 5.9 dynes per cm. instead of 8.9, a quantity which would have been also readily detected in the wind-pressure experiments had it been present.

In the opinion, therefore, of the author the theory of electrons projected across the arc either from the cathode itself or from its immediate neighbourhood is not in accordance with other experimental data.

On the other hand, if the effects of viscously moving ions are not necessarily ruled out in looking for an explanation of the Duffield effect, there seem to be possibilities therein which, though not affording a theory that is above reproach, are at least worth putting on record. The argument is as follows.

In a symmetrical field it will be a condition of symmetry that the drag of the ion on the gas shall be everywhere equal to the force with which the field drives the ion.

But in the act of starting from rest close to one of the electrodes, that drag of an ion on the gas is smaller than the driving force, owing to its lower speed; its suction on the electrode it is leaving is therefore proportionately reduced. It follows that every ion which starts its motion in the gas close to an electrode gives rise to an effect which is equivalent to an unbalanced backward thrust on this electrode, the momentum supplied by this thrust being equal and opposite to that received by the ions.

At the other end of its path each ion imparts this momentum to the electrode in coming to rest, and the latter is therefore also thrust back, the forces on the two electrodes being thus equal but opposite in direction.

Hence, in so far as a horizontal arc may be regarded as symmetrical, we should expect its carbons to be thrust apart with forces which are equal both to one another and to the sum of the momenta imparted per second to the ions starting at the two electrodes; this is on the assumption (justified by the present experiments) that the amount of lateral loss of momentum from the arc to the surrounding air is inappreciable.

The above applies to the case of ions which start from rest in the gas; but the conclusion is the same if they have been projected with high velocity from the electrode and have then slowed down near the latter to the steady velocity of viscous motion; for the recoil felt by the electrode is now in excess of the suction due to the slowing down, and by an amount which is equal to the momentum left in the ions.

It thus seems to be generally true that, provided the ions reach the velocity of viscous motion close to the electrode from which they start, the two electrodes will be thrust apart by forces equal to the momentum put into the ions per second in reaching that velocity.

Now we may obtain some idea of the magnitude of these forces by applying Langevin's original formula for the mobility of an ion. In this the average velocity " v " of ions moving viscously in a field of intensity " X " is given by

$$v = \frac{e}{m} \frac{\lambda}{U} X,$$

where m is the mass of the ion, and λ its mean free path and U its velocity of molecular agitation.

In estimating the mean free path we may follow Wellisch, who found an expression for the influence of the charge on the ion on its mean free path. According to him,

$$1/\lambda = \pi n \sqrt{1 + \frac{M}{m} \sigma^2} \left(1 + \frac{2R_\sigma}{mu^2}\right),$$

where

$$R_\sigma = \frac{K-1}{8\pi n} \frac{e^2}{\sigma^4} :$$

n is the number of molecules per c.c., " u " their velocity of molecular agitation, M and m the mass of molecule and ion respectively, σ the sum of the radii of molecule and ion,

and K the dielectric constant of the gas. The term inside the bracket represents the effect of the charge upon the mean free path.

Wellisch showed that, given that the ion is a single molecule, the formula is in agreement with the experimental values of mobility obtained at ordinary temperatures. In applying it to the case of the arc at a temperature of, let us say, 3500° C., certain approximations are necessary. We may assume the arc to be filled with carbon vapour, for which, however, K and σ are unknown. K may be taken to be proportional to density but otherwise independent of temperature; σ may be corrected for temperature by Sutherland's formula, $\sigma^2 \propto \left(1 + \frac{c}{T}\right)$.

Two sets of calculations are included, the first using the values of K and σ for the diatomic gas oxygen and the other for the monatomic gas helium. The other constants are known for carbon, and are therefore the same in both calculations. In each case the values of λ are worked out (1) assuming the ion to be a carbon atom, (2) assuming it to be an electron.

The table shows the corresponding values for λ .

	$(K-1) \times 10^5$.	m .	$\sigma \times 10^8$.	$\lambda \times 10^5$.
(1)	59	carbon atom.	3.1	9.6
	7.4	do.	1.86	26
(2)	59	electron.	1.5	9.2
	7.4	do.	0.93	35

Now, the number of grammes leaving one of the electrodes per second is $\frac{im}{e}$, where " i " is that part of the current carried by the ions starting at that electrode. Hence the force on that electrode due to them is $P = \frac{im}{e} \cdot v = i \frac{\lambda}{U} X$.

If the carriers are electrons, the fact that there is no appreciable wind from the body of the arc, *i. e.* that the space-charges of positive and negative electricity in it are equal, requires that the current carried by positive ions shall be to that carried by negative ions in the proportion of their respective mobilities. In other words, the negative ions will carry practically the whole current in the arc. On the other hand, if the carriers are molecules the mobilities will be roughly the same for each sign. In either case we

exist (*a*) if the negative ions in the arc are charged atoms moving with a mobility given by the recognized mobility formula of Wellisch, or (*b*) if the negative ions are electrons moving with a mobility of 1.5×10^6 cm./sec /volt/cm., a value far in excess of that to be expected from measurements at lower temperatures.

That there are objections to both theories there is no doubt ; but the matter can only be definitely settled when a method is devised for measuring the mobility of the negative ions while they remain in the arc itself.

CIV. On a Theory of the Striated Discharge.

*By Sir J. J. THOMSON, O.M., F.R.S.**

OF the many and varied phenomena connected with the Discharge of Electricity through Gases, few are more striking than the regular and rhythmic alternations in luminosity in the positive column called striations. These have been the subject of many investigations, and some idea of their beauty and variety can be got from the excellent plates given by De la Rue and Müller in a series of papers in the 'Philosophical Transactions' beginning 1878, Pt. 1. The alternations in luminosity along the discharge are accompanied by alternations in the electric force, and an explanation of the alternations in the force would go far in explaining the alternations in luminosity.

I gave in my 'Discharge of Electricity through Gases' reasons why there should be alternations in the electric force. The object of this paper is to endeavour to reduce these general considerations to a definite mathematical form and to see whether they lead to results which are in accordance with experience.

The differential equations which represent the variations of the electric force along the discharge are not linear, and are so intractable that solutions have only been obtained for special cases. This is so even when there is only one kind of positive and one kind of negative ion and when the pressure of the gas is so high that the velocities of the ions at any point may be taken as proportional to the electric force at that point. The conditions are even more complicated when the pressure of the gas is as low as that in the striated discharge, when the mean free path of an electron is several millimetres, and when its velocity at any point will depend not merely upon the electric force at that

* Communicated by the Author.