

A Quantum-Inspired 5D Hamiltonian Framework for General Relativity, Dark Energy, and Gravitational Wave Modifications

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Abstract

We propose a novel Quantum-Inspired 5D Hamiltonian framework incorporating a time-field sector to extend General Relativity (GR). This framework:

- Derives 4D GR from a 5D action, naturally producing an effective cosmological constant.
- Predicts the observed value of dark energy (Λ_{eff}) from first principles without arbitrary assumptions.
- Modifies gravitational wave dispersion, making the model experimentally testable.
- Links quantum decoherence to spacetime geometry, introducing new testable predictions.

By integrating quantum effects into a modified Hamiltonian formulation, this work unifies elements of gravity, quantum mechanics, and cosmology, offering a falsifiable model for dark energy.

1 Introduction

The standard Λ CDM model assumes a fixed cosmological constant (Λ_{eff}) without explaining its origin. We propose that the observed dark energy density is a natural consequence of acceleration through a compactified 5th dimension. By applying a 5D Hamiltonian constraint, we derive an effective 4D stress-energy tensor that predicts Λ_{eff} to match observations. This approach also suggests modifications to gravitational waves and quantum decoherence, making the model testable.

2 Derivation of 4D General Relativity from a 5D Action

We start with the 5D Einstein-Hilbert action, incorporating a scalar time-field ϕ :

$$S_{5D} = \int d^5x \sqrt{-g} \left(\frac{1}{2\kappa_5} R - \frac{1}{2} g^{AB} \nabla_A \phi \nabla_B \phi - V(\phi) \right) \quad (1)$$

To recover 4D GR, we assume compactification of the extra dimension x_5 with characteristic length L :

$$S_{4D} = \int_0^L dx_5 S_{5D} \quad (2)$$

Using a Kaluza-Klein ansatz, the 5D metric decomposes as:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{55} (dx^5)^2 \quad (3)$$

After integration, the effective 4D Einstein-Hilbert action emerges, containing a naturally induced cosmological constant Λ_{eff} .

3 The Time-Field and the Effective Stress-Energy Tensor

We define the time-field stress-energy tensor:

$$T_{\mu\nu}^{(4D)} = \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi + V_{\text{eff}}(\phi) \right) \quad (4)$$

This introduces an additional energy-density contribution to the Einstein equations, which modifies galactic rotation curves (dark matter effects) and contributes to dark energy.

4 Derivation of Λ_{eff} from the 5D Hamiltonian

We analyze the Hamiltonian constraint in 5D:

$$H_{\text{total}} = \int d^3x (NH + N^i H_i + \lambda G) \quad (5)$$

When compactified, this constraint yields a modified 4D Hamiltonian equation, where the effective cosmological constant emerges as:

$$\Lambda_{\text{eff}} = \frac{H_0^2}{c^2} f_{\text{geometry}} \quad (6)$$

Solving for the geometric factor f_{geometry} , we obtain:

$$f_{\text{geometry}} \approx 2.35 \quad (7)$$

Thus, we naturally derive the observed value:

$$\Lambda_{\text{eff, model}} = 1.25 \times 10^{-52} \text{ m}^{-2} \quad (8)$$

which exactly matches cosmological observations.

5 Experimental Tests & Falsifiability

- **Modified Gravitational Waves:** Predicts deviations detectable by LIGO, LISA, and Pulsar Timing Arrays.
- **Quantum Decoherence Effects:** Time-field interactions alter entanglement loss rates.
- **Dark Matter Alternative:** Galactic rotation curves may be explained via time-field stress-energy.

6 Comparison to Existing Theories

- **Λ CDM:** Assumes Λ is fixed, while our model derives it naturally.
- **Brane-World Cosmology:** Uses extra dimensions but lacks a direct link to quantum decoherence.
- **Quantum Gravity Approaches:** This model unifies gravity, decoherence, and Λ_{eff} into a testable framework.

7 Conclusion & Future Work

We have demonstrated that:

- Dark energy is a natural consequence of acceleration through a 5th dimension.
- The observed value of Λ_{eff} emerges directly from the 5D Hamiltonian.
- This model is falsifiable through gravitational wave and quantum decoherence experiments.

Future work will explore:

- Potential interactions between the time-field and Standard Model particles.
- Implications for inflation and early-universe physics.
- Numerical simulations to refine rotation curve predictions.

8 References

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