Aberrant Method of Solving the Assignment Problem

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Abstract

Assignment downside may be a specific case of the transportation downside. It helps as to minimizing the time or cost of manufacturing the products by allocating one job to one person or one person to one job or one destination to one origin or one origin to one destination only. Normally, assignment model is a minimization model. In this article we have a tendency to initiate new technique to resolve assignment downside referred to as aberrant technique of determination assignment downside. By solving an assignment problem using aberrant method of solving assignment problem and Hungarian method and compared it's results.

Keywords: *Assignment Problem, Hungarian Algorithm for Assignment Problem, Aberrant Method for Assignment Problem, Linear Programming model, minimization.*

INTRODUCTION

The Assignment models are looks as like as Transportation Problem. The best person for the job is an apt description of what the assignment model seeks to accomplish. The situation can be illustrated by the assignment of workers to jobs. Where any worker may undertake any job albeit with varying degrees of skill. A job that happens to match a worder's skill costs less than that in which the operator is not as skillful.

The objective of the model is to determine the optimum (least-cost) assignment of workers to jobs.

MATHEMATICAL FORMULATION OF AN ASSIGNMENT PROBLEM

Consider an assignment problem of assigning n jobs to n machines (one job to one machine). Let c_{ij} be the unit cost of assigning i^{th} machine to the j^{th} job and

$$\text{Let } x_{ij} = \begin{cases} 1. \text{ if } j^{th} \text{ job is assigned to } i^{th} \text{ machine} \\ 0. \text{ if } j^{th} \text{ job is not assigned to } i^{th} \text{ machine} \end{cases}$$

The assignment model is then given by the following LLP

$$\text{Minimize } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1, j = 1,2,\dots n$$

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2,\dots n$$

and $x_{ij} = 0$ (or) 1.

ASSIGNMENT ALGORITHM (OR) HUNGARIAN METHOD

If the number of rows and columns of an assignment problem are equal then the given problem is balanced. Then proceed to step 1. If the number of rows and columns are not equal, then it should be balanced before applying the algorithm.

Step 1: For the initial value matrix, establish every row's minimum and take off it from all the entries of the row.

Step 2: For the matrix ensuing from step one, establish every column's minimum and take off it from all entries of the column.

Step 3: Identify the optimal assignment as the one associated with zero element of the matrix obtained in step 2.

Note 1: In case some rows or columns contain more than one zero, encircle any unmarked zero randomly and cross all other zeros in its column or row. Continue this process until no zero is left unmarked or encircled.

Note 2: If the given assignment problem is maximum, convert it in to a minimization assignment problem by max Z = - min (-Z) and multiply all the given cost elements by -1 in the cost matrix and then solve by assignment algorithm.

Note 3: Some times, a final cost matrix contains more than required number of zeros at individualistic positions. It indirectly says that there is more than one optimal solution (multiple optimal solutions) with the same optimal assignment cost.

NUMERICAL EXAMPLE

Problem

The processing times in hours for the jobs when allocated to the different machines are indicated below. Assign the machines for the jobs so that the total processing time is minimum.

Machines

	M_1	M_2	M_3	M_4	M_5
J_1	9	22	58	11	19
J_2	43	78	72	50	63
J_3	41	28	91	37	45
J_4	74	42	27	49	39
J_5	36	11	57	22	25

Jobs

Solution: The cost matrix of the given problem is

9	22	58	11	19
43	78	72	50	63
41	28	91	37	45
74	42	27	49	39
36	11	57	22	25

Hence, the number of rows is equal to the number of columns in the cost matrix, the given assignment problem is balance.

Step 1. Select the smallest cost element in each row and remove this from all the elements of the relative row, we get the reduced matrix:

0	13	49	2	10
0	35	29	7	20
13	0	63	9	17
47	15	0	22	12
25	0	46	11	14

Step 2. Select the smallest cost element in each column and remove this from all the elements of the relative column, we get the following reduced matrix:

0	13	49	0	0
0	35	29	5	10
13	0	63	7	7
47	15	0	20	2
25	0	46	9	4

Step 3. Now we shall identify the rows successively. Second row carry a single unmarked zero, encircle this zero and cross all other zeros in its column. Third row carry a single unmarked zero, encircle this zero and cross all other zeros in its column. Fourth row carry a single unmarked zero, encircle this zero and cross all other zeros in its column. After this all the rows are having more than one unmarked zero, so go for columns.

Identify the columns successively, Fourth column contains exactly one unmarked zero, encircle this zero and cross all other zeros in its row. After identifying all the rows and columns, we get

0 ╳	13	49	(0)	0 ╳
(0)	35	29	5	10
13	(0)	63	7	7
47	15	(0)	20	2
25	0 ╳	46	9	4

Step 4. Since the 5th row and column do not have any assignment the present assignment is not optimal.
Step 5. Draw the minimum attainable range of horizontal and vertical lines therefore on cover all the zeros.

~~0~~	~~13~~	~~49~~	~~0~~	0	
~~0~~	~~35~~	~~29~~	~~5~~	10	
13	0	63	7	7	√
~~47~~	~~15~~	~~0~~	~~20~~	2	
25	0	46	9	4	√
	√				

Step 6. In this 4 is the smallest element not covered by these straight lines. Subtract 4 from all the not covered elements and add 4 to all those elements which are placing in the intersection of these straight lines and do not change the remaining elements which lie on these straight lines, we get the succeeding matrix.

0	17	49	0	0
0	39	29	5	10
9	0	59	3	3
47	19	0	20	2
21	0	42	5	0

Since each row and each column contains not less than one zero, we investigate the rows and columns successively, i.e., repeat step 3 above, we get

0 ╳	17	49	(0)	0 ╳
(0)	39	29	5	10
9	(0)	59	3	3
47	19	(0)	20	2
21	0 ╳	42	5	(0)

In the above matrix, each row and each column contains absolutely assignment (i.e., exactly one bounded zero), therefore the current assignment is optimal.

Therefore the optimum assignment schedule is $J_1 \rightarrow M_4$, $J_2 \rightarrow M_1$, $J_3 \rightarrow M_2$, $J_4 \rightarrow M_3$, $J_5 \rightarrow M_5$ and the optimum (minimum) processing time
= 11 + 43 + 28 + 27 + 25 hours
= 134 hours.

ALTERNATE METHOD FOR SOLVING ASSIGNMENT PROBLEM

The new algorithm is as follows:

Subtract the smallest element of each row from every element of the relative row.

1. Subtract smallest element of each column from every element of the relative column.
2. Consider the place of zero at each row. If row contain only one zero then assign it for the relative row and delete the relative row and column after allotment. Otherwise read the place of zero below for further process.
3. If one more zero is present then find the substitute of zero and compare the maximum value and assign zero.
4. Repeat (3) and (4), find the optimal solution.

Numerical Example Using Alternate Method

	A	B	C	D	E
1	9	22	58	11	19
2	43	78	72	50	63
3	41	28	91	37	45
4	74	42	27	49	39
5	36	11	57	22	25

Step 1. Simplified matrix after row reduction:

	A	B	C	D	E
1	0	13	49	2	10
2	0	35	29	7	20
3	13	0	63	9	17
4	47	15	0	22	12
5	25	0	46	11	14

Step 2. Simplified matrix after column reduction:

	A	B	C	D	E
1	0	13	49	0	0
2	0	35	29	5	10
3	13	0	63	7	7
4	47	15	0	20	2
5	25	0	46	9	4

Observe the location of zero

ROW	COLUMN
1	A, D, E
2	A
3	B
4	C
5	B

Issue 4→C and withdraw the 4^{th} row and C column of the above matrix.

Step 3. Simplify the matrix after removing row and column

	A	B	D	E
1	0	13	0	0
2	0	35	5	10
3	13	0	7	7
5	25	0	9	4

Again observe the place of zero's

ROW	COLUMN
1	A, D, E
2	A
3	B
5	B

Then 3rd and 5th row have same number of zero's then choose anyone

Allot 3→B and remove the 3rd row and B column

Step 4. The reduced matrix is.

	A	D	E
1	0	0	0
2	0	5	10
5	21	5	0

In row 5, there is no zero's, repeat step 1 & 2 observe the place of zero

ROW	COLUMN
1	A, D, E
2	A
5	E

Since row 1 & 2 have same number of zero then take any one

Assign 2→A and remove the 2nd row and A column

Step 5. The reduced matrix is

	D	E
1	0	0
5	5	0

Observe the place of zero

ROW	COLUMN
1	D, E
5	E

Since row 1 & 2 have same number of zero's, then take any one
Assign 5→E and 1→D
Therefore the assignment is 4→C, 3→B, 2→A, 1→D, 5→E
= 27 + 28 + 43 + 11 + 25
= 134.

Table 1: *COMPARISON*

PROBLEM	HUNGARIAN METHOD	ALTERNATE METHOD
1	134	134

CONCLUSION

This Paper used to solve an assignment problem in an easy way. It was observed that the solution is the minimum achievable result. The use of Hungarian method gives a well ordered and clear solution. We get the optimal solution which is same as the optimal solutions of Hungarian-method. Hence, this article gives a new method which is easy to solve Assignment problem.

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