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THE CONVECTION AND CONDUCTION OF HEAT IN GASES

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In a previous* paper the author has shown that the "convection" of heat from hot wires in a gas consists essentially of *conduction* through a film of relatively stationary gas around the wire. From this theory the following method was derived for calculating the power necessary to maintain a wire at any given temperature.

The loss of energy from a wire is made up of two parts, radiation and convection. Let us call this convection W , expressed in watts per cm. of length of the wire. Then W is equal to the product of two factors, thus

$$W = s (\varphi_2 - \varphi_1) \quad (1)$$

The first factor, s , is called the "shape factor" and depends only on the ratio of the diameter of the wire, a , to the diameter, b , of the conducting film around the wire. This relation is

$$s = \frac{2 \pi}{\ln \frac{b}{a}} \quad (2)$$

But s can be calculated directly without a knowledge of the film diameter b by solving the following equation

$$\frac{a}{B} = \frac{s}{\pi} e^{-\frac{2 \pi}{s}} \quad (3)$$

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Here a is the diameter of the wire and B is the thickness of the conducting film for the case of convection from a plane surface.

The second factor in (1) is $\varphi_2 - \varphi_1$ where φ depends only on the heat conductivity of the air, k , (in watts per cm.) and the temperature of the wire and of the atmosphere. Thus

$$\varphi_1 = \int_0^{T_1} k \, dT \quad \varphi_2 = \int_0^{T_2} k \, dT \quad (4)$$

Here T_1 is the temperature of the atmosphere and T_2 the temperature of the wire.

Both of the operations involved in (3) and (4) are made very easy by plotting two curves, one giving the relation of s to

$\frac{a}{B}$ and the other relation of φ to T . Data for the plotting of these curves is given in the paper referred to above.

Thus the only data necessary for a calculation of the free convection from a horizontal wire (of given diameter and temperature) in a gas of known temperature is

1. The heat conductivity of the gas as a function of the temperature.
2. The value of B .

It was shown experimentally that for air at room temperature and atmospheric pressure, B is equal to 0.43 cm. and is independent of the temperature, T_2 , of the wire, even when this varies from slightly above room temperature up to the melting-point of platinum.

It was shown that with five different sizes of wire ranging from $a = 0.0038$ cm. up to $a = 0.0500$ cm., the energy W calculated from (1) and (3) agreed excellently with experiments. This is equivalent to saying that the experimentally determined values of B were found to be independent of the diameter of the wire used in the experiment.

The theory would indicate that B should vary with

1. The nature of the gas.
2. The pressure of the gas.
3. The temperature of the gas.
4. In the case of forced convection, B should vary with the wind velocity.

The present paper will deal with the effect of the second and

fourth of the above factors on the thickness of the conducting film.

Kennelly in his excellent paper on the "Convection of Heat from Small Copper Wires"* has investigated the effect of the following factors

1. Diameter of wire. Varied from 0.011 to 0.069 cm.
2. Temperature of wire. With the free convection tests this varied from 40 deg. cent. to 200 deg. cent., but with the forced convection temperatures as high as 325 deg. cent. were used.
3. Pressure of air. Varied from 12 up to 190 cm. of mercury.
4. Wind velocity. Varied from 0 up to 2000 cm. per second.

He did not investigate other gases than air, nor try air at other than room temperature.

Kennelly derives certain purely empirical formulas to express his results. Combining them, he gives for W , the watts lost by free convection per cm. of length,

$$W = (0.0004 + 0.0064a) (T_2 - T_1) P^{0.9} \sqrt[3]{a} \quad (5)$$

where a is the diameter of the wire

P is the pressure of air in atmospheres.

For forced convection he finds

$$W = (0.00003 + 0.00580a) (T_2 - T_1) \sqrt{v + 25} \quad (6)$$

where v is the wind velocity in cm. per sec.

Thus to express his results on free convection he needs to use four empirical constants and to express the results on forced convection three more are needed. But even with all these empirical constants, the calculated values do not agree very well with the experiments. The average difference between the calculated and observed values of W for experiments on free convection is approximately 12 per cent, while in one experiment the differences are as high as 30 per cent.

Let us now analyze Kennelly's data more closely and apply to it the theory of the conducting film, thus deriving the relation between the film thickness B and the pressure and wind velocity of the air.

KENNELLY'S DATA ON FREE CONVECTION

The experimental results on free convection are given by Kennelly in ten logarithmic plots, the coordinates being

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pressures and current through the wire. He does not give the resistances of the wires at the temperature of the experiment, but does give the resistance of each of the wires at 0 deg. cent. and gives the elevation of temperature above room temperature, calculated from the resistance. He used for this calculation the formula

$$R_T = R_0 (1 + 0.0042 T_c)$$

where R_T is the resistance at the temperature T_c (in deg. cent.) and R_0 is the resistance at 0 deg. cent. He did not measure the temperature coefficient of the wire used, but assumed that the wires were of pure copper and that the temperature coefficient was that given above. He does not state the room temperature during the experiments, so I have assumed it to be 20 deg. cent., and have thus calculated the resistance of the wires from his data on the temperature elevation above room temperature and from the given temperature coefficient. By taking the product of the resistance per cm. and the square of the current, I have then calculated **W** the watts per cm. supplied to the wire. This is given in the following tables as "**W** obs."

Kennelly is very uncertain as to how large the correction for radiation should be, and finally assumes that the copper wires radiate 94 per cent as much as a black body at the same temperature. This correction amounts to about 2 per cent for the smallest wire and becomes about 8 per cent of the total energy for the largest wire. But there is ample data in the literature to show that radiation from a bright metallic surface is very much less than 94 per cent of that from a black body. A great deal of recent work has shown conclusively that the reflectivity, r , of any metal for heat rays of wave length longer than $\lambda = 6 \mu$ is accurately given by Hagen and Ruben's formula*

$$1 - r = 0.365 \sqrt{\frac{\sigma}{\lambda}}$$

where σ = specific electrical resistance of the metal

λ = wave length of the light.

By Wien's displacement law the wave length λ_m of the light of greatest intensity in the spectrum from a black body at temperature T is

$$\lambda_m = \frac{0.29}{T} \text{ cm.}$$

**Ann Phys.* 8. 1, 1902.

Therefore for radiation of heat at room temperature, about 300 deg. K.* the wave length would be about 0.0009 cm. or 9μ

Now for copper $\sigma = 1.7 \times 10^{-6}$

and for platinum $\sigma = 11.0 \times 10^{-6}$

Whence approximately

for copper $1 - r = 0.0158$

and for platinum $1 - r = 0.040$.

That is, at room temperature copper would radiate 1.6 per cent and platinum about 4.0 per cent, as much as a black body. This result for platinum is in fairly good agreement with Lummer and Kurlbaum's† measurements on the radiation from platinum. In any case, it is a very safe conclusion that the radiation from bright copper surfaces is much less than 50 per cent and therefore that it would have been better if Kennelly had neglected radiation, instead of correcting for it.

Therefore, in using Kennelly's data, we shall use the total watts as observed, rather than the values he gives for convection. At the relatively low temperatures at which his experiments were carried on, radiation probably does not exceed 1 per cent of the total losses.

In the above table the first column gives the diameter of the wire in cm. The second column gives the temperature elevation above room temperature given by Kennelly and calculated by him from the resistance. In the second column there is also given $\varphi_2 - \varphi_1$, taken from a very large scale curve prepared from the data given in the previous paper. T_1 is assumed to be 293 deg. K. The third column gives the resistance per cm. of length of the wire, calculated from the resistance at 0 deg. cent. given by Kennelly and from the temperature coefficient 0.0042 used by him in calculating $T_2 - T_1$. Hence any error in assuming a wrong value for this temperature coefficient is eliminated.

In the fourth and fifth columns is the data obtained directly from the plots given by Kennelly. Smooth curves were drawn through the points given by Kennelly without any reference to the straight lines that he uses to express his results. The ordinates of these curves at three or four well-distributed pressures were read off and are given in column V. The pressures are given in megabars (*i.e.*, 0.987 atmospheric pressure). In column VI is given the watts per cm. obtained simply by multiplying the figures in columns III by the square of the

*Degrees Kelvin (Absolute temperature).

†*Verh. Phys. Ges.*, Berlin, 17, 106, 1898.

TABLE I
KENNELLY'S EXPERIMENTS ON FREE CONVECTION

I	II	III	IV	V	VI	VII	VIII	IX	X
Diam. of Wire cm.	$T_2 - T_1$ and $\phi_1 - \phi_2$	Resist- ance Ohms cm.	Pres- sure P mega- bars	Amps.	W obs.	W calc.	B obs. cm.	B_1 calc. cm.	n
0.0114	165.0	0.0273	0.325	1.72	0.0810	0.0811	1.04	0.46	-0.73
			1.00	1.89	0.0978	0.0980	0.44		
			1.70	1.98	0.1072	0.1017	0.31		
			2.25	2.02	0.1114	0.1142	0.27		
0.0114	95.9	0.0228	0.35	1.47	0.0494	0.0439	0.54	0.49	-0.94?
			1.00	1.50	0.0513	0.0524	0.49		
			1.50	1.56	0.0556	0.0569	0.35		
			2.25	1.65	0.0621	0.0612	0.23		
0.0114	59.6	0.0205	0.33	1.13	0.0261	0.0260	0.95	0.46	-0.80
			1.00	1.21	0.0300	0.0312	0.50		
			1.50	1.27	0.0330	0.0337	0.35		
			2.25	1.36	0.0379	0.0364	0.21		
0.0262	58.8	0.00392	0.40	2.80	0.0308	0.0320	0.99	0.45	-0.84
			1.00	3.11	0.0380	0.0381	0.44		
			2.00	3.41	0.0455	0.0446	0.24		
0.0262	46.4	0.00377	0.30	2.34	0.0206	0.0235	1.92	0.59	-0.95
			0.60	2.55	0.0245	0.0268	0.91		
			1.00	2.70	0.0275	0.0296	0.56		
			2.00	2.95	0.0328	0.0346	0.31		
0.0262	37.8	0.00367	0.70	2.18	0.0175	0.0220	1.47	0.96?	-1.28?
			1.25	2.38	0.0208	0.0249	0.73		
			2.00	2.59	0.0246	0.0277	0.39		
0.0262	15.1	0.00326	0.30	1.53	0.0076	0.0074	0.92	0.35	-0.86
			0.60	1.65	0.0089	0.0084	0.51		
			1.00	1.73	0.0098	0.0093	0.36		
			2.00	1.95	0.0124	0.0109	0.17		
0.0691	179.8	0.000770	0.33	14.3	0.1575	0.139	0.68	0.32	-0.77
			1.00	16.5	0.2100	0.181	0.34		
			1.50	17.5	0.236	0.204	0.22		
			2.25	18.4	0.261	0.227	0.17		
0.0691	77.2	0.000591	0.37	10.0	0.0591	0.0545	0.71	0.34	-0.75
			1.00	11.4	0.0770	0.0693	0.33		
			1.50	12.2	0.0880	0.0777	0.23		
			2.25	12.9	0.0984	0.0868	0.17		
0.0691	18.3	0.000489	0.50	5.3	0.0138	0.0128	0.57	0.38	-0.66
			1.00	5.7	0.0159	0.0152	0.38		
			2.00	6.25	0.0192	0.0185	0.23		

figures in column V. For the reasons already given, no correction for radiation was made.

In column VIII is given the value B calculated as follows:

The value of s is found (equation 1) by dividing W (column VI) by $(\varphi_2 - \varphi_1)$ (column II). From a curve of equation (3)

drawn from data in the previous paper, the value of $\frac{a}{B}$ corresponding to the given value of s was found. This quantity divided into, a , the diameter of the wire (column I) gives B (column VIII).

It was thought that B would be found to vary inversely proportional to the pressure. In the previous paper the hypothesis was advanced that B would vary proportionally to the viscosity and inversely proportional to the density of the gas. This, however, was not very well borne out by experiments in hydrogen and mercury vapor. According to this hypothesis, B should vary inversely with the first power of the pressure, for the viscosity is independent of the pressure.

It will be observed, however, that the product obtained by multiplying together the values in columns IV and VIII shows a very distinct tendency to increase with the pressure. The quantity B was therefore plotted on logarithmic paper, as a function of P , with the result that the points were found in nearly every case to lie in straight lines. For each of the experiments with a wire at any given temperature, a straight line was drawn in this way. The ordinate of these lines for a pressure of 1 megabar was read off and is given in the above table, in column IX. The slope of the straight line, n , is given in column X. The fact that the logarithmic plots gave practically straight lines means that B varies with the n th power of the pressure. The values of n in the different experiments do not show any distinct tendency to vary either with a or with the temperature. By a careful study of the curves, the most probable value of n was thought to be about - 0.75. It is true that the average of the values of n is numerically larger than 0.75, but the sets of experiments which seem to be most free from experimental error give values close to this.

To test the accuracy of this conclusion, the figures in column VII were calculated, based on the following assumptions

1. The thickness of the plane film, B , for air at room temperature and 760 mm. pressure, is 0.43 cm. This is the value found from the experiments on platinum wires described in the previous paper.

2. That B varies inversely as the 0.75th power of the pressure (P). At a pressure of 1 megabar B would be 0.436 cm.

By aid of the above assumptions, the values of B were calculated, and from these the ratios $\frac{a}{B}$; then s was found by the plot of (3), and this was multiplied by $(\varphi_2 - \varphi_1)$ (column II), to obtain " W calc." in column VII.

The agreement between the calculated convection and that observed by Kennelly is strikingly good. The only serious discrepancy between the two occurs with the largest wire at the highest temperature. This discrepancy may perhaps largely be accounted for by radiation. A "black" wire of 0.069 cm. diameter at a temperature of 179 deg. above room temperature would radiate about 0.050 watts per cm. A polished copper wire would radiate only about 0.001 watt, but if the surface is slightly oxidized or tarnished, it might easily radiate much more. The difference between the calculated and observed watts is only about 0.025, so that if the wire should radiate 50 per cent as much as a black body this difference would be accounted for.

It is interesting to note that with only two empirically determined constants, the equation (1) allows a much closer calculation of W than did Kennelly's equation with its four empirical constants.

KENNELLY'S DATA ON FORCED CONVECTION

By forced convection Kennelly means the convection of heat from a wire which is moving rapidly relatively to the surrounding air.

Kennelly's results on forced convection are given in two logarithmic plots. In one the amperes are plotted against the wind velocity and in the other the watts per cm. of length are plotted against wind velocity. He has corrected the watts as before for radiation, but in this case the loss by convection is so great that the radiation correction is small enough to be quite negligible. Therefore I have taken Kennelly's results for the watts directly and have not calculated them from the amperes, as in the case of free convection. The following table gives a summary of Kennelly's experimental data

The third and fourth columns were obtained, as before, by drawing smooth curves as nearly as possible through the points given by Kennelly, without any reference to the straight lines

TABLE II
KENNELLY'S EXPERIMENTS ON FORCED CONVECTION

I	II	III	IV	V	VI	VII	VIII
Diameter of wire cm.	$T_2 - T_1$ and $\phi_2 - \phi_1$	Wind velocity cm. sec.	W obs. watts. cm.	W calc. watts. cm.	B $\times 10^3$ cm.	B_{900} obs. $\times 10^3$	B_{900} calc. $\times 10^3$
0.0101	106	520	0.220	0.222	10.0	6.55	6.56
		900	0.280	0.281	6.55		
	0.0298	1800	0.388	0.390	3.90		
0.0101	179	400	0.320	0.350	16.0	8.1	7.29
		900	0.466	0.485	7.8		
	0.0550	1800	0.650	0.671	4.65		
0.0101	252	400	0.486	0.502	15.9	8.4	8.02
		900	0.690	0.693	8.13		
	0.0830	1800	0.960	0.950	4.75		
0.0159	117	330	0.250	0.262	15.5	7.1	6.68
		900	0.408	0.415	6.89		
	0.0335	1800	0.590	0.596	4.05		
0.0159	211	330	0.490	0.487	16.2	7.6	7.60
		900	0.760	0.757	7.68		
	0.0670	1800	1.100	1.079	4.45		
0.0159	305	330	0.720	0.735	18.5	8.5	8.55
		900	1.130	1.122	8.46		
	0.1060	1800	1.600	1.590	4.99		
0.0204	51	800	0.217	0.200	5.81	5.9	6.00
		1300	0.252	0.260	4.75		
	0.0134	1800	0.314	0.315	3.60		
0.0204	128	210	0.297	0.274	17.6	6.3	6.78
		400	0.380	0.362	11.45		
		900	0.570	0.536	6.25		
	0.0370	1800	0.810	0.788	3.89		
0.0204	206	210	0.475	0.453	20.8	7.2	7.56
		400	0.620	0.596	13.0		
		900	0.910	0.896	7.16		
	0.0650	1800	1.280	1.280	4.48		
0.0204	283	210	0.660	0.642	23.4	7.9	8.33
		400	0.880	0.835	14.0		
		900	1.260	1.215	7.96		
	0.0965	1800	1.780	1.758	4.87		

plotted by him. The figures in columns III and IV of the table represent simply three or four well distributed points taken from these curves.

The thickness, B , of the plane film (column VI) was calculated similarly to B in Table I.

It is seen that B decreases as the wind velocity increases. To study the law with which this varies, B was plotted against V on logarithmic paper and a series of parallel straight lines was obtained. The slope of these lines was -0.75 . In other words, B varies inversely as the 0.75 power of the wind velocity.

The ordinates of these straight lines corresponding to the abscissa $V = 900$ are given in column VII.

It is seen that B_{900} increases distinctly as the temperature difference $T_2 - T_1$ increases. But it apparently does not perceptibly depend on the diameter of the wire. By plotting B_{900} against $T_2 - T_1$ it was found that the following equation gives a fairly good approximation of B_{900}

$$B_{900} = 0.0055 + 0.000010 (T_2 - T_1) \quad (7)$$

Column VIII gives the values of B_{900} calculated from this formula.

In the case of free convection, it will be remembered that B was found to be independent of the temperature T_2 of the wire, even up to the melting-point of platinum. The reason that the temperature enters here is probably that with forced convection the viscosity of the inner and hotter portions of the gas film is a factor determining the thickness of the film, whereas in the case of free convection only the viscosity of the outer portions is of importance. Between the temperatures 300 deg. and 600 deg. K, the temperature coefficient of the viscosity, η , is 0.00219, whereas the temperature coefficient of B_{900} is $0.00001 \div 0.0055 = 0.00180$, or about 83 per cent of that of the viscosity. In other words, for forced convection B is approximately proportional to the viscosity of the gas at a point $\frac{1}{6}$ of the way from the surface of the wire to the outer edge of the film.

The values of W given in column V were calculated by first determining B for the wind velocity given in column III by means of the relation

$$\frac{B}{B_{900}} = \left(\frac{900}{V} \right)^{0.75} \quad (8)$$

From this value of B , W was calculated in the usual way.

If we assume that equation (8) holds down to very low velocities and substitute for B the value 0.43 cm. found for free convection and for B_{900} the value from (7), we find for the velocity V_0 which exists in free convection

$$V_0 = 2.7 [1 + 0.0024 (T_2 - T_1)] \text{ cm. per sec.}$$

Kennelly had concluded from his observations that the energy loss from the wires varied directly as the square root of the wind velocity. According to the present theory this could only occur for wires of a certain size.

In case the diameter of the wire is large compared to the film thickness B , we should expect the energy to be inversely proportional to B ; that is, directly proportional to the 0.75 power of the wind velocity. For wires of smaller size the energy would vary less rapidly with the wind velocity, so that for wires of a certain size it would vary approximately with the square root of the wind velocity.

Kennelly's experiments cover only such a narrow range of sizes of wire that his data furnish no way of testing this deduction from the theory. But it should be pointed out that many other observers have concluded in experiments on the rate of solutions of solids in liquids and other similar phenomena, that the thickness of the diffusion film varies inversely as the 0.70 or 0.75 power of the rate of stirring. As far as I know however, the case where the size of the wire is small compared to thickness of the film has not been handled.

SUMMARY

Kennelly's data on the "Convection of Heat from Small Copper Wires" afford strong proof of the reliability and usefulness of the author's theory of convection. According to this theory, "convection" consists essentially in conduction of heat through a film of gas of definite thickness, in which the heat carried by motion of the gas is negligible compared to that carried by conduction, and outside of which the temperature is maintained uniform because of convection currents. The thickness of the film of gas is related in a simple way to the diameter of the wire, so that from the experiments the thickness B , which the film would have in case of a plane surface, can be readily calculated.

Previous results of the author have shown that

1. The quantity B , for quiet air at room temperature and one atmosphere pressure, is equal to 0.43 cm.

2. B is independent of the temperature of the wire from room temperature up to the melting-point of platinum, 1750 deg. cent.

3. The values of B obtained from experiments on wires of different sizes are found to be the same.

In the present paper it is shown that Kennelly's results confirm the above conclusions and furthermore lead to the following new conclusions:

4. The film thickness (for plane surface) B varies inversely as the 0.75th power of the pressure of the gas.

5. The value of B varies inversely as the 0.75th power of the wind velocity.

6. Although for free convection, B was found independent of the temperature of the wire, it is found that for forced convection B increases slightly with the temperature. See equation (7).

7. For forced convection, however, the value of B is found independent of the diameter of the wire, just as in the case of free convection.

8. Radiation from small metallic wires is practically negligible compared to convection, up to temperatures of several hundred degrees.

In a series of subsequent papers it will be shown that

1. The value of B increases approximately proportional to the absolute temperature of the atmosphere surrounding the wire, even when the latter varies from -190 deg. to 300 deg. cent.

2. The value of B found from actual experiments on the convection from plane surfaces agrees excellently with the value found from small wires. In the case of plane surfaces, however, the radiation loss is usually much greater than that by convection, so that a careful study of the radiating properties of the surface has to be made. It will be shown that the convection losses from cylinders of any size, as well as from plane surfaces, can be accurately calculated from the formulas given in this paper.

3. In the case of convection between two surfaces (for example, between two concentric cylinders or two boxes one in the other), the theory of the conducting film proves extremely useful and makes it possible to calculate nearly all simple cases of heat convection with reasonable accuracy.

These papers will also contain evidence of many kinds which clearly indicate the significance of the conducting film.
