The Achievements of Great Britain in the Realm of Mathematics
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which is the same thing, by the sides of the triangle $VBC$, which is similar to the triangle $CET$.

"It comes to the same purpose to take the fluxions in the *ultimate ratio* of the evanescent parts. Draw the right line $Cc$, and produce it to $K$. Let the ordinate $bc$ return into its former place $BC$, and when the points $C$ and $c$ coalesce, the right line $CK$ will coincide with the tangent $CH$, and the evanescent triangle $CEc$ in its ultimate form will become similar to the triangle $CET$, and its evanescent sides $CE$, $Ec$ and $Cc$ will be *ultimately* among themselves as the sides $CE$, $ET$ and $CT$ of the other triangle $CET$ are, and therefore the fluxions of the lines $AB$, $BC$ and $AC$ are in this same ratio. If the points $C$ and $c$ are distant from one another by any small distance, the right line $CK$ will likewise be distant from the tangent $CH$ by a small distance. That the right line $CK$ may coincide with the tangent $CH$, and the ultimate ratios of the lines $CE$, $Ec$ and $Cc$ may be found, the points $C$ and $c$ ought to coalesce and exactly coincide. The very smallest errors in mathematical matters are not to be neglected."

The weakness of this plausible statement is clear. Berkeley comments, "It is particularly remarked and insisted upon by the great author, that the points $C$ and $c$ must not be distant one from another, by any the least interval whatsoever: but that in order to find the ultimate proportions of the lines $CE$, $Ec$ and $Cc$ (*i.e.* the proportions of the fluxions or velocities), expressed by the finite sides of the triangle $VBC$, the points $C$ and $c$ must be accurately coincident, *i.e.* one and the same. A point therefore is considered as a triangle, or a triangle is supposed to be formed in a point. Which to conceive seems quite impossible."

Although Berkeley considered that he had demolished the foundations of the Science of Fluxions, he did not altogether deny the utility of the method. "But then it must be remembered that in such case, although you may pass for an artist, computist, or analyst, yet you may not be justly esteemed a man of science and demonstration." And again: "In answer to this you will perhaps say, that the conclusions are accurately true, and that therefore the principles and methods from whence they are derived must be so too. . . . I say that in every other science men prove their conclusions by their principles, and not their principles by their conclusions. But if in yours you should allow yourselves this unnatural way of proceeding, the consequence would be that you must take up with Induction, and bid adieu to Demonstration."

The reader must be left to judge of the force of Berkeley's criticisms. To the compiler of this brief account they seem conclusive against any attempt to base the Differential Calculus on the idea of motion. Such attempts, however natural and useful, seem inevitably to lead to a logical quagmire, which the builder of an exact science must contemplate with dismay. Those who share this view will not deny to Berkeley, despite his lack of constructive ideas, some part of the praise which mathematicians of a later day have bestowed on the researches of Weierstrass. W. D. EVANS.

# THE ACHIEVEMENTS OF GREAT BRITAIN IN THE REALM OF MATHEMATICS.*

## I.

THE inhabitants of the three kingdoms which constitute Great Britain did not come into contact with the civilisation of the ancient East. Nor, as far as we know, did they have the advantage of direct relations

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with the Greeks—that race, beloved of Nature, which succeeded in placing all the sciences and all the fundamental arts on so secure a basis as to defy for two thousand years any attempt at radical innovation.

The Celtic invasion, which took place about 1000 B.C., entrusted to the Druids, the celebrated priests of the dominant religion, the task of preserving, diffusing, and extending the boundaries of knowledge, but of the substance, nature, and tendencies of the science of that period we are totally ignorant. Julius Caesar has left us the oldest description of the inhabitants that we possess, but he makes no reference to the intellectual life of the ancient Britons. A second Roman invasion about a century later, followed by a struggle extending over some fifty years, secured to Rome this remote addition to her Empire. Her occupation lasted for three centuries, during which the Britons enjoyed as some compensation for the loss of their independence a period of repose and of relative civilisation. In 410 A.D. the island was released from her allegiance. The departure of the Roman eagles exposed Britain to the cupidity of the pirates of North Germany, who made numerous more or less successful descents upon its shores between A.D. 350 and 450, and after a struggle which lasted two hundred years, completed the conquest of the country. The conquerors were unable to impose upon the native population their religion, their laws, or even their language. Under such circumstances it could not reasonably be expected that the Britons would be drawn by their unwelcome guests from the state of barbarism in which they were plunged.

A few gleams of light were thrown upon the scene by the visit of the band of missionaries sent in 597 by Pope Gregory the Great. They were the first to bring books, both sacred and profane, across the Channel. A few years later, the eloquence of St. Augustine, Archbishop of Canterbury, proved so effective that in the course of a century the last traces of Paganism had vanished from the land.

In the monasteries men found that repose and atmosphere which are congenial to the study of literature, and to quiet thought over the scientific problems that occupied the minds of that age. For a measure of the arithmetical skill of the time we look to the works of the Venerable Bede (672 or 673–735). The monk of Jarrow taught his 600 scholars in a monastery situated on the borders of England and what then was Scotland. His *De Computo vel Loquela Digitorum* and *De Ratione Unciarum* attest the intense desire of the "Father of English Learning" to communicate to every class of society all that the world had then to teach of the art of computation. England had begun her contribution to the literature of Mathematics.

## II.

The example of Bede was followed by Alcuin (735–804). In his works, which in the main are historical and theological, the mathematician discovers with pleasurable anticipations a collection of problems in Arithmetic. The title is sufficient to indicate their immediate aim: *Propositiones ad acuendos Juvenes.*

But for many a long year no addition was made to this scanty body of English mathematical literature. It may be that the whole of the national energies were absorbed in the incessant struggles consequent on the inroads of the Danish and Norwegian pirates. Their incursions ceased only when the Norsemen sang their songs of victory over the native population. It was in 1066 that began the real history of the English race as the people of Great Britain. This far-reaching political revolution did not wrest from the hands of the clergy the monopoly of learning and the control of public instruction, nor could it deprive the Latin tongue of its enviable prerogative of acting as the normal vehicle for the transmission of thought. On the other hand, the characteristic phrase, "An unlearned King is a crowned Ass"—attributed to Henry I., one of the early Norman kings—goes far to prove that culture, under the new régime, could look for aid and encouragement instead of oppression and contempt. In fact, it was not long after that the University of Cambridge came into existence, and schools and colleges were founded both there and at Oxford.

## III.

The intellectual relations established in the age of the Crusades between the southern countries of Europe and ancient Greece, through Arabic channels, found an echo even in so remote a corner of the continent as that inhabited by the Anglo-Saxons. An English monk, Athelhard of Bath, translated for his fellow-countrymen the *Elements of Euclid* from the Arabic into Latin. Consider how fortunate it was that Great Britain should possess a work of such didactic value as "The Elements"! Think of the millions in Europe who never had this as their text-book, and draw up an approximate list of the innumerable forms of scientific research and criticism which might have been undertaken from such a starting-point. We are forced to rank Athelhard's translation as an event of the first order, not unworthy to be compared in the history of Mathematics with the appearance in 1202 of the *Liber Abaci* of Leonardo of Pisa (Fibonacci). It would be of the highest interest to know what reception was accorded by English minds to the messenger who brought from across the ages the greatest achievement of Greek thought, and to ascertain the precise period at which its beneficent influence is perceptible. But alas!

These are questions none can answer;
These are the problems none can solve.

That gifted genius Roger Bacon (1214-1294), "doctor mirabilis," who made so many important contributions to the progress of the various branches of Physics, left nothing (if we confine ourselves to his published works) which serves to throw any light on his statement that he held the exact sciences in such esteem that he did not hesitate to assert that Mathematics should be regarded as the Alphabet of all Philosophy. The same may be said with even greater emphasis of his colleague, John Peckham (1240–1292), and of the justly celebrated John Holywood—Johannes de Sacro Bosco—(?–1244 or 1256). The latter was one of the most eminent personalities in England in this century, and it was his constant care to extend throughout his native land such beneficent influence as he could reasonably be expected to exert.

## IV.

Thomas Bradwardine (1290–1349), Archbishop of Canterbury, "the Profound Doctor," deserves a place in the history of Geometry for his works on perspective and on stellate polygons From his didactic works we can form a fair estimate of the quantity and quality of the Mathematics taught at the Anglo-Saxon Universities of the fourteenth century. If, as is possible, there are some of his writings lying buried in undeserved oblivion in some of the libraries of Great Britain, it is of the highest importance that they should be exhumed, examined by competent investigators, and published. The same remarks apply to the two Oxford Professors, Richard of Wallingford (about 1326) and

John Maudith (about 1340), whose works on Trigonometry are imperfectly known and deserve to be brought to the full light of day.

Such a search might very possibly bring to light other scientific writings now long forgotten, and thus fill the gap which we find at this stage in the history of English Mathematics. For while the two professors just mentioned belong to the first half of the fourteenth century, it is not till the last quarter of the fifteenth century that we find mention of a new mathematical personality. We allude to Cuthbert Tonstall (1474-1559), who, after pursuing the regular course at Cambridge and Oxford, proceeded to Padua. There he won a high and well-deserved reputation, distinguishing himself more particularly in Jurisprudence. But during his stay in Italy he was also deeply interested in mathematical investigations, and made a careful study of the works of Luca Paciuolo. So great, indeed, was his enthusiasm for our science, that before finally dedicating his life to political and religious affairs, to which he had decided to devote the whole of his energies, he bade a public farewell to the subject. This parting benediction took the form of a treatise mainly intended to set forth the principles of Algebra, then a new science, in which great strides had been made in "beautiful Italy" during the historic interval between Leonardo Fibonacci and Luca Paciuolo. Such was the origin of his *De Arte Supputandi Libri Quattuor* (1522), a work which was favourably received not only in his own country but by the whole of learned Europe.

## V.

Tonstall may have been the first, but he certainly was not the only link in the scientific field between Italy and England. In fact, in the glorious Galilean epoch the intellectual relations between the two countries were both continuous and intimate. Antonio Favaro, who is thoroughly acquainted with everything that concerns the life, work, and school of Florentine Physics, gives * the names of Segeth, Southwell, Wedderburn, White, and Willoughby as those of men who spread far and wide the ideas and works of the greatest master that scientific Europe had yet produced, and made them known to their own countrymen. Another eminent geometer of that time, James Gregory (1638–1675) made several lengthy stays in Italy, as we know from letters of his which are still extant.† Two of his works were published abroad, one at Paris and the other at Venice.

Now, if we bear in mind that the methods of the Infinitesimal Calculus, in the elaboration of which Bonaventura Cavalieri and Evangelista Torricelli played so conspicuous a part, were conceived in that brilliant circle of which Galileo was the central figure, and if we remember that England was shortly to be hailed unanimously as the creator of the Calculus of Fluxions, we see at once arising the most interesting problem in the history of our science—the extent to which the genius of Newton was influenced by the work of Galileo. Such a piece of research undertaken without reference to nationality, and with the sole desire of reaching the truth, would certainly lead to most important conclusions, inasmuch as they must inevitably throw some ray of light upon the mysterious manner in which great and novel ideas reach their complete maturity.

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[*] A. Favaro mentions the Englishmen of the School of Galileo in several chapters of his series, "Amici e correspondenti di Galilei" (published in the *Atti del R. Instituto Veneto di Scienze, Lettere ed Arti*), and in many places in his "Scampoli galileiani" (in the *Memorie della R. Accademia di Padova*).

[†] Cf. L. J. Rigaud, *Correspondence of Scientific Men of the 17th Century* (Oxford, 1841).

## VI.

It would seem that the laudable efforts of Tonstall to avail himself of his personal experiences to diffuse a knowledge of the new methods and results of continental mathematicians were crowned with but moderate success. Not long after Robert Record (1510–1558), a court physician, could write of the English that in general intelligence they were superior to most, but that they were disgracefully ignorant of what others had done in the field of science. Determined to end this deplorable state of things as far as in him lay, he published an excellent treatise covering the whole ground of Arithmetic, to which he gave the significant title: *The Grounde of Artes.* It is difficult to say too much in its praise. It has a lasting place in mathematical literature as the first printed volume in which we find the sign of equality (=) which is still in use. So flattering was the reception of this book that the author later produced two others, dealing respectively with Algebra and Geometry. But a much greater influence upon his contemporaries was exercised by a famous mathematician of the reign of Queen Elizabeth—William Oughtred (1574–1660), a contemporary of Bacon and of Shakespeare, to whom we owe the *Clavis Mathematica* (in which are used for the first time the symbols × and :: for multiplication and proportion respectively). He also wrote an excellent treatise on Arithmetic, the long and well-deserved reputation of which is attested by the many editions and translations with which it was honoured, as well as by the fact that the author was encouraged to bring out an additional volume on the subject of Trigonometry.

With Record and Oughtred we approach the great stages of the epoch during which English mathematics passed from the period of infancy to that of fertile virility, in which the humble position of the scholar was changed for that of the master. The transition is marked by two names—John Napier (1550–1617) and Thomas Harriot (1560–1621).

The former travelled much in France, Germany, and Italy. His name is indissolubly connected with the logarithmic calculus which he, with the aid of his indefatigable compatriot Henry Briggs (1556–1631), placed at the disposal of the calculators of the world. Not content with the unexpected assistance he thus gave to those whose pursuits necessitated the carrying out of long and laborious computations, he applied his genius to what he called "local Arithmetic" (depending in principle on the expression of numbers in the binary scale), and taught in his *Rabdologia* an ingenious process of instrumental calculation.* And finally, it is well known to all that Spherical Trigonometry is indebted to Napier for certain rules and formulae which are still in daily use.

The place in the science to be accorded to Harriot has been a matter of some debate. Certain writers, basing their conclusions on his *Artis Analyticae Praxis* (1631), consider him as a rival of Vieta, while others regard him as a disciple, if not a plagiarist, of the great French mathematician. The question cannot be said to be settled one way or the other. The discussion was not entirely free from national prejudices, nor perhaps can a final answer be given until Harriot's manuscripts, which are religiously preserved in the Library of the British Museum, have been published, or, at any rate, closely examined. An old friend of mine, whose name is well known in the history of mathematics,†

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* The apparatus invented by Napier is now exhibited in the South Kensington Museum.

† G. Vacca, "Sui manuscritta inediti di Thomas Harriot" (*Bolletino di bibl. e storia delle scienze matematiche*, t. v. 1902, pp. 1-6).

examined these manuscripts some ten years ago. The results of his scrutiny were such as to encourage the belief that a more complete and leisurely investigation would be productive of important results. For the moment it is sufficient to add that Harriot introduced the name "canonical equation," and the symbols still in use ($>$, $<$) for "greater than" and "less than."

## VII.

Thus far we have confined ourselves to number and measurement—no mention has been made of figured extension. But, while English work on the former is characterised by a real originality, that on the latter assumes in the main the form of mere comments on philological and critical investigations. Of the first type are the lectures delivered at Oxford by Sir Henry Savile (1549–1622), the munificent founder of the two mathematical chairs in the university, and those delivered at a later period at Cambridge by Isaac Barrow (1630–1677), the master, friend, and precursor of the greatest genius England has given to the exact sciences.

We now approach the excellent editions of Archimedes, Aristarchus, and Ptolemy which we owe to the labours of the celebrated John Wallis (1616–1703), one of the most active and original investigators known to history, and the only calculating prodigy who has ever made any contribution of real value to mathematical knowledge.*

In all probability it is to his unwearied study of the scientific writings of classical antiquity that we must trace the inspiration and power displayed by Wallis in the composition of his famous *Arithmetica Infinitorum*—a most admirable preparation for the new calculi which were shortly to come into existence. In it we find for the first time the familiar symbol, $\infty$, to denote "infinity," and here, too, the student of analysis will find Wallis's most elegant expression for $\pi$ as a product of infinite factors. To Wallis also belongs the credit of the discovery of the first algebraical surface not of revolution of a degree higher than two (I allude to the celebrated cuno-cuneus or circular wedge). Nor must it be forgotten that he shares with Lord Brouncker the honour of investigating the theory of continued fractions, and that he saved from irreparable loss John Caswell's excellent treatise on Trigonometry. He had a considerable share in the founding of the Royal Society of London, and saw that justice was done to his pupil William Neil (1637–1670), who had discovered the first curve which was algebraically rectified (the semi-cubical parabola). Christopher Wren, the famous architect of St. Paul's Cathedral, followed with the rectification of the ordinary cycloid.

But while work of this kind was suited to the genius of Wallis, he also, unlike many of his contemporaries, experienced and manifested on many occasions a keen interest in the historical evolution of scientific ideas. In his *Treatise on Algebra, both Theoretical and Historical* he showed that he could not always keep under control his predilections for his own countrymen and his dislike to foreigners, so that many of his assertions were challenged, and many of his conclusions were exposed to objections of considerable force. Nevertheless, there remains a serious attempt to blend into one organic whole the narrative of the struggle and the display of the trophies of victory. But there is ample justification for the remark, that Wallis found in his own country

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* [William Rowan Hamilton at the age of twelve was on more than one occasion a match for Zerah Colburn (v. *Encyclopaedia Britannica*, "Hamilton, W. R.") W. J. G.]

timid admirers and few followers. We must not forget or pass over in silence the fact that Wallis flourished in a period that was marked by an active and peaceful exchange of ideas across the Channel. This is most marked in the development of the Theory of Numbers, striking evidence of which exists in the correspondence which exists between Pierre Fermat, that French mathematician of unique power, and Sir Kenelm Digby (1603–1665). And the name of John Pell (1610–1685), given by Euler, though somewhat inappropriately, to the fundamental equation in indeterminate analysis of the second degree, is a further proof of the collaboration that went on at that time between the mathematicians of England and the Continent, with the object of overcoming the obstacles to the solution of the important problems that challenged the skill of the mathematical world of that period.

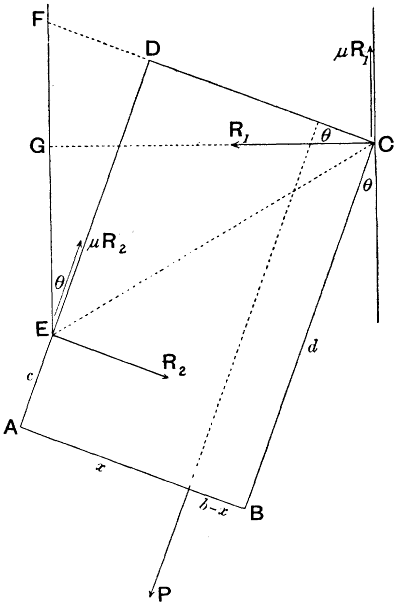
(*To be continued.*)

## ANSWERS TO QUERIES.

[**81, p. 341, vol. vi.**] The diagram gives an exaggerated picture of the state of affairs in the most general case, viz. (i) when the drawer is not tightly-fitting, and (ii) when it has been pulled out some distance before jamming. The forces acting are as shown.

$AB=b$; $BC=d$. Let the force $P$ be applied at a point in $AB$ distant $x$ from $A$ $(x \neq b/2)$, and let $AE=c$. Then $ED=d-c$, $EC=\sqrt{(d-c)^2+b^2}$, and if the distance $CG$ between the slides of the drawer be $f$,

$$EG=\sqrt{(d-c)^2+b^2-f^2}.$$



Then sin $\theta$, cos $\theta$ may be calculated; their values are found to be

$$\sin\theta=\frac{f(d-c)-b\sqrt{(d-c)^2+b^2-f^2}}{(d-c)^2+b^2}, \quad \cos\theta=\frac{bf+(d-c)\sqrt{(d-c)^2+b^2-f^2}}{(d-c)^2+b^2}.$$