

Proceedings of 7th Transport Research Arena TRA 2018, April 16-19, 2018, Vienna, Austria

Towards a microeconomic theory of For-Hire Services

Fabien Leurent a, Jaâfar Berrada b*

^a Université Paris Est, LVMT & ENPC, 6-8 Avenue Blaise Pascal, F-77455 Marne la Vallée Cedex 2, France

^b VEDECOM Institute & LVMT, 77 rue des Chantiers, 78000 Versailles, France

Abstract

For-hire services have emerged as promising solutions to meet mobility demand in customized ways. Yet, they still have to achieve profitability: this requires developing the service quality to achieve attractiveness to potential users. Among quality factors, the availability in time of space of modal resources such as vehicles and parking slots is prominent. It follows that models targeted to aid decision-making in the planning and management of for-hire services have to deal with availability as an endogenous property that varies over time and space and depends on the real-time disaggregate conditions of resource occupancy and trip demand.

This paper brings about a microeconomic model of supply and demand for a for-hire service in which availability is represented explicitly. The specific function is formulated under a particular form related to a stylized urban area. After providing the model formulation, which involves probabilistic assumptions and calculus, we use it to investigate the issues of demand-supply equilibrium, service profitability and business model optimization.

Keywords: availability time, spatial modelling, traffic equilibrium, market equilibrium, social optimum, profitability

* Corresponding author. Tel.: +33-669-017094.
E-mail address: jaafar.berrada@enpc.fr

1. Introduction

1.1. Background

The rapid development of wireless communication technologies have enabled the emergence of various taxi hailing apps (i.e. Uber, Lyft, Ola...). The transaction time is incredibly reduced and the costs for suppliers and users are significantly decreasing. As a result, the demand has been boosted and it is strongly believed that the taxi market efficiency will be improved. This transformation has then motivated several researches, which are investigating the conditions of optimizing the matching between cabs and riders while ensuring the economic and social sustainability of the service.

Therefore, researches concerned by taxi service issues have investigated the regulation and pricing strategies while integrating the waiting time of customers as a demand variable (Douglas, 1972; De Vany, 1975; Manski and al., 1976; Hackner and al., 1995; Cairns and al., 1996). In 1998, Yang and Wong introduced a taxi model applied at a network level (Yang, and al., 1998). The traffic equilibrium optimizes the utilization of taxis with reducing their searching time of customers for a given distribution of demand. The model was developed progressively to include traffic congestion (2001), elasticity demand (2001), social welfare (2002) and externalities (2005). In 2010, Yang and coauthors explicitly introduced a meeting function to investigate the searching and meeting frictions between customers and taxi drivers (Yang, and al., 2010; 2016). The meeting rate and the number of waiting customers jointly determine the customer waiting time. In other terms, they consider that the customers' waiting time is not only dependent on the taxi availability but also on the number of waiting customers. In next years, Yang and coauthors used elasticities of the meeting function to the number of vacant taxis and the number of waiting customers to analyze the service profitability and the social optimum for the monopoly. The model is applied on a simplistic network.

1.2. Objective

This paper is devoted to specify the meeting efficiency by defining the availability function of a private taxi service. The specific function is the minimum time for a customer to get a cab. We formulated the function for a stylized urban area, that we called Orbicity. The area is in form of a ring where origin and destination trips are distributed uniformly. The sensitivities of the demand to the fare level and availability time are introduced. Further, the service production process is defined to meet efficiently the demand. Based on the constructed model, we investigate the conditions of the traffic equilibrium. Then we turn to analyze the optimal business model profitability for three market's configuration: the monopoly, the social optimum and the second-best optimum. We bring out simple analytical formulas to compare these three markets.

1.3. Method

Our model combines the spatial modelling of the traffic and the microeconomic modelling of the taxi service. We built progressively the spatial model by defining the geographic parameters, the demand distribution and the technical process of the service production. Then, we construct a microeconomic model where the rules of assignment taxis to users is defined, and the production costs are defined per cab and unit of time. Finally, combining these two models under stationary operating conditions we analyze the service profitability for operators and regulators.

1.4. Structure of paper

The rest of the paper is organized in four parts. First, we describe the geographic model and present the supply-demand model of a for-hire service with emphasis on availability. We define the according fleet size, the production costs and the traffic equilibrium. Secondly, we briefly describe the problem of the monopoly. We analyze the profitability conditions and the optimal fare level, access time and fleet size. Then, we consider that the regulator ensure the social welfare and investigate the impact on the operator. Observing that the demand volume is reduced, we studied the second best optimum under budget constraint.

2. Supply demand model

2.1. Service production

Consider a city in the form of a ring with radius R ; let us call the city Orbicity. The demand volume Q is distributed uniformly along the edges of the ring. It is generated along the study period H according to the ratio $\lambda = Q/H$. Let N the fleet size. The cab busy times include the ride times, say t_R on average per trip, plus the transaction time t_T and the access time, denoted t_A . The latter is called the availability time. At instant h , let note n the number of occupied taxis and k^+ the number of vacant taxis: $k^+ \equiv N - n \geq 0$. If $k^+ = 0$ then the client is put in the waiting list. Denote $\theta \in]-\pi, +\pi[$ the angular deviation between the position of vacant taxis and the position of the new request. Specifically, assume that the vacant taxis are located at the destination of the previous rider. Since the origin and destinations are distributed uniformly, then $\forall i \in \{1, 2, \dots, k^+\}$, θ_i also distributed uniformly in $]-\pi, +\pi[$. The distance L_i between the vacant taxi i and the client is equal to $L_i = R|\theta_i|$ where $|\theta_i|$ is uniformly distributed in $]0, \pi[$. Then the cumulative distribution function is

$$F^{(i)}(x) \equiv \Pr\{L_i \leq x\} = F_{AR}(x) \text{ avec } F_{AR}(x) = \frac{\min\{x, \pi R\}}{\pi R} 1_{\{x \geq 0\}}. \quad (1)$$

The nearest vehicle is located in the distance $L_{\min} \equiv \min\{L_i : i \in \{1, 2, \dots, k^+\}\}$

Since the destination points have the same probability to be chosen by the user, then the vacant taxis have independent positions which are distributed uniformly according to the function $F_{AR}(x)$. Therefore, the cumulative distribution function $F_{\min}^{(k^+)}(x)$ of the minimal distance verify

$$\begin{aligned} 1 - F_{\min}^{(k^+)}(x) &= \Pr\{\min L_i > x\} \\ &= \bigcap_{i \in \{1, 2, \dots, k^+\}} \Pr\{L_i > x\} \\ &= \prod_{i \in \{1, 2, \dots, k^+\}} (1 - F_{AR}(x)) \end{aligned}$$

Then $F_{\min}^{(k^+)}(x) = 1 - [1 - F_{AR}(x)]^{k^+}$.

The expected value is

$$\begin{aligned} \bar{L}_{\min}^{(k^+)} &= E[L_{\min} | k^+] = \int_0^{\pi R} x dF_{\min}^{(k^+)}(x) = [x F_{\min}^{(k^+)}(x)]_0^{\pi R} - \int_0^{\pi R} F_{\min}^{(k^+)}(x) dx \\ &= \pi R - \pi R \int_0^1 [1 - (1-u)^{k^+}] du = \pi R \int_0^1 (1-u)^{k^+} du \\ &= \frac{\pi R}{k^+ + 1} \end{aligned}$$

Finally

$$\bar{t}_A^{V(k^+)} = \frac{2}{\beta(k^+ + 1)} \text{ with } \beta \equiv 2v / \pi R \quad (2)$$

We find the form of access time used in the literature (for instance, Douglas, 1972; De Vany, 1975; Cairns and Liston-Heyes, 1996; Yang et al., 2002, 2005). Thereafter, we assume that the average availability time (denoted t) is approached by the function $\bar{t}_A^{V(k^+)}$ applied to the average number of vacant taxis \hat{k} . The average number of occupied taxis \bar{n} is defined by the product of the temporal flow λ and the mean ride time $t_{ART} \equiv \bar{t} + \bar{t}_R + t_T$ (Little's law). Then we have

$$\bar{n} = \lambda(t_T + \bar{t}_R + \bar{t}) \quad (3)$$

$$\bar{t} \approx \frac{2}{\beta(N + 1 - \bar{n})} \quad (4)$$

By combining (3) and (4) while introducing for simplification $\xi \equiv \lambda(t_T + \bar{t}_R)$, the number of occupied taxis is

$$\bar{n} = \frac{N+1+\xi}{2} - \frac{N+1-\xi}{2} \sqrt{1 - \frac{8\lambda/\beta}{(N+1-\xi)^2}} \quad (5)$$

And in turn the availability function of the service is

$$\bar{t} \approx \frac{N+1-\xi}{2\lambda} \left(1 - \sqrt{1 - \frac{8\lambda/\beta}{(N+1-\xi)^2}}\right) \quad (6)$$

Furthermore, since $\bar{n} = \xi + \lambda t = \lambda t_{ART}$, then using (4) the fleet size is given by $N+1-\bar{n} = 2/(\beta t_A)$

If we neglect -1, then the fleet size is expressed by

$$\hat{N}(t, Q) = \frac{Q}{H} (t_{RT} + t) + \frac{2}{\beta t} \quad (7)$$

2.2. Costs function

The production costs on a daily basis amounts to:

$$C_P(N, Q) \equiv \chi(N) + Q c_u (t_R + t), \quad (8)$$

Wherein: C is the total production cost, χ includes the costs of depreciation, driver wages and cost of the transaction platform, and c_u is the running cost per cab and per unit of time. The function χ increases with N .

Assume that $\dot{\chi} \equiv d\chi/dN$ is constant.

Since the fleet and the availability time are linked through (7), then the production costs could be expressed with respect to the availability time and the demand volume as

$$\hat{C}_P(t, Q) \equiv C_P(\hat{N}(t, Q), Q) = \chi(\hat{N}(t, Q)) + Q c_u (t_R + t) \quad (9)$$

Thus, we observe that the availability time and the demand volume influence the production costs through their fixed and variable parts. By noting $c_u^+ \equiv c_u + \dot{\chi}/H$ and $\zeta \equiv c_u t_R + \dot{\chi} t_{RT}/H$, the derivatives of the cost with respect to the availability time and the demand volume are respectively

$$\frac{\partial}{\partial t} \hat{C}_P = \dot{\chi} \frac{\partial \hat{N}}{\partial t} + Q c_u = \dot{\chi} \left(\frac{Q}{H} - \frac{2}{\beta t^2} \right) + Q c_u = Q c_u^+ - \frac{2\dot{\chi}}{\beta t^2}. \quad (10)$$

$$\frac{\partial}{\partial Q} \hat{C}_P = \frac{\dot{\chi}}{H} (t_{RT} + t) + c_u (t_R + t) = c_u^+ t + \zeta. \quad (11)$$

The function (10) increases with t . It is positive for $t > t^* \equiv \sqrt{2\dot{\chi}/(\beta Q c_u^+)}$. The function (11) is positive and increases with t .

The average production cost per trip is higher than the marginal cost $\partial \hat{C}_P / \partial Q$. It is equal to

$$\frac{\hat{C}_P}{Q} = \frac{\chi}{Q} + c_u (t_R + t) = \zeta + c_u^+ t + \frac{2\dot{\chi}}{Q\beta t} \quad (12)$$

2.3. Demand model

Assume the demand function is

$$Q = D(\tau, t) \quad (13)$$

Wherein Q represents the quantity of passenger trips by cab on a daily basis and τ the fare of a cab trip. The demand volume decreases if the tariff and/or the availability time increases (i.e. $\partial D / \partial \tau < 0$, $\partial D / \partial t < 0$).

Let us gather the respective influences of fare level τ , run time t_R , transaction time \tilde{t}_T and availability time t onto the user into a generalized cost of trip as follows, wherein α denotes the money value of time to the user:

$$g \equiv \tau + \alpha(t_R + \tilde{t}_T + t) \quad (14)$$

Assume further that the demand volume depends on g only ($Q = D(g)$) with constant elasticity ε . Then the derivatives of D with respect to the tariff and the availability time can be expressed by

$$\partial D / \partial \tau = dD / dg = \varepsilon Q / g \quad (15)$$

$$\partial D / \partial t = \alpha dD / dg = \varepsilon \alpha Q / g \quad (16)$$

In addition, let ε_τ and ε_t be the elasticity of the demand with respect to the tariff and the availability time. That means that

$$\tau / \varepsilon_\tau = \frac{D}{\partial D / \partial \tau} = g / \varepsilon \quad (17)$$

$$t / \varepsilon_t = \frac{D}{\partial D / \partial t} = g / \alpha \varepsilon \quad (18)$$

Thereafter, we assume that the demand volume depends on g according to the relation

$$Q = Q_0 \left(\frac{g}{g_0} \right)^\varepsilon \quad (19)$$

2.4. Traffic equilibrium

The traffic equilibrium is obtained when the supply function and demand function are satisfied at the same time. In other terms, it corresponds to

$$\tilde{t} \approx \frac{N+1-\xi}{2Q/H} \left(1 - \sqrt{1 - \frac{8Q/H\beta}{(N+1-\xi)^2}} \right) ; \text{ from (6)} \quad (20)$$

$$Q = Q_0 \left(\frac{\tau + \alpha(t_{RT} + t)}{g_0} \right)^\varepsilon ; \text{ from (13)} \quad (21)$$

That constitutes a system of non-linear equations that can be solved using the fixed-point algorithm:

Step0. Set an initial value $Q^{(k)}$. Let $k = 0$.

Step1. Calculate the availability function according to $t^{(k)} = \frac{N+1-\xi}{2Q^{(k)}/H} \left(1 - \sqrt{1 - \frac{8Q^{(k)}/H\beta}{(N+1-\xi)^2}} \right)$

Step2. Update the demand volume according to $Q^{(k+1)} = Q_0 \left(\frac{\tau + \alpha(t_{RT} + t^{(k)})}{g_0} \right)^\varepsilon$

Step3. If $|Q^{(k+1)} - Q^{(k)}| \leq \theta$, then stop where θ is a predetermined convergence tolerance. Otherwise $k = k + 1$ and return to Step1.

3. Supplier behavior as a monopoly

The service supplier aims to maximize the profit which is the difference between commercial revenues and production costs

$$P_p(N, \tau, Q) = \tau \cdot Q - C_p(N, Q)$$

To maximize the profit, the service provider could act on two levers: the fleet size and the service fare. Then, the service supplier can be cast into the maximization program:

$$\max_{N, \tau} P_p(N, \tau, Q) \text{ s.t. } Q = D(\tau, t) \text{ et } t = T_A^V(N, t_T, t_R, Q)$$

To deal with the constraints simply, it is easier to embed the mutual dependency of demand volume and availability time in an adapted profit function:

$$\hat{P}_p(\tau, t) = \tau \cdot D(\tau, t) - \hat{C}_p(t, D(\tau, t))$$

To look how the adapted profit changes with tariff and availability time, we differentiate $\hat{P}_p(\tau, t)$ with respect to τ and t yielding

$$\frac{\partial}{\partial \tau} \hat{P}_P(\tau, t) = D + \tau \frac{\partial D}{\partial \tau} - \frac{\partial \hat{C}_P}{\partial Q} \frac{\partial D}{\partial \tau} \quad (22)$$

$$\frac{\partial}{\partial t} \hat{P}_P(\tau, t) = \tau \frac{\partial D}{\partial t} - \frac{\partial \hat{C}_P}{\partial t} - \frac{\partial \hat{C}_P}{\partial Q} \frac{\partial D}{\partial t} \quad (23)$$

Then, the first order optimality conditions can be expressed using (15) and (16) of the generalized cost

$$\frac{\partial}{\partial \tau} \hat{P}_P(\tau, t) = 0 \Rightarrow \left(\tau - \frac{\partial \hat{C}_P}{\partial Q} \right) \frac{\partial D}{\partial \tau} = -D \Rightarrow \tau - \frac{\partial \hat{C}_P}{\partial Q} = -\frac{g}{\varepsilon} \quad (24)$$

$$\frac{\partial}{\partial t} \hat{P}_P(\tau, t) = 0 \Rightarrow \left(\tau - \frac{\partial \hat{C}_P}{\partial Q} \right) \frac{\partial D}{\partial t} = \frac{\partial \hat{C}_P}{\partial t} \Rightarrow \tau - \frac{\partial \hat{C}_P}{\partial Q} = \frac{\partial \hat{C}_P}{\partial t} / \frac{\partial D}{\partial t} = \frac{\partial \hat{C}_P}{\partial t} \frac{g}{\alpha \varepsilon Q} \quad (25)$$

From (24), (25) and (10) we get then that at the monopoly optimum we have $-\frac{g}{\varepsilon} = \frac{g}{\alpha \varepsilon} (c_u^+ - \frac{2\dot{\chi}}{Q\beta t^2})$ so

$$\frac{2\dot{\chi}}{Q\beta t^2} = \alpha + c_u^+ \quad (26)$$

Conditions of profitability

The profitability condition for the service provider states that the tariff is higher than the mean cost per trip ($\tau \geq C_P/Q$). Thus, the tariff which optimizes the profit have to respect the following condition

$$(\zeta + c_u^+ t) - \frac{g}{\varepsilon} \geq \frac{C_P}{Q} \quad (27)$$

By substituting (12) and (17) into the condition (27)

$$Q \geq (-\varepsilon_\tau - 1) \frac{2\dot{\chi}}{\beta t(\zeta + c_u^+ t)} \text{ if } \varepsilon_\tau \leq -1 \quad (28)$$

Proving that the profit is positive only and only if $\varepsilon_\tau \leq -1$, or $\varepsilon \leq -\tau/g$. We consider thereafter that $\varepsilon = -2$

Solution of the monopoly problem

Substituting **Erreur ! Source du renvoi introuvable.** and (11) into (24), the tariff is $\tau(1 + \varepsilon) = \varepsilon \zeta + (\varepsilon c_u^+ - \alpha)t - \alpha \tilde{t}_{RT}$ which yields

$$\tau = \frac{\varepsilon \zeta - \alpha \tilde{t}_{RT}}{1 + \varepsilon} + \frac{\varepsilon c_u^+ - \alpha}{1 + \varepsilon} \hat{t}(Q) \equiv \hat{\tau}(Q) \quad (29)$$

And in turns using the availability time and the generalized cost are respectively

$$t = \sqrt{\frac{2\dot{\chi}}{(\alpha + c_u^+)\beta Q}} \equiv \hat{t}(Q) \quad (30)$$

$$\hat{g}(Q) = \frac{\varepsilon}{1 + \varepsilon} [\zeta + \alpha \tilde{t}_{RT} + \sqrt{\frac{2\dot{\chi}(c_u^+ + \alpha)}{\beta Q}}] \quad (31)$$

The demand function verify $D^{(-1)}(Q) = \alpha \tilde{t}_{RT} + \hat{\tau}(Q) + \alpha \hat{t}(Q) \equiv \hat{g}(Q)$. It is a fixed-point problem in Q only. The functions $D^{(-1)}$ and \hat{t} decrease with Q . Also, $\hat{\tau}$ decreases with Q when $\varepsilon \leq -1$ and $\alpha \geq \varepsilon c_u^+$. In addition, the existence and unicity of a solution depend on the function D . For instance, consider the favorable situation where $D^{(-1)}(Q) \cdot \sqrt{Q}$ is a decreasing function from the value Q_0 such as $D^{(-1)}(Q_0) \cdot \sqrt{Q_0} > \hat{g}(Q_0) \cdot \sqrt{Q_0}$. Since the function in the right is an increasing one and not upper bounded, then there exists one and unique intersection point between the two functions, and this solution is higher than Q_0 .

Consider $y = \frac{\zeta + \alpha \tilde{t}_{RT}}{g_0}$ et $x = \frac{1}{g_0} \sqrt{\frac{2\dot{\chi}(c_u^+ + \alpha)}{\beta Q_0}}$. Since $Q = Q_0 (\frac{g}{g_0})^\varepsilon$, then (31) is simplified as $\hat{g}(Q) = \frac{\varepsilon}{1+\varepsilon} [y g_0 + x g_0 \sqrt{Q_0/Q}]$. So using the reduced variable $g' \equiv g / g_0$, this equation could be expressed as

$$g' = \frac{\varepsilon}{1+\varepsilon} (y + x g'^{-\varepsilon/2})$$

Since $\varepsilon = -2$, these equations are simplified, $g' = y / (\frac{1}{2} - x)$ and:

$$Q^M = Q_0 \left(\frac{\frac{1}{2} - x}{y} \right)^2 \quad (32)$$

$$g^M = g_0 \frac{y}{\frac{1}{2} - x} \quad (33)$$

$$t^M = \frac{x}{\alpha + c_u^+} g_0 (Q/Q_0)^{-1/2} = \frac{g_0}{\alpha + c_u^+} \frac{xy}{\frac{1}{2} - x} \quad (34)$$

$$\tau^M + \alpha \tilde{t}_{RT} = g^M - \alpha t^M = g_0 \frac{y}{\frac{1}{2} - x} \left(1 - \frac{\alpha}{\alpha + c_u^+} x \right) \quad (35)$$

Note the unit profit $\hat{C}_P/Q = \zeta + (\alpha + 2c_u^+)t$ and the tariff is $\tau = 2\zeta + (\alpha + 2c_u^+)t + \alpha \tilde{t}_{RT}$. They increase with the availability time. The unit profit is independent on demand and access time. It is equal to

$$\tau - \hat{C}_P/Q = \zeta + \alpha \tilde{t}_{RT} = t_{RT} \dot{\chi}/H + c_u t_R + \alpha \tilde{t}_{RT} \quad (36)$$

The total profit is then directly proportional to the optimized volume of demand Q^* .

In addition, the fleet size is $\hat{N} = Q t_{ART} / H + 2 / \beta t = Q^* (\frac{1}{H} t_{RT} + (\frac{\alpha + c_u^+}{\dot{\chi}} + \frac{1}{H}) t^*)$ which increases with Q^* and t^* . The productivity by taxi is $Q^* / \hat{N} = 1 / (\frac{1}{H} t_{RT} + (\frac{1}{H} + \frac{\alpha + c_u^+}{\dot{\chi}}) t^*)$ depends only on t^* .

4. First best social optimum

The social surplus is defined as the summation of consumers' net benefits and producers' profit. Since the demand depends on the generalized cost only, then the consumers' surplus is given by $P_D(g) = \int_g^{+\infty} D(g') dg'$

The social surplus is then:

$$P_S \equiv P_D(g) + P_P = \int_g^{+\infty} D(g') dg' + \tau \cdot D(g) - C_P \quad (37)$$

Considering that the service provider have two main levers, the tariff and the accessibility time, let assess the sensitivity of the social surplus to these two variables. The derivatives of the social surplus with respect to τ and t are

$$\frac{\partial}{\partial \tau} \hat{P}_S(t, \tau) = -D + D + \tau \dot{D} - \frac{\partial \hat{C}_P}{\partial Q} \dot{D} = (\tau - \frac{\partial \hat{C}_P}{\partial Q}) \dot{D} \text{ by noting } \dot{D} \equiv dD/dg \quad (38)$$

$$\frac{\partial}{\partial t} \hat{P}_S(t, \tau) = -\alpha D + \tau \alpha \dot{D} - \frac{\partial \hat{C}_P}{\partial t} - \alpha \dot{D} \frac{\partial \hat{C}_P}{\partial Q} \quad (39)$$

Conditions of profitability

By reminding that $\hat{C}_P/Q = \partial \hat{C}_P / \partial Q + 2\dot{\chi}/(Q\beta t)$, then the service is unprofitable. The net profit is negative and decreases with the time availability. It is equal to $\hat{P}_P = Q\tau - \hat{C}_P = Q(\partial \hat{C}_P / \partial Q - \hat{C}_P/Q) = -2\dot{\chi}/\beta t$. In other terms, the subsidy required to ensure the equilibrium for the operator is equal to $-\hat{P}_P = 2\dot{\chi}/\beta \tilde{t}$.

Solution of the social optimum

The first order optimality conditions are

$$\frac{\partial}{\partial \tau} \hat{P}_S(t, \tau) = 0 \quad \text{then} \quad \tau = \frac{\partial \hat{C}_P}{\partial Q}, \quad (40)$$

$$\frac{\partial}{\partial t} \hat{P}_S(t, \tau) = 0 \quad \text{then} \quad \frac{\partial \hat{C}_P}{\partial t} = -\alpha D. \quad (41)$$

From (41), we obtain that $Q c_u^+ - 2\dot{\chi}/\beta t^2 = -\alpha Q$ then $2\dot{\chi}/\beta Q t^2 = \alpha + c_u^+$. Using x and y as defined above, then the system defined by τ , t and Q

$$t = \frac{g_0}{\alpha + c_u^+} x(Q/Q_0)^{-\frac{1}{2}} \equiv \tilde{t}(Q) \quad (42)$$

$$\tau = \zeta + g_0 \frac{c_u^+}{\alpha + c_u^+} x(Q/Q_0)^{-\frac{1}{2}} \equiv \tilde{\tau}(Q), \text{ using (40)} \quad (43)$$

$$\tilde{g}(Q) = g_0(y + x(Q/Q_0)^{-\frac{1}{2}}), \text{ since } \tilde{g}(Q) = \tilde{\tau}(Q) + \alpha(\tilde{t}_{RT} + \tilde{t}(Q)) \quad (44)$$

$$D^{(-1)}(Q) = g_0(y + x(Q/Q_0)^{-\frac{1}{2}}) \quad (45)$$

As for monopoly, we could demonstrate easily that there exists a unique solution for (45) which is higher than Q_0 . Furthermore, since the generalized cost for the system is lower than that for the monopoly, $\tilde{g} < \hat{g}$ then $\tilde{Q} > \hat{Q}$ and in turn $\tilde{t} < \hat{t}$ and $\tilde{\tau} < \hat{\tau}$. That means that the regulator needs to reduce the availability time and the tariff. Also, the fleet size should be higher than that for a monopoly. In particular, let us introduce another time the reduced variable $g' \equiv g/g_0 = y + xg'^{-\varepsilon/2}$. Then for the elasticity $\varepsilon = -2$, $g' = y/(1-x)$. The previous equations could be simplified as

$$Q^O = Q_0 \left(\frac{1-x}{y} \right)^2 \quad (46)$$

$$g^O = g_0 y / (1-x) \quad (47)$$

$$t^O = \frac{g_0}{\alpha + c_u^+} x \frac{y}{1-x} \quad (48)$$

$$\tau^O + \alpha \tilde{t}_{RT} = g^O - \alpha t^O = g_0 \frac{y}{1-x} \left(1 - \frac{\alpha}{\alpha + c_u^+} x \right) \quad (49)$$

Comparing to the monopoly profit yields

$$\frac{g^O}{g^M} = \frac{t^O}{t^M} = \frac{\tau^O + \alpha \tilde{t}_{RT}}{\tau^M + \alpha \tilde{t}_{RT}} = \frac{\frac{1}{2} - x}{1-x} < \frac{1}{2} \quad (50)$$

The ratio is less than $\frac{1}{2}$ since $x \in]0, \frac{1}{2}[$, which means that to ensure the social welfare, the generalized cost and the availability time have to be reduced by more than two times comparing to the monopoly situation. By reducing the availability time t , the unit production cost is reduced since $\hat{C}_P/Q = \zeta + (\alpha + 2c_u^+)t$

The total profit will be reduced by more than 4 times, since it depends on Q only.

In addition, the productivity by taxi is $Q/\hat{N} = 1/(\frac{1}{H} t_{RT} + (\frac{1}{H} + \frac{\alpha + c_u^+}{\dot{\chi}})t)$ decreases with t . As a result, the reduction of t involves higher productivity, so better utilization of taxis.

5. Second best optimum under budget constraint

Running a private industry while maximizing the social optimum involves substantial subsidies. To avoid subsidies, another solution consists on maximizing the social welfare given the constraint that revenues cover costs. This second-best social optimum is obtained by the following program

$$\max_{\tau, t} \hat{P}_S(\tau, t, Q) \text{ s.t. } Q = D(\tau, t) \text{ and } \tau \cdot Q \geq \hat{C}_P(t, Q). \quad (51)$$

Solution of the second-best optimum

We can form the following Lagrange function:

$$\mathcal{L}(\tau, t, \gamma) \equiv \hat{P}_S(\tau, t, Q) + \gamma(\tau \cdot Q - \hat{C}_P(t, Q)) = \int_g^{+\infty} D + (1 + \gamma)(\tau \cdot Q - \hat{C}_P(t, Q)) ; Q = D(\tau, t). \quad (52)$$

Where $\gamma \geq 0$ is the Lagrange multiplier. The conditions of the first-order optimality conditions yield to a system of three equations:

$$\partial \mathcal{L} / \partial \tau = 0 \quad \Rightarrow \gamma D + (1 + \gamma)(\tau - \partial \hat{C}_P / \partial Q) \cdot \dot{D} = 0 \quad (53)$$

$$\partial \mathcal{L} / \partial t = 0 \quad \Rightarrow -\alpha D + (1 + \gamma)[\alpha \dot{D} \cdot (\tau - \partial \hat{C}_P / \partial Q) - \partial \hat{C}_P / \partial t] = 0 \quad (54)$$

$$\partial \mathcal{L} / \partial \gamma \geq 0 \text{ and } \gamma \cdot \partial \mathcal{L} / \partial \gamma = 0 \quad \Rightarrow \tau \cdot Q - \hat{C}_P(t, Q) = 0 \quad (55)$$

And in turn

$$(1 + \gamma)(\tau - \partial \hat{C}_P / \partial Q) \cdot \dot{D} = -\gamma D \quad (56)$$

$$\frac{2\dot{\chi}}{\beta Q t^2} = \alpha + c_u^+ \quad (57)$$

$$\tau = \frac{\hat{C}_P}{Q} = \zeta + c_u^+ t + \frac{2\dot{\chi}}{Q\beta t} = \zeta + (\alpha + 2c_u^+)t \quad (58)$$

From (58) and using (11), we conclude that $\tau - \partial \hat{C}_P / \partial Q = 2\dot{\chi} / Q\beta t = (\alpha + c_u^+)t$, which by substitution in (56) gives $(\alpha + c_u^+)t = -\gamma' D / \dot{D}$, considering that $\gamma' \equiv \gamma / (1 + \gamma)$.

Let introduce the generalized cost. We have $D / \dot{D} = g / \varepsilon$ so $(\alpha + c_u^+)t = -g\gamma' / \varepsilon$. From the definition of g we have $-\varepsilon(\alpha + c_u^+)t / \gamma' = g = \alpha \tilde{t}_{RT} + \tau + \alpha t = \alpha \tilde{t}_{RT} + \zeta + 2(\alpha + c_u^+)t$. As a result,

$$t_\gamma = \frac{\zeta + \alpha \tilde{t}_{RT}}{(\alpha + c_u^+)(-\varepsilon / \gamma' - 2)} \quad (59)$$

And in turn, we can deduce τ_γ , g_γ and Q_γ :

$$\tau = \zeta + (\alpha + 2c_u^+)t_\gamma \quad (60)$$

$$g_\gamma = (\alpha + c_u^+)t_\gamma \frac{-\varepsilon}{\gamma'} = \frac{\zeta + \alpha \tilde{t}_{RT}}{1 + 2\gamma' / \varepsilon} \quad (61)$$

$$Q_\gamma = \frac{2\dot{\chi}}{\beta(\alpha + c_u^+)t^2} = \frac{2\dot{\chi}(\alpha + c_u^+)}{\beta(\zeta + \alpha \tilde{t}_{RT})^2} \left(2 + \frac{\varepsilon}{\gamma'}\right)^2 \quad (62)$$

So using x , y and the reduced variable g' , then

$$t_\gamma = \frac{g_0 y}{(\alpha + c_u^+)(-\varepsilon / \gamma' - 2)} \quad (63)$$

$$g_\gamma = g_0 \frac{y}{1 + 2\gamma' / \varepsilon} \quad (64)$$

$$Q_\gamma = \frac{2\dot{\chi}(\alpha + c_u^+)}{\beta((\alpha + c_u^+)t)^2} = Q_0 \left(\frac{x}{y}\right)^2 \left(2 + \frac{\varepsilon}{\gamma'}\right)^2 \quad (65)$$

From (61) and (64) we conclude that γ' verify

$$\left| \frac{2\gamma' + \varepsilon}{\gamma'} \right|^{2/\varepsilon} \left(\frac{2\gamma' + \varepsilon}{\varepsilon} \right) = y^{1+2/\varepsilon} x^{-2/\varepsilon} \quad (66)$$

In particular, considering that $\varepsilon = -2$, then $\left| 2 \frac{1-\gamma'}{\gamma'} \right|^{-1} (1-\gamma') = x$ so $\gamma' = 2x$, and then we conclude that

$$Q^C = Q_0((1-2x)/y)^2 \quad (67)$$

$$g^C = g_0 y / (1-2x), \quad (68)$$

$$t^C = \frac{g_0}{\alpha + c_u^+} x \frac{y}{1-2x}, \quad (69)$$

$$\tau^C + \alpha \tilde{t}_{RT} = g^C - \alpha t^C = g_0 \frac{y}{1-2x} \left(1 - \frac{\alpha}{\alpha + c_u^+} x \right). \quad (70)$$

Comparing to the monopoly problem, we observe that

$$\frac{g^C}{g^M} = \frac{t^C}{t^M} = \frac{\tau^C + \alpha \tilde{t}_{RT}}{\tau^M + \alpha \tilde{t}_{RT}} = \frac{\frac{1}{2} - x}{1-2x} = \frac{1}{2} \quad (71)$$

Comparing to the monopoly situation, the generalized cost and the availability are divided by two. The tariff level is more than two times less: $\tau^C = \frac{1}{2}(\tau^M - \alpha \tilde{t}_{RT})$. In addition, the demand volume is divided by 4 $Q^C = 4Q^M$ which means using (36) that the total profit is reduced by more than 4 times.

6. Conclusion

We have provided a microeconomic model applied on a stylized urban area to analyze the conditions of profitability and optimize the business model. The availability time is a critical factor of service quality so of the demand volume. That is in turn beneficial to the supplier in both the production of availability and the production cost. These interrelations are described through the traffic equilibrium.

Starting from a situation of monopoly, we provide the market conditions of profitability and determined the system's optimality conditions for a given demand elasticity. Then we explored the optimum conditions in order to maximize the social surplus. As for microeconomic theory, further development may be target to explore the system optimization, the duopoly and oligopoly issues, the regulation impacts.

The spatial model that we suggest "Orbicity" allows approaching results with simple relations. We could imagine that the stylized network is a transit service operating on-demand in a fixed line as a form of ring. Further development of the spatial model would permit to represent more complex networks.

7. References

- Cairns, Robert and Liston-Heyes, Catherine. 1996. Competition and regulation in the taxi industry. *Journal of Public Economics*. Vol. 59, pp. 1-15.
- De Vany, Arthur. 1975. Capacity Utilization under Alternative Regulatory Restraints: An Analysis of Taxi Markets. *Journal of Political Economy*. Vol. 83, 1, pp. 83-94.
- Douglas, George. 1972. Price regulation and optimal service standards. *Journal of Transport Economics and Policy*. pp. 116-127.
- Hackner, Jonas and Nyberg, Sten. 1995. Deregulating taxi services: a word of caution. *Journal of Transport Economics and Policy*. 1995, Vol. 29, 2.
- Manski, Charles and Wright, David. 1976. Nature of equilibrium in the market for taxi services. *Transportation Research Record* pp. 296-306.
- Wang, Xiaolei, He, Fang and Gao, Oliver. 2016. Pricing strategies for e-hailing platform in taxi service. *Transportation Research Part E Logistics and Transportation Review*. 93, pp. 212-231.
- Wong, K.I, Wong, S.C. and Yang, Hai. 2001. Modeling urban taxi services in congested road networks with elastic demand. *Transport Research B*. Vol. 35, pp. 819-842.
- Wong, K.I. and Wong, S.C. 2002. A sensitivity-based solution algorithm for the network model of urban taxi services. *Transportation and Traffic Theory in the 21st Century*.
- Yang, H., et al. 1998. A network model of urban taxi services. *Transport Research B*. Vol. 32, pp. 235-246.
- Yang, H., et al. 2010. Equilibria of bilateral taxi-customer searching and meeting on networks. *Transport Research B*. Vol. 44, pp. 1067-1083.
- Yang, H., et al. 2005. Regulating taxi services in the presence of congestion externality. *Transport Research A*. Vol. 39, pp. 17-40.