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Loyd A. Jones

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VIII. *A Method and Instrument for the Measurement of the Visibility of Objects.* By LOYD A. JONES *.

Introduction.

DURING the summer of 1917 when the ravages of the submarine began to be a serious menace to marine shipping, various schemes of painting were devised, designed to protect the surface craft from attack by the submarine. The object of these earlier systems of painting was to render the vessel thus painted as invisible as possible and thus to elude observation by the submarine operator. Extravagant and conflicting claims were made as to the efficiency of the various methods and no satisfactory means of deciding upon their relative merits appeared to exist. It was at this time that Mr. Lindon W. Bates, Chairman of the Engineering Committee of the Submarine Defence Association, requested us to take up the problem of finding some precise means of measuring the visibility of ships painted according to various of these low visibility schemes and further to devise, if possible, a system of painting giving the lowest possible visibility. Some means of precise measurement was highly desirable in order that the claims of the various systems might be correctly evaluated by a method not involving a personal judgment.

When the work on this subject was taken up a survey of the field showed that at that time no method for the numerical specification of visibility, or instrument for its quantitative measurement, was available. The first step, therefore, was the working out, from the theoretical standpoint, of the fundamental laws upon which the quantitative evaluation of visibility could be based. Following this an instrument, operating upon the proper principle as indicated by the theoretical equations and suitable either for the measurement of visibility values of small models under known conditions of illumination and background or of actual boats at sea, was designed and built.

The method and instrument proved to be entirely satisfactory and some very interesting practical results were obtained in the examination of various so-called low visibility systems of painting. However, it is not the intention of this paper to deal with practical results but rather to confine ourselves to a treatment of those phases of the problem

* Communication No. 79 from the Research Laboratory of the Eastman Kodak Company. Communicated by the Director.

of greatest interest from a purely scientific point of view, namely, the theoretical analysis of the visibility problem, the measurement and numerical specification of visibility, and the instrument designed for this purpose.

Nomenclature and Definitions.

The intensity factor of the sensation resulting from the incidence of radiant energy upon the retina is expressed in terms of brightness. The brightness, B , of an element of luminous surface from any point of view may be expressed in terms of the luminous intensity per unit area of that surface projected on a plane perpendicular to the line of sight. When expressed in this manner it is measured in candles per unit area of the projected area. Brightness may also be expressed, and perhaps more logically, in terms of the specific luminous radiation of an ideal perfectly diffusing surface, that is a surface obeying Lambert's law. The brightness unit in this case is the lambert, which is defined as equal to the brightness of an ideal surface radiating or reflecting one lumen per square centimetre. In practice this unit is too large for convenience and hence the millilambert or '001 lambert is used. An ideal, perfectly diffusing surface emitting one lumen per square foot will have a brightness of 1.076 millilamberts. Measurements of brightness are made by means of a suitable form of photometer calibrated to read directly in the desired brightness units. The brightness of a surface depends upon two factors, the illumination of that surface and its reflecting power.

The illumination, E , of a surface at any point is the luminous flux density incident on the surface at that point, or the flux per unit of intercepting area. "Luminous Flux," F , is the rate of flow of radiant energy evaluated with reference to the visual sensation, and is expressed in lumens: the lumen being defined as equal to the flux emitted in a unit solid angle (steradian) by a point source of unit candle-power. The c.g.s. unit of illumination is the phot, which is defined as one lumen per square centimetre. The practical unit in most common use, however, is the foot candle, which is defined as one lumen per square foot, and which is equal to 1.0764 milliphots. For a uniformly illuminated surface,

$$E = \frac{F}{S}, \quad S \text{ being the area.}$$

Illumination is measured by a special type of photometer,
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usually referred to as a lumeter or illuminometer, these being calibrated to read directly in some suitable illumination units.

The coefficient of reflexion or total reflecting power of a surface is defined as the ratio of the total reflected luminous flux to the total incident luminous flux. In most practical work this value is not of great importance, the value desired being that of the reflecting power of the surface measured under certain specified conditions, such as the angle of incidence of the flux and the position from which the surface is viewed. The term "Reflexion Factor," R , is used to indicate this particular value and is defined as the ratio of the reflected to the incident flux. Reflexion from a surface may be either specular, diffuse, or a mixture of the two. In the case of pure specular reflexion all of the incident flux is reflected in such a way that the angle of reflexion is equal to the angle of incidence; while in the case of completely diffuse reflexion the reflected flux is equal in all directions regardless of the angle of incidence, the distribution being in accord with Lambert's cosine law. Very few cases of pure specular or diffuse reflexion are found in practice, there being generally a superposition of the two. The reflexion factor is measured by the use of a reflectometer, a photometer of special design, care being taken that conditions of illumination and angle of view are such as to give correct values for application in the particular case under consideration. This value is purely numeric and is usually expressed as a percentage value. If, with a specified condition of illumination, the reflexion factor, R , and the brightness, B , of a surface are measured from the same position, then $B = E \cdot R$, and hence the value of E may be determined; or in any case where two of these factors are known the third can be computed.

The quality factor of the luminous flux is that property which depends upon the spectral distribution of that flux, colour being defined as the subjective evaluation as expressed in terms of hue and purity or saturation. Hue is that property of colour which depends upon the variation in the sensation due to the variation of the wave-length of the luminous flux, while saturation expresses the proximity of the colour to a condition of monochromatism. Monochromatic spectral light has a saturation of 100 per cent., while pure white light has a saturation of zero. White, therefore, is a limiting colour having no hue and zero saturation. In practice it has been found convenient in many cases to express the saturation factor in the inverse order, that is as impurity rather than purity. The term used

in such expression is called the "per cent. white," for which the symbol I is used. Thus a colour for which $I=100$ per cent. is equivalent to zero saturation and if $I=0$ per cent., saturation is 100 per cent.

It has been demonstrated experimentally that any colour can be matched by the mixture, in the proper proportions, of white light with monochromatic spectral light of the proper wave-length. In this way a direct measurement of the fundamental sensation properties of a colour may be made. The hue is specified by the wave-length of monochromatic light used (wave-length of the dominant hue). The saturation is specified either as the purity (per cent. hue) or as the impurity (per cent. white), the former value being obtained from the ratio of the intensity of the monochromatic to the total intensity (monochromatic plus white) of the mixture, while the latter value (per cent. white) is given by the ratio of the intensity of the white to the total intensity of the mixture. These values are pure numerics. The usual unit used in expressing the wave-length of light is the millimicron, which is equal to $\cdot 0000001$ centimetre and is designated by the symbol $\mu\mu$.

In the foregoing paragraphs have been defined the various terms that will be used in the following discussion of the subject of visibility. These are summarized briefly in the following table for convenience of reference :—

<i>Symbol.</i>	<i>Quantity.</i>	<i>Unit.</i>
F	Luminous Flux.	Lumen.
E	Illumination.	Foot Candle.
B	Brightness.	Lambert.
R	Reflexion Factor.	Per cent.
H	Hue.	Wave-length ($\mu\mu$).
S	Saturation	
	Purity.	Per cent. Hue.
	Impurity.	Per cent. White.

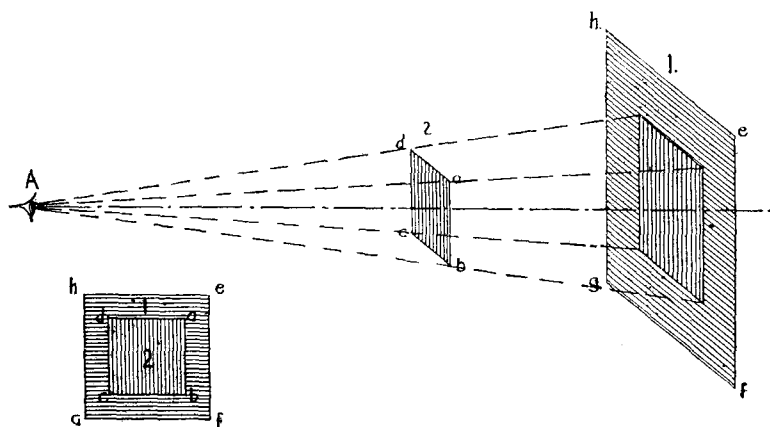
Theoretical Analysis of the Visibility Problem.

In general it may be said that non-luminous objects are visible by virtue of the light reflected from them. However, any particular object in the field of vision becomes visible as such only by contrast with its surroundings—that is, when the light emanating from that object (either by reflexion or emission) differs in some respect from the light

flux which enters the eye from the projected space immediately surrounding that object. The sensation caused by the incidence of radiant energy, which we call light, upon the retina of the eye may be said to consist of two factors, brightness and colour, the former being dependent upon the intensity and the latter upon the quality of the incident radiation. This second factor of the sensation may be said also to consist of two parts, hue and purity or saturation. Hue refers to the position, in the spectrum, of the dominant wave-length, and saturation expresses the proximity of the colour to monochromatism. It is evident, therefore, that a sensation due to the impingement of radiant energy upon the retina may vary in three respects, that is, with respect to brightness, hue, and saturation. A contrast in the visual field resulting in the visibility of an object may be due, therefore, to brightness contrast, to hue contrast, or to saturation contrast; or to a combination of any two or all three of these factors.

For the purposes of the theoretical treatment of this problem it will be necessary to make certain simplifying hypotheses. Begin first with the problem of the determination of the visibility of an object uniform in colour and brightness viewed against a background also of uniform

Fig. 1.



Diagrammatic Illustration of the Relation of Object to Background.

colour and brightness. In fig. 1 this case is shown in perspective, 1 being the background, 2 the object, and A the eye or view-point. The visual field will appear as shown in

the lower left-hand drawing of fig. 1, 2 representing the object (rectangle $abcd$) and, 1, the background (rectangle $efgh$).

The terms used in the discussion and the symbols employed are as indicated previously except that a subscript number or letter attached to a given symbol indicates that the term applies to the object or surface designated by that number or letter. Thus E_1 is the symbol used for the illumination on the surface 1, the background.

Assume for the moment that the object and background are illuminated by light of the same quality and also that this quality be specified as white, which is defined as light from the noon sun on a clear day or its spectral equivalent. Now, the visibility, V , of the object as seen against this background is dependent upon the total contrast existing between the two. This total contrast is made up of three factors: (1) Brightness contrast, C_b ; (2) Hue contrast, C_h ; and (3) Saturation contrast, C_s . These three factors of the total contrast may be evaluated as follows:—

$$C_b = f(B_1, B_2),$$

$$C_h = f(H_1, H_2),$$

$$C_s = f(S_1, S_2).$$

The total visibility may then be expressed in the general form,

$$V = f(C_b, C_h, C_s).$$

The laws governing the reaction of the retina to the various brightness stimuli are so well established that it is comparatively easy to evaluate the term $C_b = f(B_1, B_2)$ directly in terms of visibility, to make quantitative measurements of visibility as such, and to correlate such determination made under widely different conditions. Unfortunately, the other terms, C_h and C_s , cannot be so readily evaluated. This is due to the lack of knowledge concerning the fundamental reactions of the retina to these stimuli. However, it is possible by a direct method of measurement to determine the total visibility of an object against a given background. Such a value includes in a single term the visibility due to all three kinds of contrast. Since the part of total visibility due to brightness contrast may be determined independently, a means is thus available of evaluating that part of visibility due to the combined effect of hue and saturation contrast. Since hue and saturation are the two factors of quality, the part of total visibility due to hue and saturation may for convenience be designated

as quality contrast, C_q . It should be borne in mind, however, that this term includes two independent variables, both of which must be considered in any evaluation of visibility due to quality contrast. It is entirely possible that the visibility due to C_h and C_s could be evaluated separately, but this would require a large amount of fundamental research which it will not be advisable or necessary to go into at this time.

The visibility resulting from brightness contrast is directly proportional to the subjective contrast and hence for an eye adapted to a fixed brightness level to the ratio of the two brightnesses, B_1 and B_2 . Therefore, we may write

$$V_b = f\left(\frac{B_1}{B_2}\right).$$

A direct measurement of B_1 and B_2 will therefore determine V_b . This method is satisfactory and applicable if there is no hue or saturation contrast between object and background, and if the object and background are each uniform in brightness, hue, and saturation. These conditions, however, do not exist in practice. Brightness measurements in the presence of colour differences are difficult to make and subject to great errors. Moreover, in practice it is frequently necessary to determine visibility in cases where the object or background is not uniform in brightness, the variation being irregular and practically indeterminate. In such cases the determination of the effective values of B_1 and B_2 is extremely difficult and, if not impossible, entirely impracticable. These objections to this method of measuring visibility are so serious as to make it almost useless from the practical standpoint, and hence it is necessary to consider other methods for the accomplishment of the desired result.

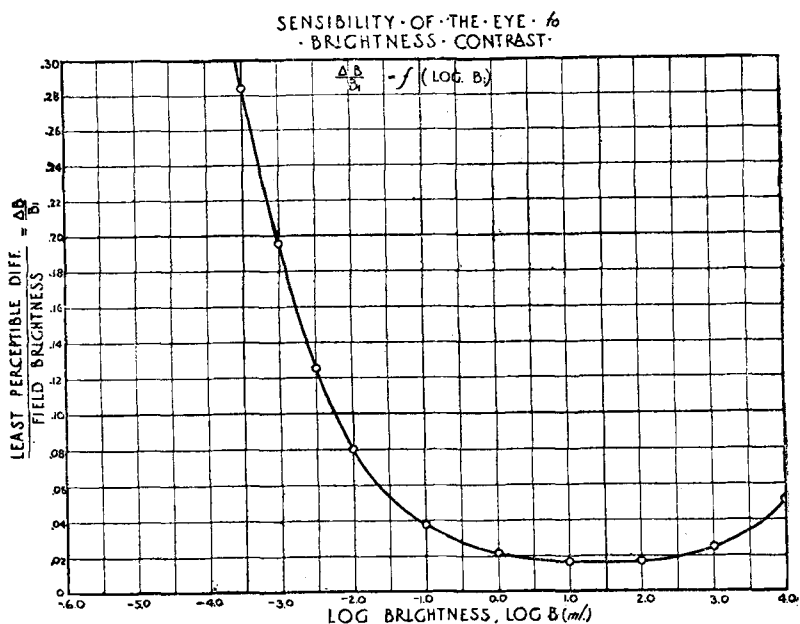
A careful consideration of the causes of lowered visibility reveals the fact that loss of visibility results from the introduction of veiling glare between the eye and the object and also to a lowering in the brightness of the object and its background. In order to explain the action of this veiling glare in producing a diminution or in some cases an entire loss of visibility, it will be helpful to present a curve, fig. 2, which shows in graphic form the contrast sensibility of the eye*. The abscissæ values of this curve are the field or background brightness, the ordinates are the ratio of ΔB to B_1 . The term ΔB , the least perceptible difference, is the brightness difference between B_1 and B_2 when that difference

* P. G. Nutting, Trans. Ill. Eng. Soc. vol. xi. p. 945 (1916).

is just perceptible to the eye when adapted to the field brightness B_1 . The sensibility is inversely proportional to ΔB and hence to

$$\frac{\Delta B}{B_1}.$$

Fig. 2.



The Sensibility of the Eye to Brightness Contrast.

The curve shown is therefore the reciprocal of the sensibility curve and is given in this form for the sake of convenience in application to this problem. It will be noted that the sensibility at low values of B is relatively low (ΔB large) increasing to a maximum (ΔB small), which remains sensibly constant over a considerable range of brightness values and then again decreasing for very high values of B_1 .

The term k , which will be called the "contrast constant" of the eye for any given value of B , is defined by the equation,

$$k = 1 + \frac{\Delta B}{B_1}.$$

Since $\triangle B = B_1 - B_2$ or $B_2 - B_1$, depending upon the absolute magnitude of the values, it follows that

$$k = \frac{B_1}{B_2} \text{ or } \frac{B_2}{B_1},$$

that ratio being used which results in a value of k greater than unity.

It is evident that if $\frac{B_1}{B_2} = k$,

the object, 2, of fig. 1 will be just visible against the background, 1;

while if $\frac{B_1}{B_2} < k$,

the object will be invisible.

Now, in case $\frac{B_1}{B_2} > k$,

if it is possible by any means to reduce the value of the ratio to the point where

$$\frac{B_1}{B_2}$$

is just less than k , the object will be rendered just invisible. This can be accomplished by adding the same brightness, B_v , to both B_1 and B_2 . B_v being of the magnitude required to satisfy the equation

$$\frac{B_1 + B_v}{B_2 + B_v} = k.$$

This superposed brightness, B_v , will be referred to as "veiling glare."

Now, the magnitude of B_v required to satisfy the equation just given may be taken as a direct measure of the visibility of objects under constant conditions of illumination. It is evident, however, that the value of B_v required by the equation will depend not only upon the ratio of B_1 to B_2 , but also upon the absolute values of those terms. Now, assuming that k is constant, and not dependent upon B_1 , visibility must be independent of the absolute value of either B_1 or B_2 , and a function only of their ratio. Such evaluation may be accomplished by writing,

$$V_b = \frac{B_v}{B_1}.$$

In order to determine visibility, therefore, it is only

necessary to measure the values B_1 and B_v in any particular case. In practice the veiling glare, B_v , is produced artificially by inserting at some convenient point between the eye and the object a semi-transparent surface of some sort, such as a half-silvered mirror, this surface being illuminated to the desired brightness, B_v , by any convenient method. The values of B_1 and B_v may be measured by means of a suitable brightness photometer. The visibility of any object, under a fixed set of conditions, may now be specified in definite units directly comparable with the visibility of different objects under the same conditions.

It is evident that a variation in value of any one of the four terms, E_1 , E_2 , R_1 , and R_2 , will cause a corresponding change in the value of V_b . Visibility may be evaluated, therefore, as a function of any one of these four terms as a variable and the remaining three as constants, thus ;

$$\begin{array}{ll} V_b = f(R_1) & R_2, E_1, \text{ and } E_2 \text{ constants,} \\ V_b = f(R_2) & R_1, E_1, \text{ and } E_2 \text{ constants,} \\ V_b = f(E_1) & E_2, R_1, \text{ and } R_2 \text{ constants,} \\ V_b = f(E_2) & E_1, R_1, \text{ and } R_2 \text{ constants.} \end{array}$$

It is entirely possible to evaluate each of these functions, but a consideration of the problem to which the theory is later to be applied shows that this is not necessary and that a different method of evaluation is more directly applicable.

In the problems to which the results of this theoretical treatment are to be applied the sky forms the background in most of the cases to be considered, and it is impossible in general to treat the sky as a surface. It is not possible, nor is it necessary, to determine independently the values of the reflexion factor and illumination in dealing with this sky background. However, its brightness can easily be measured and from the standpoint of brightness the sky may therefore be regarded and treated as a surface. The variables E_1 and R_1 are therefore eliminated from this problem, being replaced by a single term, B_1 . This leaves for consideration the three variables, E_2 , R_2 , and B_1 , for each of which as a variable an evaluation of V_b may be formulated. Again, considering conditions in nature, it will be seen that E_2 and B_1 are not in general independent variables but more or less dependent one upon the other. It is desirable, therefore, to combine these two into a single term, as a function of which visibility may be expressed. This combination of B_1 and E_2 is best accomplished by taking the ratio of the former to the latter. This ratio is a complete specification of the lighting conditions

at any instant and is therefore in the practical problem of an object illuminated by natural light a complete specification of the weather conditions at a given time. This term will be referred to as the "Weather Coefficient," W , its evaluation in terms of the other quantities being expressed by the equation

$$W = \frac{B_1}{E_2}.$$

The variables to be considered are therefore R_2 and W , and the necessary evaluations of visibility are of the form, $V_b = f(R_2)$, $W = \text{a constant}$; and $V_b = f(W)$, $R_2 = \text{a constant}$. The first equation when properly formulated will make possible the computation of the variation in visibility due to a variation in the value of the reflexion factor, for any specified value of the weather coefficient; the second equation will give the variation in visibility with the value of the weather coefficient for any object of definite reflexion power.

Before proceeding with the formulation of these visibility functions it is desirable, for the sake of clearness, to summarize briefly the terms thus far defined which must be used in the subsequent development of the theory.

B_1 = Brightness of background.

B_2 = Brightness of object.

E_2 = Illumination of object.

R_2 = Reflexion factor of object.

B_v = Brightness of veiling glare which when superposed over object and background will reduce the contrast to a just imperceptible value.

V_b = Brightness visibility.

W = Weather Coefficient.

k = Constant contrast of the eye.

Now, B_v must satisfy the equation,

$$\frac{B_1 + B_v}{B_2 + B_v} = c, \quad \dots \dots \dots (1)$$

where c is a constant depending upon whether B_1 is greater or less than B_2 .

If $\frac{B_1}{B_2} > k, c = k; \quad \dots \dots \dots (2)$

or if $\frac{B_1}{B_2} < k, c = \frac{1}{k}, \quad \dots \dots \dots (3)$

solving equation (1), for B_v we obtain,

$$B_v = \frac{B_1 - c B_2}{c - 1} \quad (4)$$

Now, $B_2 = E_2 \cdot R_2$ (5)

$$\therefore B_v = \frac{B_1 - c E_2 R_2}{c - 1} \quad (6)$$

Also, $V_b = \frac{B_v}{B_1}$ (7)

Therefore,
$$V_b = \frac{B_1 - c E_2 R_2}{B_1(c - 1)}$$

$$= \frac{1}{c - 1} - \left(\frac{E_2}{B_1} \cdot R_2 \cdot \frac{c}{c - 1} \right) \quad (8)$$

As previously shown,

$$W = \frac{B_1}{E_2}, \quad (9)$$

therefore

$$V_b = \frac{1}{c - 1} - \left(\frac{1}{W} R_2 \cdot \frac{c}{c - 1} \right),$$

$$V_b = \left(\frac{1}{W} \cdot R_2 \cdot \frac{c}{1 - c} \right) - \frac{1}{1 - c} \quad (10)$$

Equation (10) is a general expression of brightness visibility as a function of both W and R_2 .

Now, in evaluating the constant c it is found that there are two possible values, one in case $\frac{B_1}{B_2} > k$, and one for the

case where $\frac{B_1}{B_2} < k$.

Case I., $\frac{B_1}{B_2} > k$, $c = k$,

$$V_b = \left(\frac{1}{W} \cdot R_2 \cdot \frac{k}{1 - k} \right) - \frac{1}{1 - k} \quad (11)$$

Case II., $\frac{B_1}{B_2} < k$, $c = \frac{1}{k}$,

$$V_b = \left(\frac{1}{W} \cdot R_2 \cdot \frac{1}{k - 1} \right) - \frac{k}{k - 1} \quad (12)$$

Equations (11) and (12) now express visibility as a function

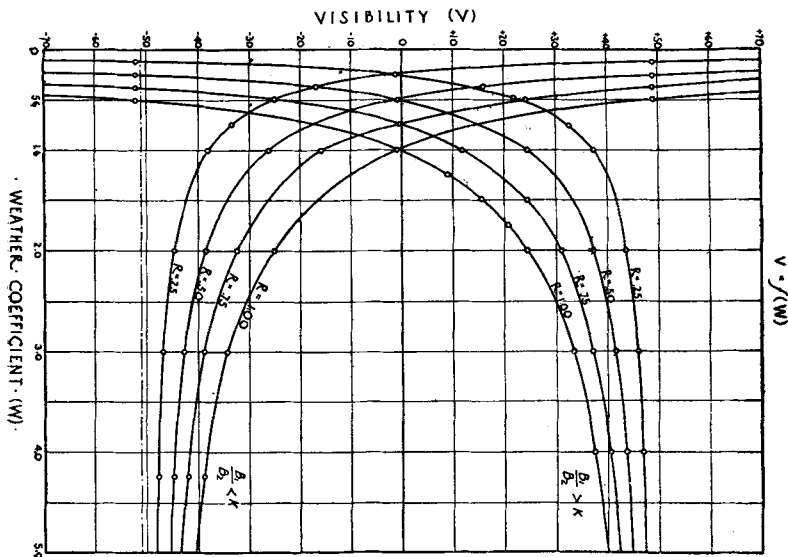
of the variables R_2 and W for the two cases covering all values of the ratio

$$\frac{B_1}{B_2}.$$

If in these equations R_2 be treated as a constant they then express visibility in the form $V_b = f(W)$; while if W be treated as a constant they express visibility in the form $V_b = f(R_2)$.

Now the value of the contrast constant k can be determined from the sensibility curve, fig. 2, for any specified value of B_1 . Let a value of $k=1.02$ be assumed and then by assuming a series of values for R_2 a family of curves showing V as a function of W can be plotted. These curves are given in fig. 3, the ordinate values being visibility, V_b ,

Fig. 3.



Visibility as a Function of the Weather Coefficient.

and the abscissæ values the weather coefficient, W . It will be seen from fig. 3 that

$$\text{Case I., } \frac{B_1}{B_2} > k,$$

is represented by a group of rectangular hyperbolæ having

as asymptotes the lines $y = +50$ and $x = 0$; while

$$\text{Case II.,} \quad \frac{B_1}{B_2} < k,$$

gives a family of hyperbolæ having as asymptotes the lines $y = -51$ and $x = 0$.

It will be seen that considerable portions of these curves lie in the fourth quadrant where values of V_b are negative. It is obviously impossible to have values of visibility less than zero, hence all such values may be considered as unreal or imaginary. The only portions of the curves of interest in this problem are those lying in the first quadrant. It will be noted also that in general visibility is high for very low values of W , decreasing to zero at a value of W which depends upon the assumed value of R_2 , and then rising (approaching $V = +50$ as a limit) for high values of W . It will be noted that for any given value of R_2 there is a small range of W values for which $W = 0$. If we express this range by the symbol ΔW its value may be expressed by the equation,

$$\Delta W = R_2 \frac{k-1}{k}.$$

Thus the range of weather conditions for which V can be zero is a function of both R_2 and k . It will be seen by examination of the curves that $V = 0$ when $R_2 = W$. An object becomes invisible against a given background when the reflexion factor, R_2 , of that object is equal to the ratio of background brightness, B_1 , to the illumination, E_2 , on the object plane, that is when $B_1 = B_2$.

Now, passing on to a consideration of visibility as a function of the reflexion factor, R_2 , as a variable quantity and W as constant, the curves shown in fig. 4 are obtained by solution of equations (11) and (12) for various fixed values of W and the same value of k ($k = 1.02$) as was used previously. It will be seen that

$$\text{Case I.,} \quad \frac{B_1}{B_2} > k,$$

is represented by a series of straight lines, all passing through the point $x = 0$, $y = +50$. The slope of the line for any particular assumed value of W is given by the expression

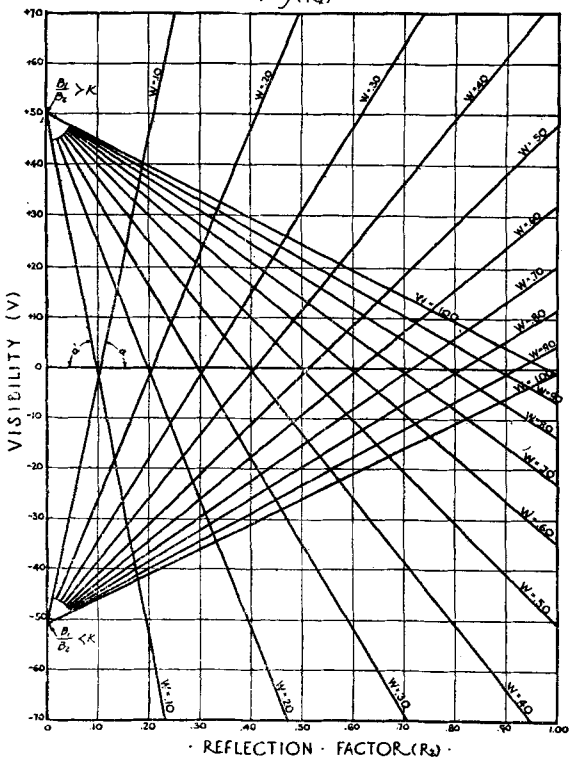
$$\tan \alpha = \frac{k}{W(1-k)}.$$

Case II., $\frac{B_1}{B_2} < k,$

leads to family of straight lines all passing through the point $x=0, y=-51$, the slope of any one being given by the equation

$$\tan a = \frac{1}{W(k-1)}.$$

Fig. 4.

$$V = f(R_2)$$


Visibility as a Function of the Reflexion Factor.

It will be noticed in solving for the value of $\tan a$ that in

Case I., where $\frac{B_1}{B_2} > k$,

the value is negative, while the value of $\tan a$ in Case II. is positive. This offers a convenient method for indicating

whether the object to which any visibility value applies is lighter or darker than the background against which it is measured. The visibility value itself must from the very nature of the term be always positive, but by specifying the sign of the first derivative of the visibility function at a given point it can be determined whether such visibility is due to the object being brighter than the background or *vice versa*. The first derivative of the function

$$V=f(R_2) \text{ is } \frac{dV}{dR_2},$$

which, since the function is a straight line, is equal to the tangent, thus

$$\frac{dV}{dR_2} = \tan \alpha.$$

If $\frac{dV}{dR_2}$ is negative it indicates that $\frac{B_1}{B_2} > k$

and therefore that the object is darker than the background.

In case $\frac{dV}{dR_2}$ is positive it indicates that $\frac{B_1}{B_2} < k$

and therefore that the object is brighter than the background.

In the data presented later in the paper the visibility values will be followed by plus or minus signs, which will be understood to indicate the sign of the first derivative of the function at that point and hence show the relative magnitude of B_1 and B_2 . A convenient way of remembering the significance of these signs will be to consider that the plus sign indicates that the addition of brightness to the background is needed in order to make it match the object, while the minus sign indicates that brightness must be subtracted from the background. In case a visibility value is represented by a point lying upon the branch of negative slope and also upon the branch of positive slope, that is, at the point of intersection of the two lines forming a complete curve of V for all values of R_2 , $\frac{dV}{dR}$ for that visibility value is either negative or positive.

In such a case $B_1=B_2$, and if the visibility value is greater than zero such visibility must be ascribed to either hue or saturation contrast or to the combined effect of these factors. In order to designate such conditions, the visibility value will be followed by the sign plus or minus, \pm .

As in the previous set of curves, those of fig. 3, only the values of V lying in the first quadrant are real. The curves of fig. 4 show in graphic form the variation of visibility with

the reflexion factor of the object for a fixed value of the weather coefficient. It will be again noted that $V_b=0$ when $R_2=W$.

This completes the theoretical treatment of visibility due to brightness contrast. In closing the discussion it may be well to review briefly the most important points. Visibility due to brightness contrast, V_b , is measured by a determination of B_v , the veiling glare which when superposed upon the object and background will reduce the contrast to a just perceptible value. The fundamental equation is

$$\frac{B_1 + B_v}{B_2 + B_v} = c,$$

where c is a constant depending for its value upon the sensibility functions of the eye. Formulations of the functions $V_b=f(R_2)$ and $V_b=f(W)$ lead to equations (11) and (12), by solution of which curves given in figs. 3 and 4 are obtained. It is also shown that for $V_b=0$, $R_2=W$. As stated previously, the term V_q is composed of two factors, V_h , hue visibility, and V_s , saturation visibility. Notwithstanding the fact that the sensibility of the eye to hue and purity differences is fairly well known, it is not possible to formulate directly the visibility functions in these cases. It may be said in general that $V_h=f(H_1, H_2)$, but just what form the function will take it is impossible to say. It is probable that the maximum visibility will be found when H_1 and H_2 are complementary hues, diminishing to a zero value as H_1-H_2 approaches zero. It is equally impossible in the light of our present knowledge to evaluate the expression $V_s=f(S_1, S_2)$. However, it is evident that V_s can be zero only when $S_1=S_2$, that is, when the saturation of the object is equal to the saturation of the background. It will probably be found that V_s is directly proportional to the saturation difference, in which case the expression would be of the form, $V_s=a(S_1-S_2)$, where a is a constant of proportionality which may or may not vary for different values of H . At present it is sufficient to give the general form of the evaluation of V_q , which includes V_h and V_s . The total visibility, V , can be measured by the superposition of a veiling glare, B_v ; and by measuring B_1 and B_2 , V_b can be computed by means of the theoretical equations. In this way V_b and V_q may be separated if desired. It is evident that the expression for V (total visibility) must be of the form

$$V=f(V_b, V_h, V_s).$$

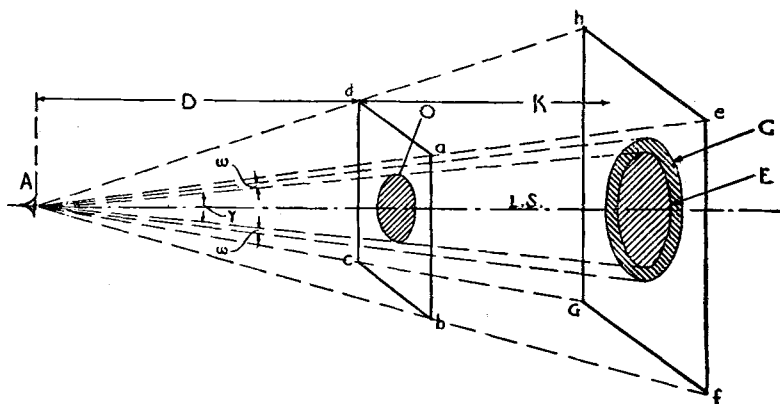
Since none of the terms V_b , V_h , and V_s can ever be less than zero, it is evident that in order to obtain a value of $V=0$, V_b , V_h , and V_s must each be equal to zero. That is, no object can be invisible against a given background unless it matches that background in brightness, hue, and saturation, these conditions being expressed symbolically by the equations $B_1=B_2$, $H_1=H_2$, and $S_1=S_2$.

Natural Causes of Lowered Visibility.

In considering the visibility of objects at relatively great distances from the observer, situated in natural surroundings and subjected to lighting conditions which depend upon the state of the weather, it becomes necessary to introduce some new factors into the problem. Thus far we have dealt with the brightness of a surface and have assumed that if measured in a given direction the value will be independent of the distance between the surface and the point of observation. This is true in case there is no absorption or emission of light within the space between the surface and the point of observation. In general, this condition cannot be assumed to exist in nature, for the air through which an object is viewed may carry in suspension particles of matter in a more or less finely divided state which may absorb or scatter the light reflected or emitted by the objects viewed, thus causing the apparent brightness from a given point of observation to be less than the real brightness determined at the surface. Such particles, if illuminated either by light reflected or emitted by the objects viewed or by light from other sources, may, by reflecting, refracting, or diffracting this incident light, cause it to enter the eye of the observer. Such particles act, therefore, as sources of light within the space between the object and the observer, thus causing the apparent brightness of the surface to be greater than its real value. It is necessary to deal, therefore, with objects distributed in three-dimensional spaces which may be filled with minute particles acting as sources or sinks of light causing the apparent brightness of a surface from a given point of observation at some distance from that surface to be different from the real brightness measured from a point near or at that surface. Thus far we have dealt only with the real values of surface brightness of objects distributed in a space free from such sources and sinks of light. In order to treat the more complex problem it is necessary to expand the nomenclature to include the spatial distribution of light as well as of objects.

Let us take a specific case of an object uniform in reflexion factor and colour (hue and saturation), the visible surface of which occupies a position in an approximately vertical plane perpendicular to the line of sight. Let us assume further that this object is viewed against a sky background uniform in brightness and colour. This condition is shown in perspective in fig. 5. The eye or viewpoint is at A,

Fig. 5.



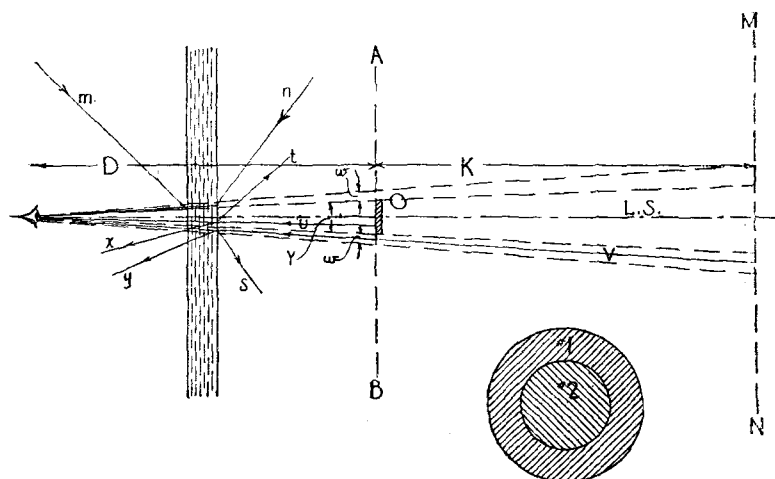
A Diagrammatic Illustration of the Relation of Object to Background.

the line of sight being the line LS . The object, 2, is represented by the circle O lying in the object plane $abcd$, which is approximately perpendicular to the line of sight. The background, 1, is represented by the imaginary plane $efgh$, also perpendicular to LS and situated at infinity. The angle subtended by the object at the eye is γ , and is assumed to be small.

In fig. 6 is shown a cross-section obtained by passing a horizontal plane through the line of sight. The object is at O , the line AB is the trace of the object plane and the line MN is that of the background plane. For the purposes of this discussion we need deal only with that part of the background immediately surrounding the object which is included in the angle ω , that is the space lying within the circle C and outside of circle E , of fig. 5, the projection of the object on the background plane. The visual field in this case is as shown in the lower right-hand drawing of fig. 6, the area marked 2 being the object and that marked 1 the background. It should be understood that the background is not limited as indicated, but that for

the purposes of this discussion only that portion of it which surrounds the object need be considered. It will be convenient for the sake of reference to divide the space into two parts: (a) the space lying between A and the object

Fig. 6.



Horizontal Section through the Line of Sight.

plane will be termed the "foreground space," D; (b) the space between the object plane and the background plane will be called the "background space," K.

Material particles in these spaces may react in four ways upon light traversing this space. If very small, such particles will cause diffraction; if larger, reflexion, absorption, and refraction. Thus the intensity of a beam of light travelling in a given direction may be diminished by any one of these four effects, also light may appear to originate in the space between the point of observation and the object viewed from any one of three of these four causes. It is unnecessary for the purposes of this discussion to evaluate the losses or gains in brightness for each of these different causes, it being sufficient to evaluate the total gain or loss under any particular condition. From the standpoint of apparent brightness any diminution in the brightness value may be regarded as due to absorption whether the light is absorbed or scattered by the particles present in the space lying between the point of observation and the surface being considered. Likewise any increase in the apparent brightness may be regarded as a

superposed brightness arising from any or all of the possible causes. The action of a given space containing material particles upon a brightness viewed through that space may effectively be defined by two factors, the transmission, T , and the effective brightness, P . The transmission may be defined as the ratio of the transmitted to the incident intensity, and effective brightness as the apparent brightness of a perfectly black surface when viewed through the space under consideration. Consider the case of a surface of which the real brightness is B viewed through specified space of which the transmission is T , and the effective brightness P . The apparent brightness, L , of that surface will be given by the expression $L = (B \cdot T) + P$.

Turning now to a consideration of conditions existing in nature, if neither the background nor foreground space contained any material particles capable of producing a scattering of light, the sky would be perfectly black and B_1 , the brightness of the background, would be zero, while the apparent brightness of the object would be equal to its real brightness, $L_2 = B_2$. Such conditions, of course, never exist since the air itself and the very small particles of matter always carried in suspension cause scatter by diffraction, thus giving rise to the blue light that we call the sky. Such conditions we term a perfectly clear atmosphere, the sky being a dark blue, practically uniform in colour and brightness from zenith to horizon. There is practically no emission or absorption of light within the foreground space, hence $L_1 = B_1$ and $L_2 = B_2 = E_2 \cdot R_2$.

Consider next the case where the background space, K , contains, in addition to the scattering material always present, larger particles which produce scatter by reflexion or refraction or both; $L_1 = (B_1 \cdot T_k) + P_k$. In most cases the fog or clouds formed by the particles in the space K are so dense that T_k is very small and L_1 is practically $= P_k$. That is, the sky is obscured and the entire brightness is due to light reflected and refracted from the cloud structure within the background space. Since there is no scattering material within the space D , $L_2 = B_2$. Finally, assume that both space D and K are filled with scattering particles, then

$$L_1 = [(B_1 \cdot T_k) + P_k] \cdot T_d + P_d.$$

It is unnecessary in practice to separate the factors of the brightness due to the various causes in the background space, and hence for the sake of simplicity the symbol B_1 (background brightness) is used to include all such factors. The symbol B_1 will therefore be used for the term $(B_1 \cdot T_d + P_k)$.

The special expression for apparent background brightness L_1 therefore becomes $L_1 = (B_1 \cdot T_d) + P_d$.

The expression for visibility due to brightness contrast given in a previous section of this paper was

$$V_b = \frac{B_v}{B_1} \dots \dots \dots (7)$$

B_v being defined by the equation

$$\frac{B_1 + B_v}{B_2 + B_v} = c. \dots \dots \dots (1)$$

By superposing the veiling brightness B_v over both B_1 and B_2 , the apparent brightness of each is changed until the ratio of the apparent brightness is equal to the constant c . $B_1 + B_v$, therefore, is the apparent brightness of the background. Hence

$$B_1 + B_v = L_1 = (B_1 \cdot T_d) + P_d,$$

and likewise

$$B_2 + B_v = L_2 = (B_2 \cdot T_d) + P_d.$$

These equations are satisfied if $B_v = P_d$ and $T_d = \text{unity}$. These equations show the exact analogy existing between the conditions assumed in the theoretical evaluation of visibility and those conditions that actually occur in nature resulting in lowered visibility.

It will be well at this point to discuss the direct effect upon visibility of various distributions of the scattering material. Take first the ideal case of no scattering material at any point. The sky will be black, $L_1 = B_1 = 0$. The only case of low visibility would be for a perfectly black object, and as such do not exist, visibility would in general be high. In case of a clear atmosphere visibility is low only in case the object matches the sky (dark blue) in colour and brightness. Since there is no veiling brightness or absorption of light within the foreground space, visibility would in general be high. Consider now the presence of other scattering material, such as clouds, mist, fog, dust, &c., located entirely in the background space, and either localized in a given region or uniformly distributed over a considerable distance in the direction of the line of sight. This will operate to change the apparent brightness and colour of the area 1 of the visual field in the lower right hand drawing of fig. 6, but will not change the value of the brightness or colour of the object, 2. The change produced may be in any direction, depending upon the nature, amount, and spatial distribution of the scattering material. The presence of such material in

this space behind the object plane may either raise or lower the visibility of the object, depending on the precise existing conditions. Hence the presence of such material cannot in general be termed a cause of lowered visibility, although in some cases it may operate in that direction. The brightness B_1 is in this case also a result of the summation of the light entering the eye from any and all points within the angle ω , and from between A and infinity. This light is scattered sunlight, the scattering being due to diffraction, reflexion, and refraction, the last two factors being predominant in the case of cloudy sky or of mist in the atmosphere.

Now let us consider the case where scattering material is found also in the space between the eye, A, and the object plane. This material may be localized in a particular region or uniformly distributed throughout the space from A to the object plane. The presence of this finely divided material, such as dust, mist, &c., will operate in two ways upon the apparent values of the brightness of object and background. A decrease in the intensity of the light emanating from the object and background space will be accompanied by an increase in the intensity of the light apparently emitted by the foreground space. That is, T_d and P_d in general vary according to some inverse law. In order to separate the action of these factors let us take it that $T_d = \text{unity}$ and allow P_d to vary. Some value of P_d is thus added to both B_1 and B_2 , and it will be seen that whatever the value of P_d the ratio of B_1 to B_2 will be decreased, thus causing a lowering of visibility. Assuming a small constant value of P_d , it will be seen that an increase in T_d will change both B_1 and B_2 in the same proportion, thus keeping their ratio constant. A variation in T_d , therefore, affects the visibility only in so far as the change in B_1 governs the value of k (the contrast factor of the eye). Since the highest values of B_1 found in nature are not above the point where

$$\frac{\Delta B}{B_1}$$

begins to increase (due to very high values of B_1), it follows that if a variation in T_d has any effect upon visibility it will be to lower the value of that term.

As stated previously, the relation between the terms T (transmission) and P (effective brightness) is in general expressed by some inverse function. Thus as P increases in magnitude, T usually diminishes. No general expression

for the relation between these two terms can be given without an exact knowledge of the physical characteristics of the scattering and absorbing particles. Thus, in case the particles are opaque and of low reflecting power, the term P may be almost negligible while T is very low, so that the entire effect upon the apparent brightness of an object viewed through the space filled with such particles is due to the absorption of light by the particles, the expression for apparent brightness being $L=B.T$. On the other hand, if the particles are highly reflecting or transparent, the predominant factor may be P (the effective brightness). The relative magnitude of P and T depends to some extent also upon the conditions of illumination prevailing in the space considered. It is not feasible, therefore, to formulate the relation between P and T for all cases since this relation involves a consideration of the nature, size, and spatial distribution of the scattering particles and also the conditions of illumination.

In closing this section of the discussion it will be well to summarize briefly the conclusions reached regarding the visibility of an object under natural atmospheric conditions and let its visibility have an appreciable value which may be termed its initial visibility. A change of this initial value is in general due to the presence in the atmosphere of material particles which may either diffract, absorb, reflect, or refract the light travelling through the space occupied by such particles. If these particles are confined to the background space the change in initial visibility may be either positive, negative, or zero, depending upon the particular conditions. If the particles exist in the foreground space the change in the initial visibility will in general be either zero or negative, although it is possible under certain conditions to produce an increase in visibility by means of absorbing elements in the foreground. This latter case is very unusual. The final conclusion may therefore be drawn that in the great majority of cases low visibility is due to the presence of absorbing or scattering material in the foreground space which operates in such a way as to lower the apparent contrast between the object and its background.

The Measurement and Specification of Visibility.

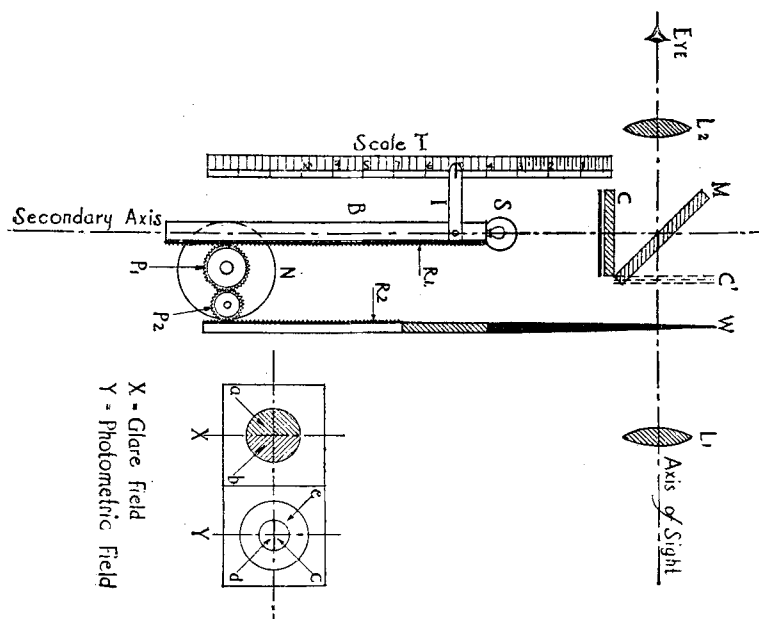
The general principles upon which the specifications of visibility must be based have been outlined in the section dealing with the theory of the subject. It will be remembered that visibility may be specified in terms of the brightness of a veiling glare, B_v , and the brightness of

the background, B_1 , against which the object is viewed. This veiling glare was defined as that brightness which, when superposed upon the visual field composed of an object and its background, will reduce the apparent contrast between object and background to the limit of visibility. It will be remembered that the discussion referred to dealt with the valuation of the visibility due to brightness contrast, V_b . Now suppose that in addition to a brightness contrast, a contrast in either hue or saturation also exists. The amount of veiling glare required to reduce the total contrast to a just perceptible value will in general be greater than in case the colour contrast was not present. It is entirely logical, therefore, to apply the same general method for the evaluation of total visibility, that is, the superposition over the object and background of a veiling glare of sufficient brightness to reduce the total contrast to zero. The distinction between the terms V_b and V , the total visibility, should, however, be borne in mind. The total visibility in any case, whether that visibility be due either to brightness, hue, or saturation contrast, or to any combination of these terms, is evaluated in terms of the equivalent brightness contrast which would produce the same degree of visibility. The validity of such a method is strongly supported by the fact that loss of visibility in nature is almost entirely due to the presence of a veiling glare which is quite constant in quality, its colour being approximately white, but variable in intensity. This natural veiling glare, arising from the presence of diffusing material in the foreground space, produces a lowering of the visibility value regardless of whether the initial visibility is due to brightness, hue, or saturation contrast.

It was necessary to design and build an instrument and to develop methods for the precise measurement of these qualities, B_o and B_1 , under practical conditions. After extensive preliminary trials a satisfactory instrument was developed. Several different types, all operating upon the same basic principles, having been designed, the type which appeared to be most convenient for practical work was chosen and a complete instrument constructed. This instrument is called a "Visibility-Meter," and patents covering the basic principles upon which it is constructed and several particular designs have been applied for by the Eastman Kodak Company, in whose Research Laboratory these experiments were conducted. In fig. 7 is given a diagrammatic sketch showing the arrangement of the essential parts of the instrument, and figs. 8 and 9 show photographs of the completed instrument.

The letter M, fig. 7, indicates a semi-transparent mirror set at 45° to the axis of the instrument, which is coincident with the axis of sight. This mirror reflects about 50 per cent. of the incident light and transmits approximately 20 per cent., the remaining 20 per cent. being absorbed. The

Fig. 7.

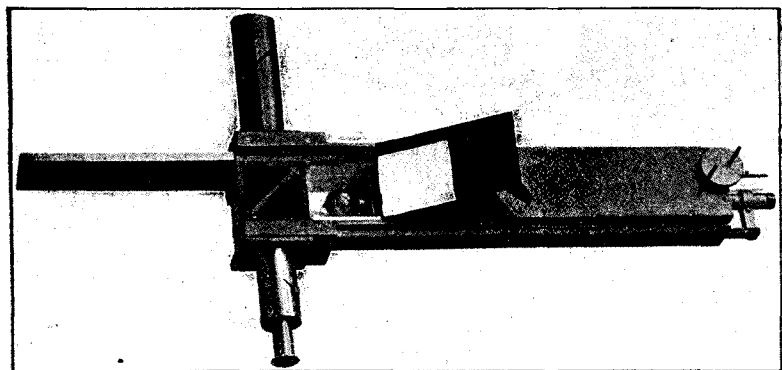


Diagrammatic Illustration of the Visibility Meter.

secondary axis of the instrument is a line perpendicular to the axis of sight at the point where the reflecting surface intersects the axis of sight. C is a diffusing member composed of pot or flashed opal glass placed perpendicular to the secondary axis. This diffusing surface is illuminated by a light source, S, mounted so as to move along the secondary axis. This source is mounted at one end of a brass tube, B, on one side of which is the rack, R_1 . A knurled hand wheel, N, is mounted rigidly on a shaft carrying also the pinion P_1 , which engages the rack R_1 , thus providing for the movement of the source along the secondary axis by a rotation of the hand wheel, N. A second pinion, P_2 , in mesh with P_1 , engages the rack R_2 , which is rigidly attached to the frame carrying the neutral gray non-diffusing optical wedge, W.

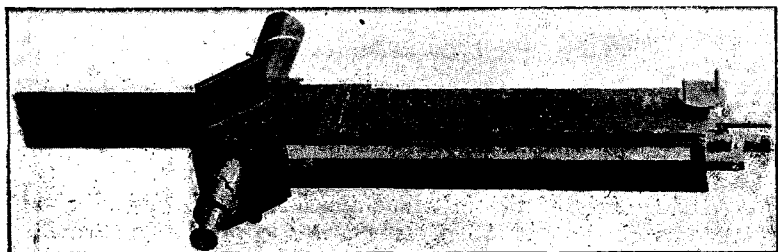
This wedge varies continually in opacity from one end to the other, and is so arranged as to move across the axis of sight. The object lens, L_1 , is of such focal length that the object of which the visibility is to be measured is imaged sharply at

Fig. 8.



The Visibility Meter Open.

Fig. 9.



The Visibility-Meter Closed.

the point where the reflecting surface, M , intersects the axis of sight. This lens is adjustable in position, so that either near or far objects may be imaged in the proper plane. The eye lens, L_2 , is of such power that the magnification of the system is unity, thus giving a retinal image of the same size as when the object is viewed with the naked eye. The lens L_2 enables the eye to see clearly defined the image formed by the lens L_1 . The mirror M may be replaced by a simple photometric field for the purpose of measuring brightness, it

being necessary to measure B_1 in order to obtain the visibility value. The details of the interchangeable field are shown in the small right hand drawing of fig. 7.

In order to increase the sensibility of the instrument in the measurement of B_v , a strip of clear gelatine film is placed over one-half of the field, thus increasing the reflecting power of that half by about 8 per cent. If a be the half of the field thus covered, it will for a given position of the source, S , cause a brighter veiling glare to appear as superposed over the object and background than the other half of the field, b . Now, if the object be so adjusted in the field of the instrument that the line between the two halves of the field bisects (approximately) the object, that part of the object in the field b will still be visible when the other part (that in field a) has been completely obscured by the veiling glare of that part of the field. In this way certain limits are set between which every setting must be made. If it is desired to decrease the difference in brightness between the two parts of the field (in order to narrow the limits of the setting), this may be done by making the reflecting powers of one-half of the field any desired amount greater than the other. This is easily accomplished if the reflecting surface is made by depositing the reflecting metal on the glass by a cathodic discharge.

The photometric field is made by covering one-half, c , of the small circular opening with a piece of brightly reflecting matt paper. The white paper is illuminated by the source, S , and the other half, d , being open, permits the eye to see the image of the object or background formed by the lens L_1 . The line between c and d (when the photometric field is in a position for use) lies at the intersection of the axis of sight with the secondary axis and hence is the plane in which the object is imaged by the lens L_1 . The circular field e surrounding the photometric fields, c and d , is made of grey matt paper. This is the type of field actually used in the instrument, but any of the well-known types, such as a Lummer-Brodhun cube, may be substituted if desired, the chief requirement being a sharp dividing line lying in the image plane. These two fields are mounted in a small metal frame sliding in milled metal ways placed perpendicular to the plane through the axis of the instrument. Stops are provided so that each field may easily and quickly be brought into position with the axis of sight passing through the centre of the field being used. A light filter is placed at F . This filter is of such quality that the light from the source S after passing through the filter matches in colour

the light which illuminates the object and background. A graduated scale, T , is fastened rigidly to the case which encloses the source. An index, I , attached to the source or its supporting member moves along this scale indicating at all times the position of the source, S , and wedge, W , with reference to the axis of sight. The scale is so calibrated that from the position of the index at any instant the brightness of the glare field B_1 and the transmission of the wedge (T_a) on the axis of sight can be determined. By turning the hand wheel N the source S and the wedge W are caused to move simultaneously and in such fashion that an increase in the brightness of the veiling glare, B_1 , is accompanied by a decrease in the transmission of the wedge of the axis of sight. This decrease in transmission causes a diminution in the intensity of the light which reaches the eye from the object and background.

Now the light transmitted by the diffusing member C is reflected into the eye, appearing to come from the image of that surface. Thus diffuse white light is caused to enter the eye from a point between the eye and the object being observed, increasing the apparent brightness of both object and background by the same amount and causing the ratio of B_1 to B_2 to be lowered. At the same time the wedge W is introduced causing a proportionate reduction in the apparent brightness of both B_1 and B_2 .

It will be seen, therefore, that the action of the instrument is exactly analogous to the action of material particles distributed through the foreground space, *i. e.*, the production of a veiling glare between the eye and the object and the absorption of a certain percentage of the light reflected or emitted by the object and background. Now, if the source and wedge be moved to such a position that the object is just visible in one part of the field and not visible in the other a setting is obtained from which the values of B_v and T_a can be determined.

In our fundamental equations, B_v was defined as the brightness of the veiling glare which when superposed over object and background will reduce the visibility to zero or a just perceptible value. It will be noted that as no term covering the decrease in brightness due to absorption of light in the foreground space appears in those equations, it is inferred that the entire loss of visibility is produced by the veiling glare, B_v . Now, the effect of the introduction of an absorbing member such as the wedge, W , is merely to decrease the amount of veiling glare required to reduce the visibility to zero. It is not feasible in practice to produce the extinction

of visibility by a veiling glare alone, due to the fact that the values of B_1 and B_2 are so high that a source of very high intensity would be required to give the required value of B_v . As it is desirable to make the instrument as portable as possible such sources cannot conveniently be used on account of the excessive weight of storage batteries required to operate them. By using an absorbing wedge in the axis of sight a much smaller lamp may be used. Such procedure does not in any way interfere with the correct determination of B_v . In order to obtain the maximum possible illumination on the diffusing member with a lamp of given energy consumption, the interior walls of the chamber inclosing the source, S , are painted white. This tends to increase the brightness of the diffusing member and also to increase the uniformity of illumination on this surface, which is extremely desirable. This painting of the interior walls prevents the use of the inverse square law in computing the illumination on the diffusing surface from known values of intensity of source and distance between source and surface. This, however, does not interfere in any way with the operation of the instrument.

For a given ratio of B_1 to B_2 the value of B_v required to produce a loss of visibility is directly proportional to the absolute values of B_1 and B_2 . Thus, by reducing the apparent brightness of B_1 and B_2 to one-tenth of their actual values by means of the member W , only one-tenth of the amount of veiling glare from the surface of M will be required to produce a given lowering of visibility. The brightness of the glare field of the instrument will be designated by B_v' and should not be confused with the term B_v appearing in the equations. The value of B_v is computed from those of B_v' and T_a , the transmission of the wedge W at the point through which passes the axis of sight.

The statement that the value of B_v , appearing in the fundamental equations, is directly proportional to B_1 or B_2 and hence inversely proportional to T_a rests upon a basic assumption which should be mentioned at this point. In order for this to be true, k , the contrast factor of the eye must remain constant. That is, the total field of brightness to which the eye is subjected must not change sufficiently to cause an accompanying change in k . This factor is satisfied in the instrument by so adjusting the density gradient of the wedge W and the linear velocity of the wedge relative to that of the source, that the total field of brightness B_v , of the instrument remains sensibly constant regardless of the

position of the members relative to the axis of sight. It is not possible to obtain exact constancy of B_t for all values of B_1 , but B_t can be kept within the range for which k is constant. However, in case B_t should vary beyond the specified range of values it is still possible to compute B_e , provided the resulting change in the value of k is known. If B_t is measured, which can easily be done, the corresponding values of k may be read from the curve in fig. 2. In practice it is found that B_t can be kept within the required limits in almost all cases by choosing a wedge of proper density gradient and by adjusting the number of teeth on the pinions so that the desired relative motions of S and W are obtained.

The above consideration shows that it is not necessary to exactly simulate in the instrument the relations existing in nature between the values of the veiling glare brightness and the opacity arising from the particles suspended in the foreground space. Since this relation is not constant for all natural conditions, being dependent up the nature, size, and spatial distribution of such scattering and absorbing particles, it would be quite impossible to make a single instrument exactly simulating all possible conditions resulting in lowered visibility.

Now it will be noted by referring to the theoretical treatment that

$$V = \frac{B_v}{B_1}.$$

It is necessary, therefore, to determine the value of B_1 . This is done by substituting for the glare field X, in fig. 7, the photometric field Y. The instrument being calibrated as a brightness photometer, the value of B_1 is read directly from the scale when a photometric balance exists between the fields c and d . The field d is filled by the image of the background.

In order to obtain a value of W , the weather coefficient, which is defined by the expression

$$W = \frac{B_1}{E_2}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (9)$$

it is necessary also to determine the value of E_2 , the illumination on the object plane. This may be done by measurement of the brightness, B_0 , of a surface on the object plane of which the reflexion factor R_0 is known. Such a surface is termed a test plane and is made by covering a frame of the proper size with canvas or sail cloth painted with several coats of a matt white paint. The reflexion factor, R_0 , of this surface is carefully determined by suitable laboratory methods. When determinations of visibility are to be made this test plane is fixed in the object

plane so that its brightness may be read from the designated point of observation.

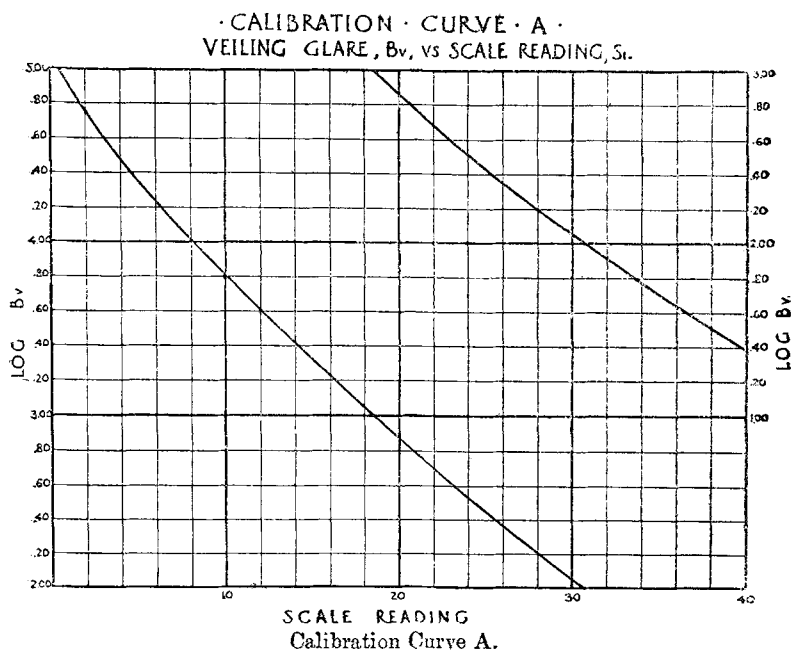
The procedure in taking a complete visibility reading then consists of three steps :—

(1) With the glare field in position the instrument is so set that the image of the object occupies a position in the field X, fig. 7, such that the dividing line between a and b approximately bisects the image. The hand wheel N is then turned until the object is just visible in field b , and is invisible in field a . The position of the index on the scale S is then read giving the scale reading S_1 .

(2) The photometric field is thrown into position and the instrument so aligned that the image of the background fills the portion d of the field Y, fig. 7. N is then turned until d and c are equal in brightness and the position of the index being read gives the scale reading S_2 .

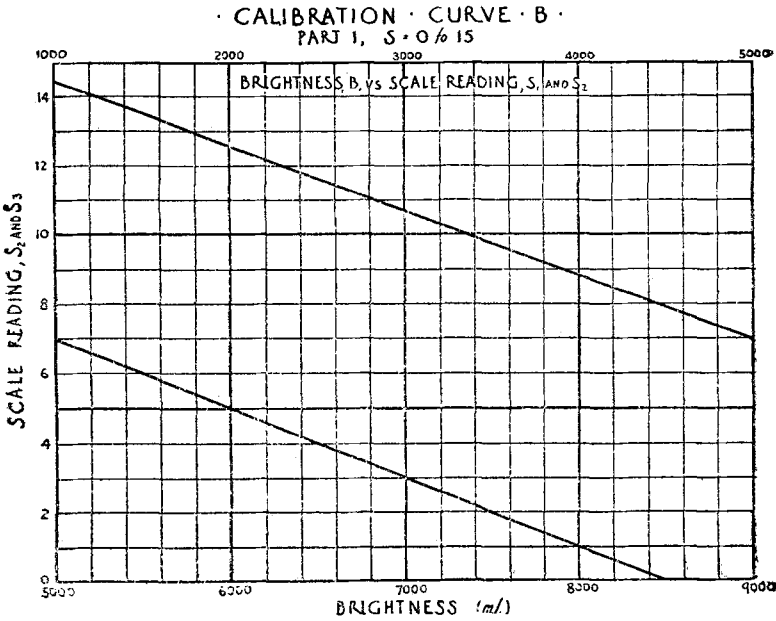
(3) The alignment of the instrument is changed so that the image of the test plane fills the field d and a photometric balance is again made by turning N. The position of the index now gives the third scale reading S_3 .

Fig. 10.



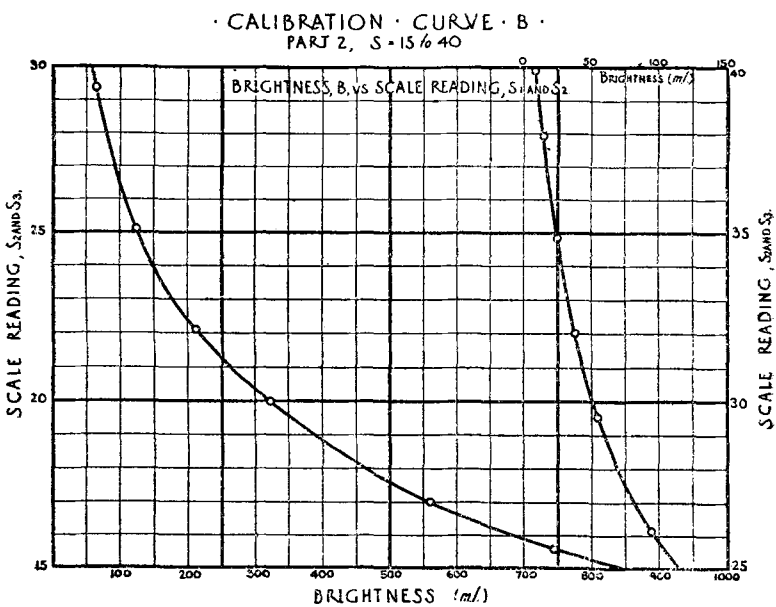
Two calibration curves for the instrument must be prepared, shown in figs. 10, 11, and 12. Curve A is for veiling

Fig. 11.



Calibration Curve B, Part I.

Fig. 12.



Calibration Curve B, Part II.

glare, the curve having $\log B_v$ values as ordinates and scale readings as abscissæ. Curve B is for brightness. It has scale readings as ordinates and brightness values as abscissæ. By the use of S_1 and Curve A, B_v is obtained, and by using Curve S_2 and Curve B, the value of B_1 is found. S_3 in conjunction with Curve B gives B_0 .

The relations from which the Curves A and B are obtained and the methods of computing the final values of visibility from the observed values are given below. The values read directly are the scale readings :

S_1 = Visibility setting,

S_2 = Photometric setting on background,

S_3 = Photometric setting on test plane.

By the use of the Curves A and B :

S_1 gives B_v = Brightness of veiling glare,

S_2 gives B_1 = Brightness of background,

S_3 gives B_0 = Brightness of test plane.

Other terms, the values of which must be previously determined, are :

T_a = Transmission of wedge, W,

T_m = Transmission of mirror, M,

R_m = Reflexion factor of mirror, M,

R_0 = Reflexion factor of test plane,

B_c = Brightness of upper surface of diffusing member, C.

The terms of which the values may be computed from the available data are :

V = Total visibility,

W = Weather Coefficient,

B_t = Total field brightness,

B_v' = Brightness of glare field instrument.

E_2 = Illumination on object plane.

Some of the relations existing between the various terms are given below :

$$B_v = \frac{B_v'}{T_a \cdot T_m},$$

$$B_v' = B_c \cdot R_m,$$

$$B_v = \frac{B_c \cdot R_m}{T_a \cdot T_m} \quad \dots \quad (13)$$

T_m and R_m are constants, while B_c and T_a are variable with the value of the scale reading. The relation between the scale reading and B_c and R_m must be determined, after which equation (13) is used for obtaining the values of $\log B_v$, from which in turn the Calibration Curve A is plotted.

$$\begin{aligned} V &= \frac{B_1}{B_v}, \\ W &= \frac{B_1}{E_2}, \\ E_2 &= \frac{B_0}{R_0} \dots \dots \dots (14) \\ \therefore W &= \frac{B_1 \cdot R_0}{B_0}. \end{aligned}$$

R_0 is a constant. B_1 and B_0 are determined from S_2 and S_3 in connexion with the Calibration Curve B. Therefore W may be computed. W and V are thus determined and provide the necessary data for plotting the total visibility as a function of the weather condition, which may then be compared with the theoretical curves shown in fig. 3. If the objects on which the measurements were made were of the same colour as the background (the visibility being entirely due to brightness contrast) those experimental curves should coincide with the theoretical ones applying to the same conditions. If a colour (either hue or saturation) difference exist, the difference between the measured and computed values will be the part of the total visibility due to quality contrast. In this way the form of the expression $V_q = f(W)$ may be evaluated graphically,

$$B_t = B_e' + B_1 \cdot (T_a \cdot T_m) \dots \dots \dots (15)$$

By computing the value of B_t by this equation for various scale readings its constancy or variation can be determined. This will show, in connexion with fig. 2, whether or not k remains constant and if not will provide the required data for the determination of its value. Thus any error introduced by a variation of k may be eliminated.

Attention should again be called to the analogy between the method employed in the instrument for reducing the visibility to a zero value and the phenomenon of lowered visibility in nature. Given an object seen against the sky and at some distance from the observer. Now let a cloud or mist or any collection of small material particles be formed

in the foreground space of such concentration or density that the object is just visible. In the terminology previously adopted we may state this condition by the equation

$$\frac{(B_1 \cdot T_d) + P_d}{(B_2 \cdot T_d) + P_d} = c. \quad . \quad . \quad . \quad . \quad (16)$$

In the instrument the condition when a visibility setting is made is given by

$$\frac{B_1 + B_v}{B_2 + B_v} = c,$$

where $B_v = \frac{B_v'}{T_a \cdot T_m},$

$$\therefore \frac{B_1(T_a \cdot T_m) + B_v'}{B_2(T_a \cdot T_m) + B_v'} = c. \quad . \quad . \quad . \quad . \quad (17)$$

Equations (16) and (17) are of exactly the same form. The term T_d in (16), which is the transmission of the foreground space, is replaced in (17) by $(T_a \cdot T_m)$, the transmission of the wedge and mirror of the instrument. The term P_d in (16), which is the brightness due to light reflected or refracted by the particles in the foreground space, is replaced in (17) by B_v' , the brightness of the glare field of the instrument. The constant c is the same in both cases, being equal to

$$k \quad \text{or} \quad \frac{1}{k},$$

depending upon whether B_1 is greater or less than B_2 .

As was pointed out previously the exact relation between the values of T and P varies greatly with the nature of the particles producing the diffusion in the foreground space.

Since the relation is subject to so much variation no attempt was made in the design of the instrument to imitate any particular kind of natural fog or haze. Another factor, however, must be considered in fixing the relation of B_v' to $(T_a \cdot T_m)$, this being the necessity for keeping the total field brightness $(B_t = B_v' + B_1(T_a \cdot T_m))$ practically constant for all possible values of the scale reading. Another point requiring consideration was the necessity for keeping the light source sufficiently small, with respect to voltage and current consumption, to permit of convenient operation by easily portable batteries.

A large amount of data relative to the visibility of objects

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has been obtained; the greater part of this refers to the visibility of small boat models camouflaged in various ways. As was previously stated, it is not the object of this paper to present the results of this work. However, in order to convey a more definite idea of the performance of the visibility meter a single set of data and one pair of curves will be included. In Table I., under the heading F 20, are

TABLE I.

F 20.		$R_2 = .43.$	
W.	V.	W.	V.
.40	1.5+	.10	81.5+
.47	2.5—	.20	27.8+
.70	10.0—	.25	17.0+
.90	11.5—	.30	9.9+
.30	12.5+	.35	4.6+
.34	5.0+	.40	0.9+
.25	2.5+	.414	0.0
.42	2.5+	+0	0.0
.38	4.0+	.447	0.0
.55	5.5—	.50	2.6—
.42	0.9+	.60	6.3—
.45	1.0—	.70	9.0—
.78	9.0—	.80	11.0—
.35	8.5+	.90	12.6—
.50	3.0—	1.00	13.8—

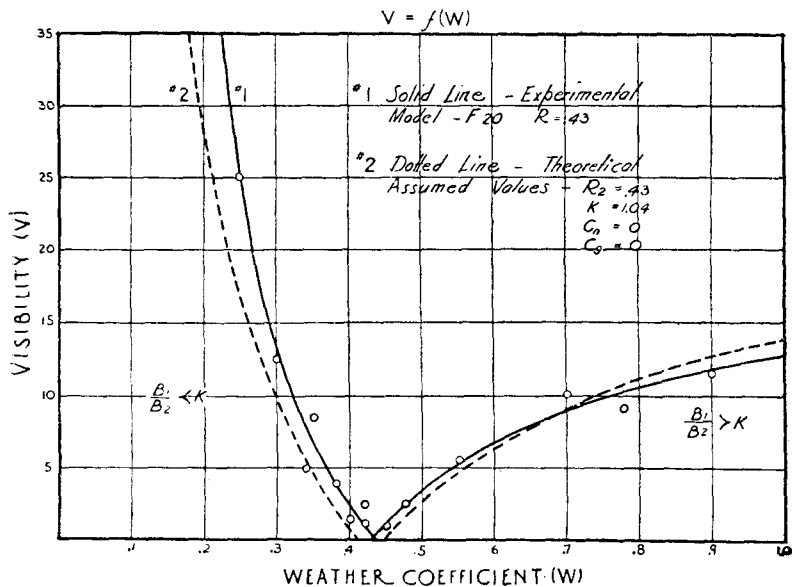
given the visibility values, V, for various weather coefficients, W. The model used was painted a flat bluish-green gray and was of the lowest average visibility for the period covered by the observations. The specifications of colour and reflecting power for this model are :

Reflexion Factor.....	.43
Wave-Length of Dominant Hue ...	488 $\mu\mu$
Saturation	88 p. c. white.

Many more observations than are given in Table I. were made but those presented are considered as typical and cover the maximum range of W values encountered. In the columns under the heading $R_2 = .43$ are the theoretical values of V corresponding to a series of assumed values for W, and $R_2 = .43$. The observed and computed values are plotted in fig. 13. The fact that the minimum visibility of the model F 20 was practically zero shows that the hue contrast, C_h , and saturation contrast, C_s , between model and background must

have been very small. Hence the curve plotted from the observed values should agree quite closely with the theoretical curve $V=f(W)$ for brightness contrast alone. It will be noted that the curves agree fairly well in the region

Fig. 13.



Comparison of Observed with Computed Values.

$W=.45$ to $W=.90$, but that more marked differences exist for lower values of W . This is probably due to the presence of quality contrast which existed to a greater extent under those weather conditions resulting in low values of W . Such values of W usually denote clear sky and bright sunlight, and although the colour of the background may be constant the apparent colour of the object will be changed, due to the difference in quality of the incident illumination.

Although the greater part of the work done with this instrument relates to the visibility of ships as seen against a sky background it is evident that the same method can be applied to the evaluation of the visibility of other objects under various conditions of background and illumination. While it is considered that the general principles of the method are applicable to all problems of this nature it is

recognized that its use in other cases may require modification of the details of the instrument, experimental procedure, and of the reduction of the data to the most useful form.

One problem to which this method has already been applied with considerable success is the measurement of the increase or decrease in the visibility of objects resulting from the use of colour filters. The change of visibility in such cases is obtained by the adjustment of the selective absorption to fit the requirements and the condition under which they are used. The enhancement of visibility by use of colour filters usually depends upon their ability to increase either the hue or saturation contrast or both. The increase of visibility in such cases can be determined quantitatively by first making one reading in the usual way without a filter and one after having inserted the filter between W and L_1 (fig. 7).

The complete interpretation of such results requires an extension of the theory to cover the evaluation of visibility in terms of hue and saturation contrast and cannot be presented at this time. In conclusion, the author desires to express his sincere thanks to Dr. C. E. K. Mees and to Mr. Lindon W. Bates for their many helpful suggestions and constant encouragement given through the course of this investigation.

Research Laboratory,
Eastman Kodak Company,
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IX. *A Note on the Equivalent Shell of a Circular Current.*

By SATYENDRA RAY, M.Sc., B.A., *Lecturer in Physics, Canning College, and Senior Science Master, La Martiniere College, Lucknow* *.

§ 1. *Introduction.*

THE magnetic induction of a circuit at any point is identical in magnitude and direction with that due to a magnetic shell bounded by the circuit, the strength of the shell being numerically equal to the current. The equivalence is, however, true only for points not close to the magnetic shell. [*Vide* Maxwell's 'Electricity and Magnetism,' vol. ii. §§ 482-484.]

The shell is defined as magnetic matter magnetized in a direction everywhere normal to its surface. This together with the law of refraction of lines of induction, viz. $\frac{\tan \theta_1}{\mu_1} = \frac{\tan \theta_2}{\mu_2}$, and the law that for a tube of induction

* Communicated by Prof. D. N. Mallik, Sc.D., F.R.S.E.