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393. Mechanical Construction for the Trisection of an Angle

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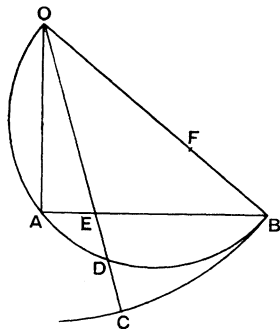
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**393. [K. 21. b.]** *Mechanical Construction for the Trisection of an Angle.*

Let  $\angle AOB$  be the angle. Describe a semicircle on  $OB$ . Join  $AB$ . With  $O$  as centre, draw the arc  $BC$ . Through  $O$  draw  $OC$  in such a way that the part intercepted between  $BA$  and arc  $BC$  is bisected by the arc  $BD$ . (This can be effected most accurately by means of a flat rule.)



Then  $OC$  is the required trisector.

$$OC \cdot OE = OE^2 + OE \cdot EC,$$

$$\text{or } OB \cdot OE = OE^2 + 2OE \cdot DE$$

$$= OE^2 + 2AE \cdot EB = OA^2 + AE^2 + 2AE \cdot EB$$

$$= OA^2 + AB^2 - EB^2 = OB^2 - EB^2,$$

$$\text{i.e. } OB(OB - OE) = EB^2.$$

Make  $OF = OE$ , and we have

$$OB \cdot BF = BE^2.$$

The circle round  $OEF$  will touch  $AB$  at  $E$ .

$$\text{Hence } \angle OEA = \angle OFE = \angle OEF.$$

The supplement of twice  $\angle OEF$  (which is  $\angle EOF$ ) is therefore twice the complement of  $\angle AEO$  (which is  $\angle AOE$ ); hence  $\angle EOF$  is twice  $\angle AOE$ ; that is,  $OC$  is a trisector.

I have not seen this mechanical construction anywhere in print. It appears to be simple as compared with the constructions I have come across, and I publish it after a hesitation of some years.

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INAYAT ULLAHKHÁN.

**394. [K. 13. a.]** The following method of finding the length and equation and the coordinates of the extremities of the shortest distance between two lines whose equations are given is perhaps as short as any for numerical examples, although it leads to heavy work in the general case.

$$\text{Let the given lines be } \frac{x-1}{1} = \frac{y-9}{-2} = \frac{z-5}{1} \dots\dots\dots(1)$$

$$\text{and } \frac{x-6}{7} = \frac{y+7}{-6} = \frac{z}{1} \dots\dots\dots(2)$$

The coordinates of any two points on (1) and (2) respectively may be written

$$(1+r, \quad 9-2r, \quad 5+r)$$

$$\text{and } (6+7s, \quad -7-6s, \quad s).$$

Hence the direction-cosines of the line joining them are proportional to

$$r-7s-5, \quad -2r+6s+16, \quad r-s+5.$$

If this line is the shortest distance, the conditions of perpendicularity give

$$(r-7s-5) - 2(-2r+6s+16) + (r-s+5) = 0$$

$$\text{and } 7(r-7s-5) - 6(-2r+6s+16) + (r-s+5) = 0.$$

Solving these, we find  $r=2$ ,  $s=-1$ , so the extremities of the shortest distance are  $(3, 5, 7)$  and  $(-1, -1, -1)$ .

Hence the length of the shortest distance is  $\sqrt{116}$ , and its equation is

$$\frac{x-3}{2} = \frac{y-5}{3} = \frac{z-7}{4}.$$

H. PIAGGIO.