

MATHEMATICAL MODELLING AND SIMULATION OF FULL CAR SUSPENSION SYSTEM

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ABSTRACT

The objective of this paper is to mathematical modeling in relation with state space and eight degree freedom system of car seat and suspension system to determine set of parameters to maintain to achieve best condition of driver and occupants in vehicle. This index eliminates the need for further subjective estimations and can be a useful parameter in various correlation analyses and vibration comfort predictions.

The vertical acceleration depends upon suspension parameters such as sprung mass, unsprung mass, stiffness of spring, damping coefficient and road profile. We have consider the passive type, constraints comes from practical and comfort ability considerations such as limit of vertical acceleration of passenger seat, tire displacement and suspension working space.

This current study is an objective evaluation of vehicle comfort characteristics based on mathematical modeling and simulation analysis. A variety type of road types will be selected to assess transmitted vibrations transmitted to the passengers. The displacements for respective direction are main account parameter Gausdian input is most efficient. In future scope it will preferable to use optimization technique to evaluate parameters to optimum jerk to passenger with experimentation.

KEYWORDS: Mathematical modeling, road profile, state space, passive suspension.

INTRODUCTION

Suspension system plays an important role for a comfortable ride for passengers besides protecting the chassis and other working parts from getting damaged due to road shocks. If in a vehicle both front and rear axles are rigidly fixed to the frame, while vehicle is moving on the road, the wheels will be thrown up and down due to the irregularities of road, as such there will be much strain on the component as well as the journey for the passengers in the vehicle will also be very uncomfortable. This is the system that provide comfortable ride and also prevent damage to the working parts.

The primary function of the suspension system is in a car to isolate the road excitations experienced by the wheels from being transferred to the passengers. The mathematical models are able to convert the system into mathematical equations so the equations will be solved and some rigid conclusions can be drawn for proper and optimized performance.

Alkhatib et al. (2004) used genetic algorithm method to the optimization problem of a linear 1DOF vibration isolator mount and the method is extended to the optimization of a linear 2DOF car suspension model and an optimal relationship between the RMS of the absolute acceleration and the RMS of the relative displacement was found.

Hassan Navi(2009) examined account of daily exposure to vibration A(8) and Vibration Dose Value (VDV) exposed to the passengers travelling in the train and the effects produced by the exposure towards human body. From the results shown, the whole-body vibration exposure level can be determined.

Gobbi (2001) used a 2DOF vehicle model and introduced an optimization method, based on Multi-Objective Programming of analytical formulae featuring the best compromise among conflicting performance indices pertaining to the vehicle suspension system, i.e. discomfort, road holding and working space.

Huijun Gao(2005) presents a load-dependent controller design approach to solve the problem of multi-objective control for vehicle active suspension systems by using linear matrix inequalities. A quarter-car model with active suspension system is considered. The usefulness and the advantages of the proposed controller design methodology are demonstrated via numerical simulations.

Sagar Deshpande(2005) a comprehensive optimal design solution is presented for piecewise-linear vibration isolation systems. First, primary suspension optimum parameters are established, followed by an investigation of jump-avoidance conditions for the secondary suspension Averaging method is employed to obtain an implicit function for frequency response of a bilinear system under steady-state conditions

O Gundogdu(2006) presented suspension system using genetic algorithms to determine a set of parameters to achieve the best performance of the driver. The optimization results are compared through step and frequency responses of the seat and suspension system for the optimum and currently used suspension systems
 Abdolvahab Agharkakli (2012) studied to obtain a mathematical model for the passive and active suspensions systems for quarter car model. Active suspension poses the ability to reduce the traditional design as a compromise between handling and comfort by directly controlling the suspensions force actuators.
 Analysis of prior research shows suspension parameters are optimally designed to attain the best compromise between ride quality and suspension deflections.

EIGHT DEGREES OF FREEDOM (DOF) FULL CAR MODEL

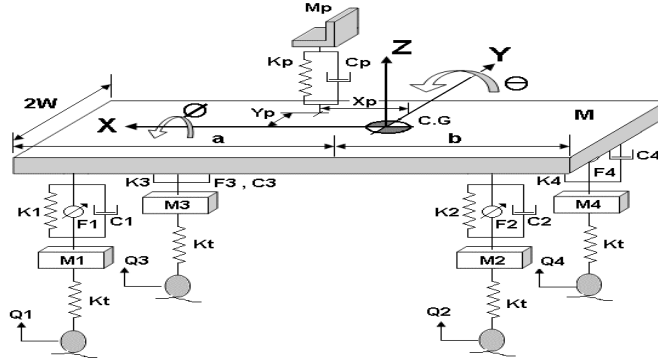


Fig.1. Eight DOF model

- | | |
|--|---|
| Mp: Passenger seat mass (kg) | M: Sprung mass (kg) |
| M1 & M3: Front left and front right side unsprung mass respectively (kg) | Xp & Yp: Distance of seat position from CG of sprung mass (m) |
| M2 & M4: Rear left and rear right side unsprung mass respectively (kg) | K1 & K3: Front left and front right side spring stiffness respectively (N/m) |
| Kp: Passenger Seat Stiffness (N/m) | K2 & K4: Rear left and rear right side spring stiffness respectively (N/m) |
| Kt: Tyre stiffness (N/m) | Cp: Passenger seat damping coefficient (Ns/m) |
| Q1 & Q3: Road input at front left and front right side respectively | C1 & C3: Front left and front right side suspension damping co-eff. respectively (Ns/m) |
| Q2 & Q4: Road input at rear left and rear right side respectively | C2 & C4: Rear left and rear right side suspension damping co-eff. respectively (Ns/m) |
| Ix: Mass moment of inertia for roll (kg-m ²) | a & b: C.G location from front and rear axle respectively (m) |
| Iy: Mass moment of inertia for roll (kg-m ²) | 2W: Wheel track (m) |

Using the Newton's second law of motion and free-body diagram concept, the following equations of motion are derived,

$$\begin{aligned}
 M\ddot{Z}_p + K_p(Z_p - Z) + C_p(\dot{Z}_p - \dot{Z}) &= 0 \\
 M\ddot{Z}_1 + K_1(Z_1 - Z) + C_1(\dot{Z}_1 - \dot{Z}) &= 0 \\
 M\ddot{Z}_2 + K_2(Z_2 - Z) + C_2(\dot{Z}_2 - \dot{Z}) &= 0 \\
 M\ddot{Z}_3 + K_3(Z_3 - Z) + C_3(\dot{Z}_3 - \dot{Z}) &= 0 \\
 M\ddot{Z}_4 + K_4(Z_4 - Z) + C_4(\dot{Z}_4 - \dot{Z}) &= 0 \\
 I_x \ddot{\theta} + K_t(Z_1 - Z - X_0\theta - Y_0\dot{\theta}) &+ K_t(Z_2 - Z - X_0\theta - Y_0\dot{\theta}) \\
 &+ K_t(Z_3 - Z - X_0\theta - Y_0\dot{\theta}) + K_t(Z_4 - Z - X_0\theta - Y_0\dot{\theta}) = 0
 \end{aligned}
 \tag{2}$$

$$A13 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0], A15 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1]$$

$$A4 = -1/M \begin{bmatrix} -K_p \\ -C_p \\ K1 + K2 + K3 + K4 + K_p \\ C1 + C2 + C3 + C4 + C_p \\ W(K1 + K2 + K3 + K4) + Y_p K_p \\ W(C1 + C2 + C3 + C4) + Y_p C_p \\ (-aK1 + bK2 - aK3 - bK4) + X_p K_p \\ (-aC1 + bC2 - aC3 - bC4) + X_p C_p \\ -K1 \\ -C1 \\ -K2 \\ -C2 \\ -K3 \\ -C3 \\ -K4 \\ -C4 \end{bmatrix}^T$$

$$A6 = -1/Ix \begin{bmatrix} K_p Y \\ C_p Y \\ W(K1 + K2 + K3 + K4) + Y_p K_p \\ W(C1 + C2 + C3 + C4) + Y_p C_p \\ W^2(K1 + K2 + K3 + K4) - Y_p^2 K_p \\ W^2(C1 + C2 + C3 + C4) - Y_p^2 C_p \\ (-aK1W + bK2W + aK3W - bK4W) + X_p Y_p K_p \\ (-aC1W + bC2W + aC3W - bC4W) + X_p Y_p C_p \\ -K1W \\ -C1W \\ -K2W \\ -C2W \\ -K3W \\ -C3W \\ -K4W \\ -C4W \end{bmatrix}^T$$

$$A8 = -1/Iy \begin{bmatrix} K_p Y \\ (-aK1 + bK2 - aK3 + bK4) - X_p K_p \\ (-aC1 + bC2 - aC3 + bC4) - X_p C_p \\ (-aK1W + bK2W + aK3W - bK4W) - X_p Y_p K_p \\ (-aC1W + bC2W + aC3W - bC4W) - X_p Y_p C_p \\ a2(K1 + K2 + K3 + K4) - X_p^2 K_p \\ a2(C1 + C2 + C3 + C4) - X_p^2 C_p \\ (-aK1W + bK2W + aK3W - bK4W) + X_p Y_p K_p \\ (-aC1W + bC2W + aC3W - bC4W) + X_p Y_p C_p \\ aK1 \\ aC1 \\ -bK2 \\ -bC2 \\ aK3 \\ aC3 \\ -bK4 \\ -bC4 \end{bmatrix}^T$$

$$A10 = \frac{-1}{M1} \begin{bmatrix} 0 \\ 0 \\ -K1 \\ -C1 \\ K1W \\ C1W \\ aK1 \\ aC1 \\ Kt + K1 \\ C1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$$

$$A14 = \frac{-1}{M2} \begin{bmatrix} 0 \\ 0 \\ -K3 \\ -C3 \\ K3W \\ C3W \\ aK3 \\ aC3 \\ 0 \\ 0 \\ 0 \\ 0 \\ Kt + K2 \\ C3 \\ 0 \\ 0 \end{bmatrix}^T$$

ROAD PROFILE

Road irregularity or unevenness represents the main disturbing source for either the rider or vehicle structure itself.

$$Q_{1,3} = \begin{cases} \frac{h}{2} & (1 - \cos \omega t) & \text{if } 0 \leq t \leq \frac{2l}{v} \\ 0 & & \end{cases}$$

$$Q_{2,4} = \begin{cases} \frac{h}{2} & (1 - \cos \omega t) & \text{if } 0 \leq t \leq \frac{2l}{v} \\ 0 & & \end{cases} \quad (10)$$

3.1 PASSIVE SUSPENSION SYSTEM

For passive suspension system as there is no actuator force i.e. [F] =0 and Eq.(9) becomes

$$\dot{X} = AX + BQ \quad (11)$$

3.2 SIMULATION

Table 1. Fixed parameters

Parameter	Values	Parameter	Values
K1	18760	M	1000
K3	18760	M1	85
K2	37520	M2	60
K4	32000	M3	85
C1	900	M4	60
C3	900	Mp	100
C2	936	a	1.07
C4	936	b	1.32
C3	900	w	3
Kt	150000	Xp	0.234
Ct	2000	Yp	0.375

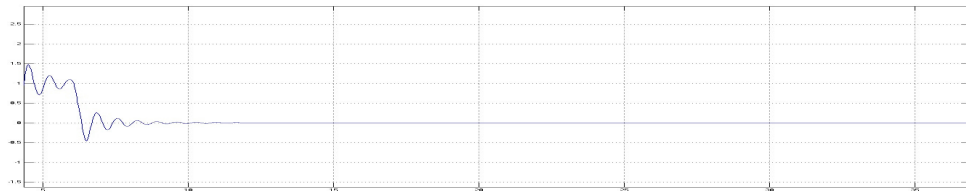
Simulink software is tightly integrated with the MATLAB environment. It requires MATLAB to run, depending on it to define and evaluate model and block parameters

1. Define model inputs.
2. Store model outputs for analysis and visualization.
3. Perform functions within a model, through integrated calls to MATLAB operators and functions.

RESULTS AND DISCUSSION

The model was simulated using different inputs to them in profiles, thus graph were plotted for the different inputs to predict behavior of system for inputs.

Step input –



Magnitude

Time

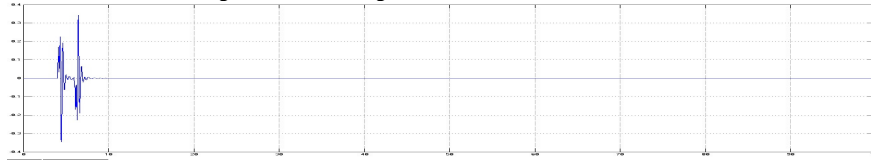
Graph 1 .Heave Direction displacement



Time

Magnitude

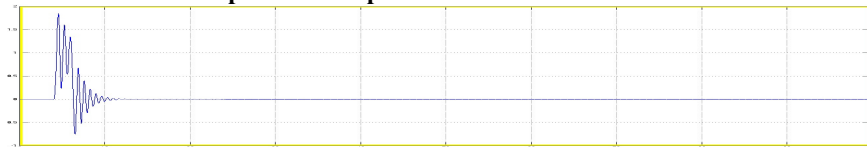
Graph 2. Pitch Displacement



Time

Magnitude

Graph 3. Roll Displacement



Time

Magnitude

Graph 4. Seat Displacement

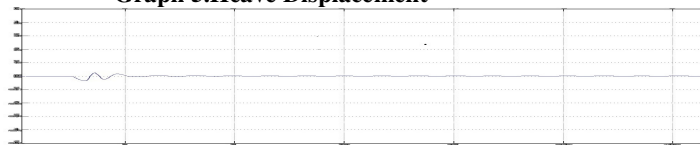
INPUT SIGNAL -PULSE



Time

Magnitude

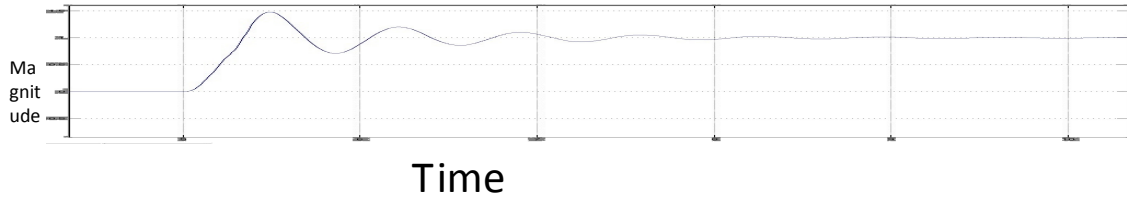
Graph 5. Heave Displacement



Time

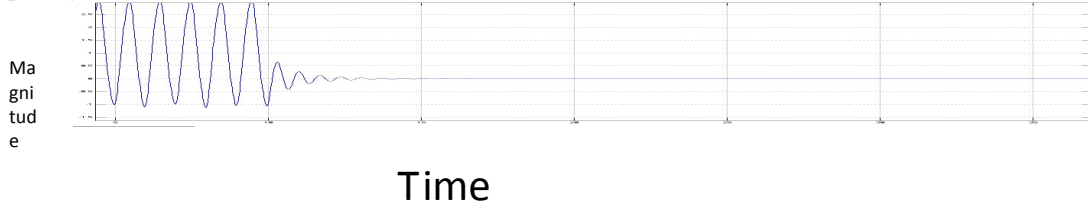
Magnitude

Graph 6. Pitch Displacement

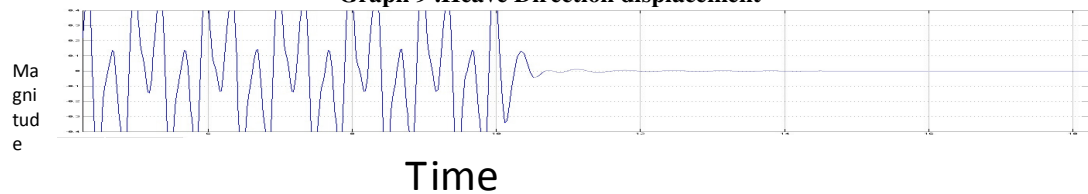


Graph8. Seat Displacement

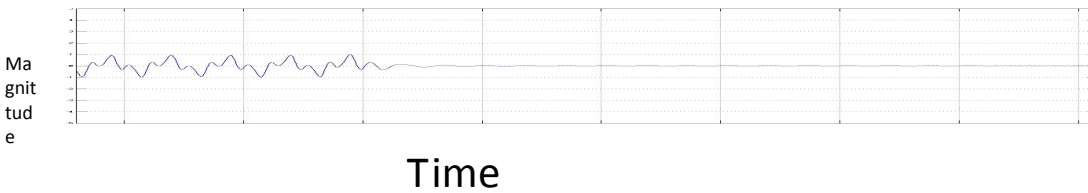
Input signal -Square



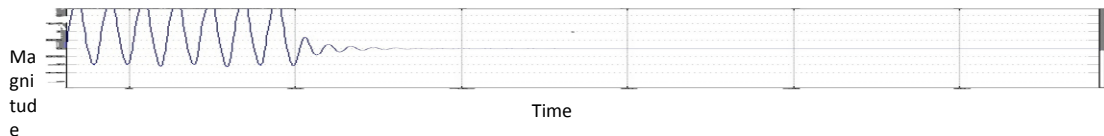
Graph 9 .Heave Direction displacement



Graph 10 .Roll Direction displacement



Graph 11 .Pitch Direction displacement



Graph 12 .Pitch Direction displacement

As the input signal is varied as per instant condition with model. The output generated graph has shown in above pages. as varying the input signal from step the input was given to it displacement in all respective four parameters i.e. Heave, Roll, Pitch, Seat is to be counted as rising at starting but goes to decreasing as time passes to the step in put function, Where as in pulse generated function output was associated with mid range values s for all the four parameters .In shortly speaking associated with mid plane peak values only.

The second group of input section was associated with Triangular signal it was clearly showing no displacements in seat and roll whereas it showed in sheave and pitch directions. For similar case was clearly seen for Gausdian type of input signal except with roll.

With improved signal type to actual road profile may maintain similarity for sinusoidal type only absence in seat displacement at certain speed of extent of level but as approaching to increase the overall speed of simulation it will goes to increase the amplitude of displacements for respective directions .

CONCLUSION

The total analysis of the conventional full car model is done using mathematical modeling. The full car model is dependent on the simple suspension system. But the actual car model uses the different model of suspension system. The strut used in this full car model is Macpherson's strut. To measure the various performance parameters like vertical acceleration, frequency of suspension vibration the test rig can be manufactured, which is the future scope part of the project with appropriate optimization tool must be implemented. From above all work it can be concluded that, the comfort to the rider can be given when suspensions are good. It can be possible to find the range for which the vertical acceleration can be reduced to the minimum value. The comfort is not dependent only on the spring or the on the damper. It should be selected such that the stability and comfort both are achieved simultaneously.

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