Metamodels based on deterministic and stochastic radial basis functions for engine noise shielding of innovative aircraft

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Scope of the work

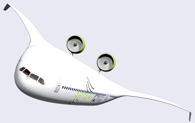
- Analyse the potential of meta–modelling techniques based on Radial Basis Functions (RBF) in **aeroacoustics**
- Develop dynamic meta–models for high-efficiency **optimisation** in presence of **aeroacustic objectives and constraints**.
- Estimate the **uncertainty related to breakthrough technologies** in general–purpose analysis tools.

The context

- Sustainable development of civil aviation is strongly **noise–constrained**
- Aeroacoustics must be considered in the conceptual design phase
- Simple noise models are not available for innovative concepts

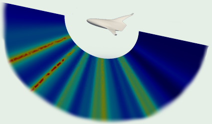
ARTEM (Aircraft noise Reduction Technologies and related Environmental iMpact)

Robust MOCDO of unconventional configurations including low–noise objectives and/or constraints



ANIMA (Aviation Noise Impact Management through Novel Approaches)

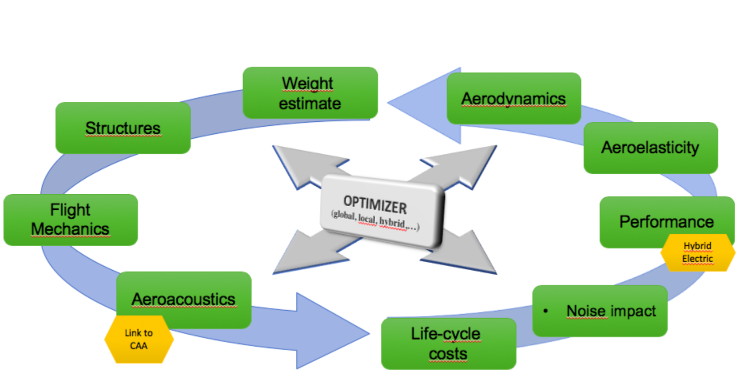
Stand–alone models to include new technologies and concepts in impact management and analysis tools



The tool

FRIDA (FRamework for Innovative Design in Aeronautics)

Multi–Objective, Multi–disciplinary Robust Design Optimization environment developed by **Roma Tre Aircraft Design Group** for classic (T&W) and innovative (BWB, PP) configurations

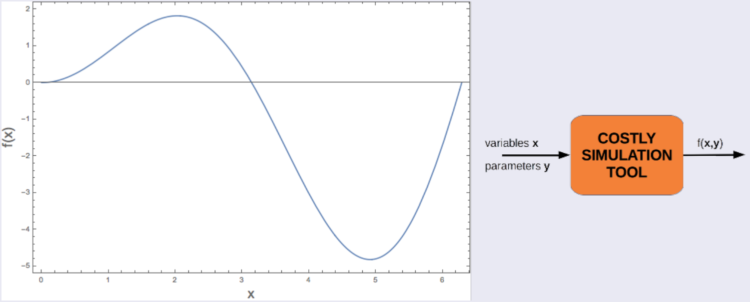


Summary

- Meta–Models (MM) definition
- RBF–based deterministic and adaptive–stochastic MM
- Simple 1D benchmark
- An early application to shielding (1D and 2D)
- Current activity

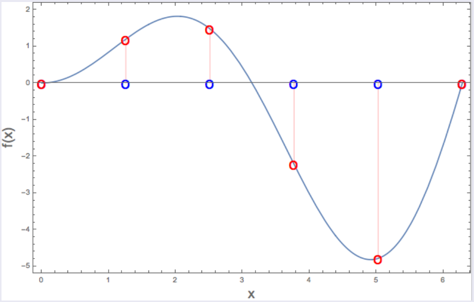
Meta-model = the model of a model

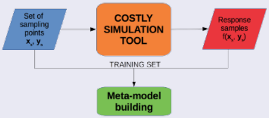
In our context: a fast model reproducing the response of a costly simulation



Meta-model = the model of a model

The *Training Set (TS)* gives the response at a set of points

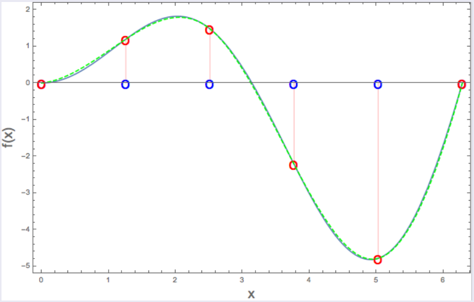


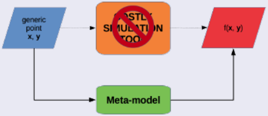


Meta-models for aeroacoustic Simulation–Based Design

Meta-model = the model of a model

The *Meta–Model (MM)* reproduces the response at any $\mathbf{x} \in \mathcal{D}_{TS}$





Meta-models for aeroacoustic Simulation–Based Design

Accuracy is strongly application–dependent

- Location and number of TS points
- Properties of the target response $f(\mathbf{x}, \mathbf{y})$
- Characteristics of the surrogate model $\hat{f}(\mathbf{x}, \mathbf{y})$

Many different approaches are available . . .

In the present work we focus on **Radial Basis Fuctions (RBF)**

- Simple implementation
- Demonstrated effectiveness in medium– to high–dimensional problems
- Versatility: the choice of the RBF kernel makes possible the *tailoring* of the MM

Deterministic RBF MM

RBF MM

Given a training set TS of M points $[\boldsymbol{\xi}_i, f(\boldsymbol{\xi}_i)]_{i=1}^{M}$, with $Dim(\boldsymbol{\xi}) = N$, the RBF model of the sampled response is

$$\hat{f}(\boldsymbol{\xi}) = \sum_{i=1}^{M} w_i \, \varphi\left(|\boldsymbol{\xi} - \boldsymbol{\xi}_i|\right)$$

Weights w_i are obtained by imposing the reproduction of TS, $\mathbf{A}\,\mathbf{w} = \mathbf{f}$, with $[\mathbf{A}]_{ij} = \varphi\left(\left|\boldsymbol{\xi}_i - \boldsymbol{\xi}_j\right|\right)$.

RBF Kernels

Kernel choice is a key point (Gaussian $\varphi(r) = e^{-(\gamma\, r)^2}$, Inverse quadratic $\varphi(r) = 1/\left[1 + (\gamma\, r)^2\right] \ldots$). For the moment, let's start with simple polyharmonic splines

$$\varphi(r) = r^{\epsilon}, \qquad \epsilon = 1, 3, 5, \ldots$$

Deterministic RBF MM

RBF tuning

Specifically, we will use the cubed Euclidean distance

$$\varphi\left(|\boldsymbol{\xi}-\boldsymbol{\xi}_i|\right)=\left[\sqrt{\sum_{k=1}^{N}\left(\xi^k-\xi_i^k\right)^2}\right]^3$$

The RBF sensitivity to local curvature can be mitigated with an auto–tuning procedure

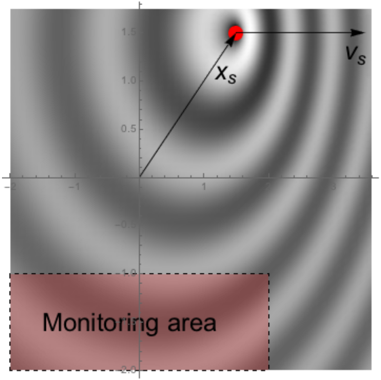
$$\varphi\left(|\boldsymbol{\xi}-\boldsymbol{\xi}_i|\right)=\left[\sqrt{\sum_{k=1}^{N}c_k^2\left(\xi^k-\xi_i^k\right)^2}\right]^3$$

where c_k is a function of max local curvature.

A simple benchmark

The problem

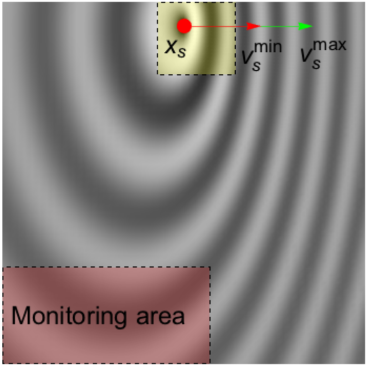
- **Target**: field induced by a moving isotropic point source in a co–moving region
- **Design variable**: position of the source $\mathbf{x}_s$
- **Parameters**: Mach number M_s, observer location $\mathbf{x}_M$

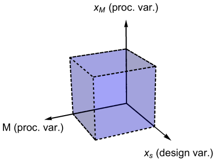


A simple benchmark

Design and Training spaces

- **Design space**: region of the physical space where the source can be located
- **Training space**: region of the abstract space of all the possible *experiments*

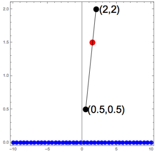




$$\xi = \left\{ \begin{array}{c} \mathbf{x}_s \\ M_s \\ \mathbf{x}_M \end{array} \right\}$$

A simple benchmark

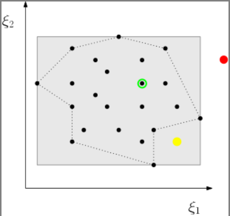
- $\mathbf{x}_s \in [(0.5, 0.5), (2, 2)]$
- A line of N microphones along $z = 0$
- $M_s \in (0.2, 0.4)$



Here, the training set comprises $N_s = 5$ source positions, $N = 40$ monitoring points and 3 values for Mach.

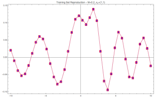
Number of training experiments is $N_p = 600$

Off–set and off–domain evaluations

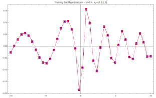


- TS reproduction
- Off-set prediction
- Off-domain prediction

A simple benchmark

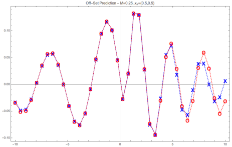


$M_s = 0.2$, $\mathbf{x}_s \equiv (1.5, 1.5)$

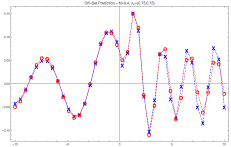


$M_s = 0.2$, $\mathbf{x}_s \equiv (0.5, 0.5)$

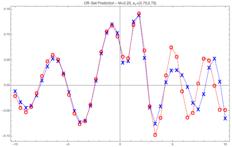
Training set reproduction



$M_s \notin TS$, $\mathbf{x}_s \in TS$

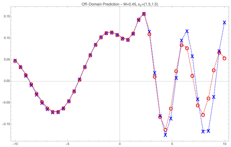


$M_s \in TS$, $\mathbf{x}_s \notin TS$

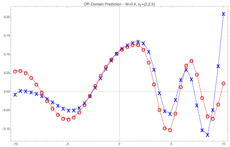


$M_s \notin TS$, $\mathbf{x}_s \notin TS$

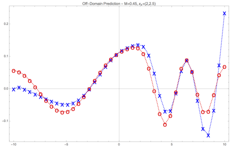
Off set prediction



$M_s \notin \mathcal{D}_T$, $\mathbf{x}_s \in \mathcal{D}_T$



$M_s \in \mathcal{D}_T$, $\mathbf{x}_s \notin \mathcal{D}_T$



$M_s \notin \mathcal{D}_T$, $\mathbf{x}_s \notin \mathcal{D}_T$

Off domain prediction

So, for deterministic MM…

- Definition of the *best* TS is not a trivial task (D.O.E.? …EXPENSIVE !)
- Verification of MM accuracy needs the time–consuming model to be run
- Improvement of the MM can be a resource–draining task

So, for deterministic MM. . .

- Definition of the *best* TS is not a trivial task (D.O.E.? . . . EXPENSIVE !)
- Verification of MM accuracy needs the time–consuming model to be run
- Improvement of the MM can be a resource–draining task

Let's go
DYNAMIC, ADAPTIVE and STOCHASTIC!

Stochastic RBF MM

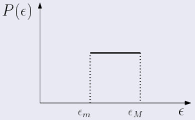
Stochastic RBF

$$\varphi\left(|\boldsymbol{\xi}-\boldsymbol{\xi}_i|\right)=\left[\sqrt{\sum_{k=1}^{N}\left(\xi^k-\xi_i^k\right)^2}\,\right]^{\epsilon}, \quad \epsilon\sim Unif\left[\epsilon_{min},\epsilon_{max}\right]\equiv\mathcal{D}_\epsilon$$

Stochastic MM

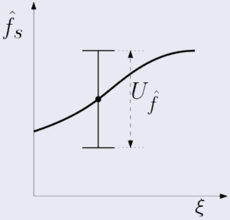
Is the expected value EV of $\hat{f}$ over ϵ

$$\hat{f}_s(\boldsymbol{\xi})=EV\left[\hat{f}(\boldsymbol{\xi},\epsilon)\right]=\int_{\mathcal{D}_\epsilon}\hat{f}(\boldsymbol{\xi},\epsilon)\,P(\epsilon)\,\mathrm{d}\epsilon$$



MM Uncertainty

Each estimate of $\hat{f}_s(\xi)$ is associated to an uncertainty $U_{\hat{f}}(\xi)$



It is defined as the difference of the relevant α–quantiles

$$U_{\hat{f}}(\xi) = q(\alpha_1, \xi) - q(\alpha_2, \xi) = CDF^{-1}(\alpha_1, \xi) - CDF^{-1}(\alpha_2, \xi)$$

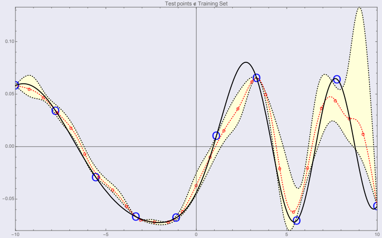
with

$$CDF(y, \xi) = \int_{\mathcal{D}_\epsilon} H[y - \hat{f}(\xi, \epsilon)]\, P(\epsilon)\, \mathrm{d}\epsilon$$

Dynamic–Adaptive MM

MM quality

$U_{\hat{f}}(\xi)$ can be used to measure the local reliability of the MM



A dynamically adaptive MM can be built

1. Build the MM on the current TS
2. Search for $Max[U_{\hat{f}}(\xi)], \xi \in \mathcal{D}_T$
3. Increase TS with new point at U_{max} and update MM (with the costly model)
4. Stop when $U_{max} \leq U_{conv}$

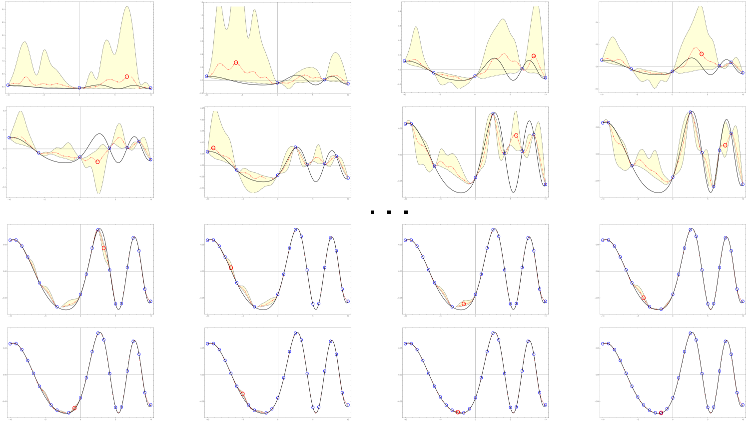
The simple benchmark

Same as before, but now with dynamic, stochastic approach

- $U_{\hat{f}}(\xi) = q(0.975, \xi) - q(0.025, \xi)$ (95% confidence band)
- $U_{conv} = 10^{-5}$
- Initial TS with $M = 3$
- Monte Carlo method with 15 random samples for $\epsilon \in [1, 3]$

A simple benchmark

Progressive update of TS



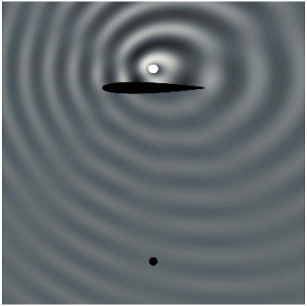
A simple benchmark

- Additional samples only where needed (high uncertainty)
- Uncertainty quantification using the MM $\Longrightarrow$ FAST !
- Minimises the calls to the high–fidelity model (only TS update)
- Once that $U_{\hat{f}} < \epsilon$ a deterministic model (faster, no Monte Carlo) can be built on the converged TS

A simple 1D shielding exercise

The problem

- **Target**: ΔSEL at a monitoring point located 2 chords underneath a NACA 0012 foil
- **Design variable**: position of the source along the chord, x_s at 0.1 chord above the foil



The TS is one–dimensional and coincides with $\mathcal{D}$

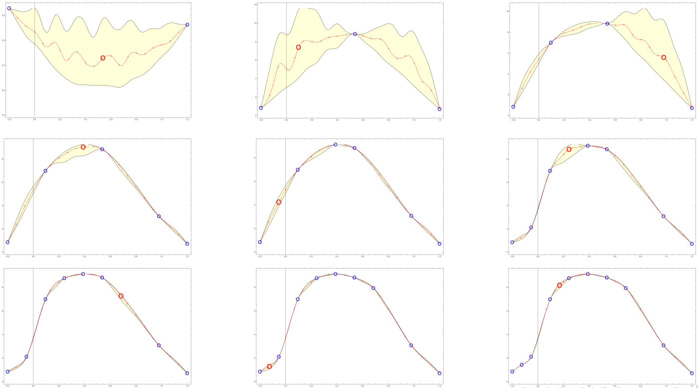
$$\xi = x_s$$

A simple 1D shielding exercise

The TS is updated when $U_{max} \leq 0.001$

- $U_{\hat{f}}(\boldsymbol{\xi}) = q(0.975, \boldsymbol{\xi}) - q(0.025, \boldsymbol{\xi})$ (95% confidence band), $U_{conv} = 10^{-5}$
- Monte Carlo method with 15 random samples for $\epsilon \in [1, 3]$
- Airfoil scattering calculated with in-house convective 2D BEM code

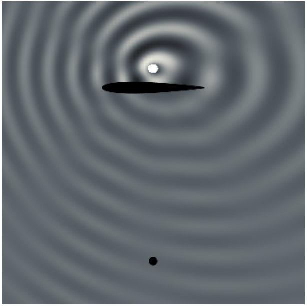
Progressive update of TS

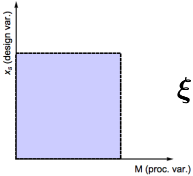


A simple 2D shielding exercise

The problem

- **Target**: ΔSEL at a monitoring point located 2 chords underneath a NACA 0012 foil
- **Design variable**: position of the source along the chord, x_s at 0.1 chord above the foil
- **Parameter**: Mach number M_s of the uniform stream



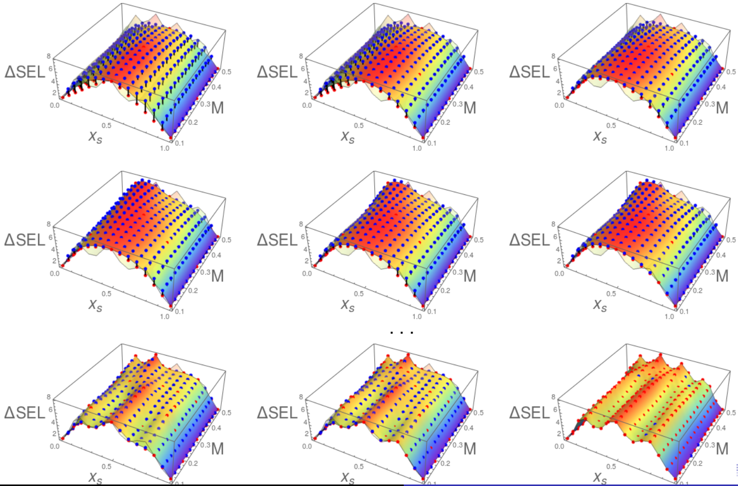


$$\xi = \left\{ \begin{array}{c} x_s \\ M_s \end{array} \right\}$$

A simple 2D shielding exercise

Same procedure: the TS is updated when $U_{max} \leq 0.001$

- $U_{\hat{f}}(\xi) = q(0.975, \xi) - q(0.025, \xi)$ (95% confidence band), $U_{conv} = 10^{-5}$
- Monte Carlo method with 15 random samples for $\epsilon \in [1, 3]$



Current activity

- Tailored RBF kernel (oscillating, decaying, complex . . .)
- Selection of appropriate stochastic parameters
- High–dimensional training spaces
- Adaptive strategies for dynamic update

Concluding remarks

- The work is a preliminary analysis of modern meta–modelling techniques applied to aeroacoustic problems
- the general approach adopting RBF with standard polyharmonic kernels appears to be promising
- the potentiality of tailored RBF kernels deserves a careful investigation to be completely disclosed

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- The work is a preliminary analysis of modern meta–modelling techniques applied to aeroacoustic problems
- the general approach adopting RBF with standard polyharmonic kernels appears to be promising
- the potentiality of tailored RBF kernels deserves a careful investigation to be completely disclosed

Thank you !