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A Comparison of Parallel and Perpendicular Wave Propagations at Low and High Frequencies in Magnetized Plasma

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ABSTRACT

Plasma Waves may propagate in different frequency levels. This study justifies the reason for the difference in dispersion curve at low and high frequencies of plasma wave propagations. At low frequencies i.e. ($\omega \ll \Omega_i$), the right handed and the left handed waves tend to the Alfven wave. The fast and slow Alfvén waves are indistinguishable for parallel propagation whereas the shear-Alfvén wave does not propagate perpendicular to the magnetic field. At high-frequency the right-handed waves, propagating parallel to the equilibrium magnetic field, and oscillating at the frequency Ω_e are absorbed by electrons. By this paper it is proved that the low-frequency branch of the dispersion curve differs fundamentally from the high-frequency branch, because the former branch corresponds to a wave which can only propagate through the plasma in the presence of an equilibrium magnetic field, whereas the high-frequency branch corresponds to a wave which can propagate in the absence of an equilibrium field.

Keyword: wave propagation, Alfven wave, magnetized plasma, frequency

INTRODUCTION

In plasma physics, waves in plasmas are an interconnected set of particles and fields which propagate in a periodically repeating fashion. Plasma is a quasi neutral, electrically conductive fluid. In the simplest case, it is composed of electrons and a single species of positive ions, but it may also contain multiple ion species including negative ions as well as neutral particles. Due to its electrical conductivity, a plasma couples to electric and magnetic fields. This complex of particles and fields supports a wide variety of wave phenomena [1].

Waves in plasmas can be classified as electromagnetic or electrostatic according to whether or not there is an oscillating magnetic field. Applying Faraday's law of induction to plane waves, I find $k \times E = \omega B$ implying that an electrostatic wave must be purely longitudinal. An electromagnetic wave, in contrast, must have a transverse component, but may also be partially longitudinal [2].

Waves can be further classified by the oscillating species. In most plasmas of interest, the electron temperature is comparable to or larger than the ion temperature. This fact, coupled with the much smaller mass of the electron, implies that the electrons move much faster than the ions. An electron mode depends on the mass of the electrons, but the ions may be assumed to be infinitely massive, i.e. stationary. An ion mode depends on the ion mass, but the electrons are assumed to be mass less and to redistribute themselves instantaneously according to the Boltzmann relation. Only rarely, e.g. in the lower hybrid oscillation, will a mode depend on both the electron and the ion mass [1, 3].

The various modes can also be classified according to whether they propagate in an un magnetized plasma or parallel, perpendicular, or oblique to the stationary magnetic field. Finally, for perpendicular electromagnetic electron waves, the perturbed electric field can be parallel or perpendicular to the stationary magnetic field. Hence this paper reviews the comparison between plasma wave propagations parallel and perpendicular to the magnetic field in the magnetized plasma.

The Wave Equation

I can start with Maxwell's equations, which I can write as [8].

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad 1$$

$$\nabla \times B = \mu_0 \left(j + \varepsilon_0 \frac{\partial E}{\partial t} \right) = i\mu_0 \omega \varepsilon_0 \left(\frac{j}{i\omega\varepsilon_0} - E \right) = -i\mu_0\omega\varepsilon_0 \left(\overleftrightarrow{I} + \frac{i\overleftrightarrow{\sigma}}{\omega\varepsilon_0} \right) \mathbf{E} = -i\mu_0\omega\overleftrightarrow{\varepsilon}\mathbf{E} \qquad \mathbf{2}$$

Where I have used the relation $j = \overleftrightarrow{\sigma} E$ (Ohm's law for high frequency behavior). To generate the wave equation, I take the curl of Eq. 1 and substitute from Eq. 2. :

$$\nabla \times \nabla \times E = -\frac{\partial}{\partial t} \nabla \times B = \mu_0 \omega^2 \varepsilon_0 \overleftrightarrow{K} \cdot E = \frac{\omega^2}{c^2} \overleftrightarrow{K} \cdot E \qquad 3$$

Where

$$\overleftrightarrow{K} \equiv \frac{\overleftrightarrow{\varepsilon}}{\varepsilon_0} = relative\ susceptibility\ tensor$$

For plane wave solutions the wave equation gives

$$ik \times (ik \times E) - \frac{\omega^2}{c^2} \overleftrightarrow{K} \cdot E = 0 \qquad 4$$

Or

$$n \times (n \times E) + \overleftrightarrow{K} \cdot E = 0 \qquad 5$$

Where

$$n = \frac{c}{\omega} k \qquad 6$$

is the refractive index vector.

The dielectric Susceptibility Tensor

The dielectric tensor for the plasma can be expressed as

$$\overleftrightarrow{\varepsilon} = \varepsilon_0 (\overleftrightarrow{I} + \frac{i}{\varepsilon_0 \omega} \overleftrightarrow{\sigma}) \qquad 7$$

$$\overleftrightarrow{\varepsilon} = \varepsilon_0 \begin{pmatrix} \epsilon_1 & \epsilon_2 & 0 \\ \epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix} \qquad 8$$

With

$$\epsilon_1 = 1 + \frac{i}{\omega\varepsilon_0}\sigma_\perp \quad 9$$

$$\epsilon_2 = \frac{i}{\omega\varepsilon_0}\sigma_H \quad 10$$

$$\epsilon_3 = 1 + \frac{i}{\omega\varepsilon_0}\sigma_0 \quad 11$$

From equation 8 and equation 11 we can write the dielectric susceptibility tensor in the form

$$\overleftrightarrow{K} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \quad 12$$

Where the reason for using the nomenclatures S, P, D as opposed to $\epsilon_1,\ \epsilon_2 and\ \epsilon_3$ will be apparent later.

$$S = 1 + \frac{i}{\omega\varepsilon_0}\sigma_\perp \quad 13$$

$$D = \mp\frac{i}{\omega\varepsilon_0}\sigma_H \quad 14$$

$$P = 1 + \frac{i}{\omega\varepsilon_0}\sigma_0 \quad 15$$

Where the minus sign is for ions and positive for electrons and where we use the conductivity components for high frequency electric field are defined as

$$\sigma_\perp = \sigma_0\frac{v^2}{v^2+\omega^2{}_c} \qquad \text{Perpendicular conductivity}$$

$$\sigma_H = \sigma_0\frac{\mp v\omega_c}{v^2+\omega^2{}_c} \qquad \text{Hall conductivity}$$

$$\sigma_\parallel = \sigma_0 = \frac{ne^2}{mv} \qquad \text{Longitudinal conductivity}$$

With v replaced by $-i\omega$.

Thus for S, I obtain (for electrons)

$$S = 1 + \frac{i}{\omega\varepsilon_0}\frac{ine^2}{m_e\omega}\frac{\omega^2}{\omega^2-\omega^2{}_{ce}}$$

$$= 1 + \left(\frac{ne^2}{m_e\varepsilon_0}\right)\frac{1}{\omega^2-\omega^2{}_{ce}}$$

$$= 1 - \frac{\omega^2{}_{pe}}{\omega^2-\omega^2{}_{ce}}$$

Including both ions and electrons gives

$$S = 1 - \sum_{i,e}\frac{\omega^2{}_p}{\omega^2-\omega^2{}_c} \quad 16$$

In a similar way, we obtain

$$D = \sum_{i,e}\pm\frac{\omega^2{}_p\omega_c}{\omega(\omega^2-\omega^2{}_c)} \quad 17$$

$$P = 1 - \sum_{i,e}\frac{\omega^2{}_p}{\omega^2} \quad 18$$

Where the plus sign is for ions and minus for electrons. Let me now simplify the first term and so develop a dispersion relation for wave characterized by the tensor $\overleftrightarrow{K}$. I assume the wave is propagating at an angle θ to the ambient magnetic field $B = B_0\hat{k}$ and without lose of generality, that the propagation vector lie in the $x - z$ plane.

The Dispersion Relations of parallel and perpendicular wave propagations

Parallel Wave Propagation

Let me now consider wave propagation, at arbitrary frequencies, parallel to the equilibrium magnetic field. When $\theta = 0$, the Eigen mode equation,

$$\begin{pmatrix} S-n^2cos^2\theta & -iD & n^2cos\theta sin\theta \\ iD & S-n^2 & 0 \\ n^2cos\theta sin\theta & 0 & P-n^2sin^2\theta \end{pmatrix}\begin{pmatrix} E_X \\ E_Y \\ E_Z \end{pmatrix} = 0 \quad 19$$

Simplifies to

$$\begin{pmatrix} S-n^2 & -iD & 0 \\ iD & S-n^2 & 0 \\ 0 & 0 & P \end{pmatrix}\begin{pmatrix} E_X \\ E_Y \\ E_Z \end{pmatrix} = 0 \quad 20$$

One obvious way of solving this equation is to have

$$P \simeq \frac{\pi_e{}^2}{\omega^2} \quad 21$$

With the eigen vector $(0, 0, E_z)$. This mode is longitudinal in nature, and, therefore, causes particles to oscillate parallel to B_0. It follows that the particles experience zero Lorentz force due to the presence of the equilibrium magnetic field, with the result that this field has no effect on the mode dynamics [4].

The other two solutions to Eq. (20) are obtained by setting the 2×2 determinant involving the $-\ x$ and y - components of the electric field to zero. The first wave has the dispersion relation

$$\pi^2 = R \simeq 1 - \frac{\pi_e{}^2}{(\omega+\Omega_e)(\omega+\Omega_i)} \quad 22$$

And the eigen vector $(E_x,\ iE_x,\ 0)$. This is evidently a right-handed circularly polarized wave. The second wave has the dispersion relation

$$\pi^2 = L \simeq 1 - \frac{\pi_e{}^2}{(\omega-\Omega_e)(\omega-\Omega_i)} \quad 23$$

And the eigen vector $(E_x,\ -iE_x,\ 0)$. This is evidently a left-handed circularly polarized wave. At low frequencies. i.e. ($\omega \ll \Omega_i$), both waves tend to the Alfven wave. Note that the fast and slow Alfvén waves are indistinguishable for parallel propagation. Let me now examine the high-frequency behavior of the right- and left-handed waves.

For the right-handed wave, it is evident, since Ω_e is negative, that $\pi^2 \to \infty$ as $\omega \to |\Omega_e|$. This resonance, which corresponds to $R \to \infty$, is termed the electron cyclotron resonance. At the electron cyclotron resonance the transverse electric field associated with a right-handed wave rotates at the same velocity, and in the same direction, as electrons gyrating around the equilibrium magnetic field. Thus, the electrons experience a continuous acceleration from the electric field, which tends to increase their perpendicular energy. It is, therefore, not surprising those right-handed waves, propagating parallel to the equilibrium magnetic field, and oscillating at the frequency Ω_e are absorbed by electrons [3, 5].

When ω is just above $|\Omega_e|$ I find that π^2 is negative and so there is no wave propagation in this frequency range. However, for frequencies much greater than the electron cyclotron or plasma frequencies, the solution to Eq. (22) is approximately $\pi^2 = 1$. In other words $\omega^2 = k^2c^2$: the dispersion relation of a right-handed vacuum electromagnetic wave. Evidently, at some frequency above $|\Omega_e|$ the solution for π^2 must pass through zero, and become positive again. Putting $\pi^2 = 0$ in Eq. (22), I find that the equation reduces to

$$\omega^2 + \Omega_e\omega - \pi_e{}^2 \simeq 0 \quad 23$$

assuming that $V_A \ll c$. The above equation has only one positive root, at $\omega = \omega_1$, where

$$\omega_1 \simeq \frac{|\Omega_e|}{2} + \sqrt{\left(\frac{\Omega_e{}^2}{4} + \pi_e{}^2\right)} > |\Omega_e|\ . \quad 24$$

Above this frequency, the wave propagates once again as in the following manner.

Graphical representation of right handed parallel propagation to the magnetic field

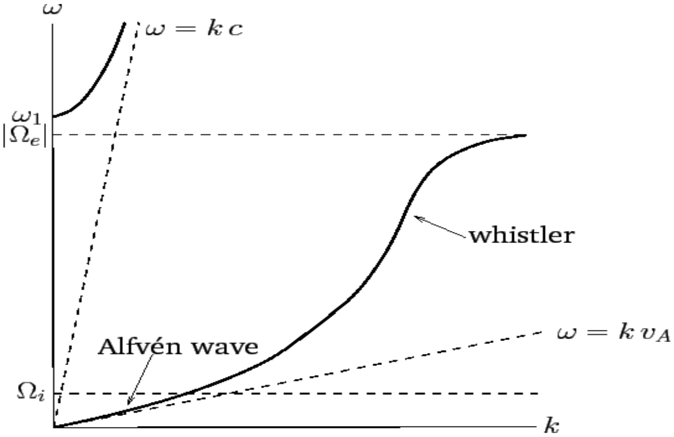

Fig. 1 Dispersion relation for a right-handed wave propagating parallel to the magnetic field in magnetized plasma.

This figure shows the sketch of dispersion curve for a right-handed wave propagating parallel to the equilibrium magnetic field. The continuation of the Alfvén wave above the ion cyclotron frequency is called the electron cyclotron wave, or sometimes the whistler wave. The latter terminology is prevalent in ionosphere and space plasma physics contexts. The wave which propagates above the cutoff frequency ω_1 , is a standard right-handed circularly polarized electromagnetic wave, somewhat modified by the presence of the plasma. Note that the low-frequency branch of the dispersion curve differs fundamentally from the high-frequency branch, because the former branch corresponds to a wave which can only propagate through the plasma in the presence of an equilibrium magnetic field, whereas the high-frequency branch corresponds to a wave which can propagate in the absence of an equilibrium field [6].

The curious name ``whistler wave" for the branch of the dispersion relation lying between the ion and electron cyclotron frequencies is originally derived from ionospheric physics. Whistler waves are a very characteristic type of audio-frequency radio interference, most commonly encountered at high latitudes, which take the form of brief, intermittent pulses, starting at high frequencies, and rapidly descending in pitch. Whistlers were discovered in the early days of radio communication, but were not explained until much later. Whistler waves start off as ``instantaneous" radio pulses, generated by lightning flashes at high latitudes [7, 8]. The pulses are channeled along the Earth's dipolar magnetic field, and eventually return to ground level in the opposite hemisphere. Now, in the frequency range $\Omega_i \ll \omega \ll |\Omega_e|$, the dispersion relation (22) reduces to

$$n^2 = \frac{k^2c^2}{\omega^2} \backsimeq \frac{\pi_e{}^2}{\omega|\Omega_e|} \qquad 25$$

As is well-known, pulses propagate at the group-velocity,

$$v_g = \frac{d\omega}{dk} = 2c\frac{\sqrt{\omega|\Omega_e|}}{\pi_e}$$

Clearly, the low-frequency components of a pulse propagate more slowly than the high-frequency components. It follows that by the time a pulse returns to ground level it has been stretched out temporally, because the high-frequency components of the pulse arrive slightly before the low-frequency components. This also accounts for the characteristic whistling-down effect observed at ground level.

For a left-handed circularly polarized wave, similar considerations to the above give a dispersion curve of the form sketched in Fig. 2. In this case, π^2 goes to infinity at the ion cyclotron frequency, Ω_i corresponding to the so-called ion cyclotron resonance (at $L \to \infty$) At this resonance, the rotating electric field associated with a left-handed wave resonates with the gyro motion of the ions, allowing wave energy to be converted into perpendicular kinetic energy of the ions. There is a band of frequencies, lying above the ion cyclotron frequency, in which the left-handed wave does not propagate. At very high frequencies a propagating mode exists, which is basically a standard left-handed circularly polarized electromagnetic wave, somewhat modified by the presence of the plasma. The cutoff frequency for this wave is

$$\omega_2 \backsimeq -\frac{|\Omega_e|}{2} + \sqrt{\frac{\Omega_e{}^2}{4} + \pi_e{}^2}$$

Graphical representation of left handed parallel propagation to the magnetic field

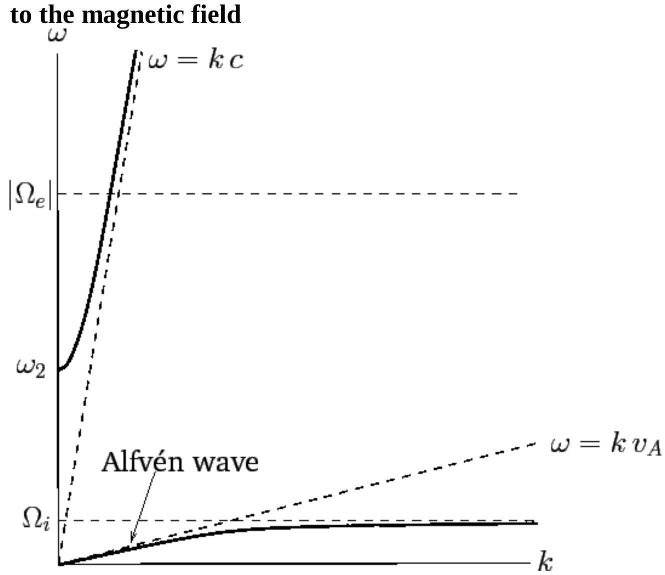

Fig. 2 Dispersion relation for a left-handed wave propagating parallel to the magnetic field in magnetized plasma.

The lower branch in Fig. 2 describes a wave that can only propagate in the presence of an equilibrium magnetic field, whereas the upper branch describes a wave that can propagate in the absence an equilibrium field. The continuation of the Alfvén wave to just below the ion cyclotron frequency is generally called the ion cyclotron wave.

Perpendicular Wave Propagation

Let me now consider wave propagation, at arbitrary frequencies, perpendicular to the equilibrium magnetic field. When $\theta = \frac{\pi}{2}$ the eigen mode equation (19) simplifies to

$$\begin{pmatrix} S & -iD & 0 \\ iD & S-n^2 & 0 \\ 0 & 0 & P-n^2 \end{pmatrix} \begin{pmatrix} E_X \\ E_Y \\ E_Z \end{pmatrix} = 0 \qquad 26$$

One obvious way of solving this equation is to have $P - n^2 = 0$ or

$$\omega^2 = \pi_e{}^2 + k^2c^2 \qquad 27$$

with the eigenvector (0, 0, E_z). Since the wave-vector now points in the x -direction, this is clearly a transverse wave polarized with its electric field parallel to the equilibrium magnetic field. Particle motions are along the magnetic field, so the mode dynamics are completely unaffected by this field. Thus, the wave is identical to the electromagnetic plasma wave in an un magnetized plasma. This wave is known as the ordinary, or O-, mode [9].

The other solution to Eq. (26) is obtained by setting the 2×2 determinant involving the x and y components of the electric field to zero. The dispersion relation reduces to

$$n^2 = \frac{RL}{S} \qquad 28$$

with the associated eigenvector $E_x(1, -\frac{iS}{D}, 0)$. Let me, first of all, search for the cutoff frequencies, at which π^2 goes to infinity. According to Eq. (28), the resonant frequencies are solutions of

$$S = 1 - \frac{\pi_e{}^2}{\omega^2 - \Omega_e{}^2} - \frac{\pi_i{}^2}{\omega^2 - \Omega_i{}^2} = 0 \qquad 29$$

The roots of this equations can be obtained as follows. First, I note that if the first two terms are equated to zero, I obtain

$$\omega_{UH} = \sqrt{\pi_e{}^2 + \Omega_e{}^2} \quad 30$$

If this frequency is substituted into the third term, the result is far less than unity. I conclude that ω_{UH} is a good approximation to one of the roots of Eq. (29). To obtain the second root, I make use of the fact that the product of the square of the roots is

$$\Omega_e{}^2\Omega_i{}^2 + \pi_e{}^2\Omega_i{}^2 + \pi_i{}^2\Omega_e{}^2 \simeq \Omega_e{}^2\Omega_i{}^2 + \pi_i{}^2\Omega_e{}^2 \quad 31$$

I, thus, obtain $\omega = \omega_{LH}$ where

$$\omega_{LH} = \sqrt{\frac{\Omega_e{}^2\Omega_i{}^2 + \pi_i{}^2\Omega_e{}^2}{\pi_e{}^2 + \Omega_e{}^2}} \quad 32$$

The first resonant frequency, ω_{uH} is greater than the electron cyclotron or plasma frequencies, and is called the upper hybrid frequency. The second resonant frequency, ω_{LH}, lies between the electron and ion cyclotron frequencies, and is called the lower hybrid frequency.

Unfortunately, there is no simple explanation of the origins of the two hybrid resonances in terms of the motions of individual particles. At low frequencies, the mode in question reverts to the compressional-Alfvén wave discussed previously. Note that the shear-Alfvén wave does not propagate perpendicular to the magnetic field.

Using the above information, and the easily demonstrated fact that

$$\omega_{LH} < \omega_2 < \omega_{UH} < \omega_1 \quad 33$$

I can deduce that the dispersion curve for the mode in question takes the form sketched in Fig. 3.

Graphical representation of perpendicular wave propagation to the magnetic field

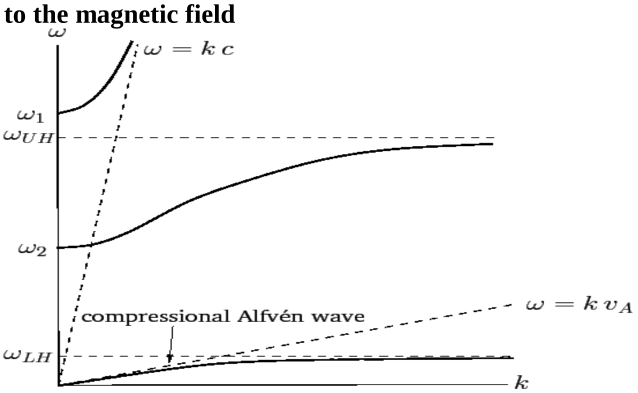

Figure 3: Dispersion relation for a wave propagating perpendicular to the magnetic field in a magnetized plasma.

The lowest frequency branch corresponds to the compressional-Alfvén wave. The other two branches constitute the extraordinary, or X- wave. The upper branch is basically a linearly polarized (in the y direction) electromagnetic wave, somewhat modified by the presence of the plasma. This branch corresponds to a wave which propagates in the absence of an equilibrium magnetic field. The lowest branch corresponds to a wave which does not propagate in the absence of an equilibrium field. Finally, the middle branch corresponds to a wave which converts into an electrostatic plasma wave in the absence of an equilibrium magnetic field.

Wave propagation at oblique angles is generally more complicated than propagation parallel or perpendicular to the equilibrium magnetic field, but does not involve any new physical effects.

CONCLUSION

The dispersion curve that expresses the criteria of the wave in magnetized plasma is quite different at low and high frequency levels and for parallel and perpendicular propagations. At low frequencies i.e. ($\omega \ll \Omega_i$), the right and left handed waves tend to the Alfven wave. Note that the fast and slow Alfvén waves are indistinguishable for parallel propagation. At high frequency the right-handed wave is termed the electron cyclotron resonance. At the electron cyclotron resonance the transverse electric field associated with a right-handed wave rotates at the same velocity, and in the same direction, as electrons gyrating around the equilibrium magnetic field. Thus, the electrons experience a continuous acceleration from the electric field, which tends to increase their perpendicular energy. Therefore those right-handed waves, propagating parallel to the equilibrium magnetic field, and oscillating at the frequency Ω_e are absorbed by electrons. Hence we can conclude that the low-frequency branch of the dispersion curve differs fundamentally from the high-frequency branch, because the former branch corresponds to a wave which can only propagate through the plasma in the presence of an equilibrium magnetic field, whereas the high-frequency branch corresponds to a wave which can propagate in the absence of an equilibrium field.

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