Experimentation of the time response of the cantilever beam

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Abstract

This paper is devoted to the study of the time response of the cantilever beam. The experimental results, obtained by Malatkar (2003), for the steady state response of the beam are presented first. These experimental results are compared with the corresponding numerical results, obtained with the *NLB* [8] program. Then the *NLB* program is used to calculate the transient response of the beam, which is compared against results from a finite element model developed in ANSYS®.

Introduction
Experimental Steady State Response

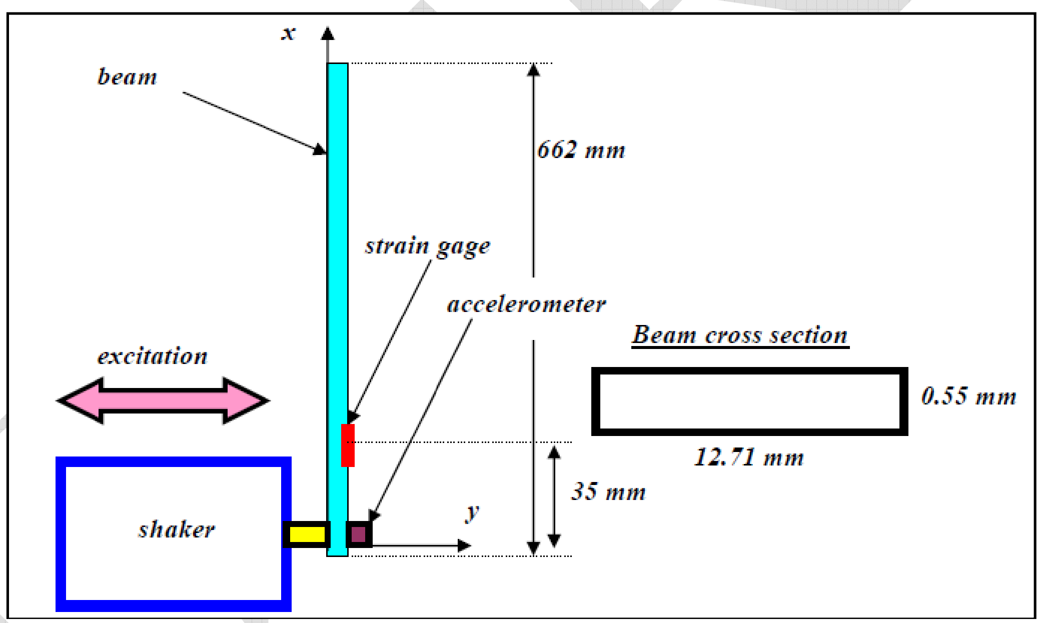

Figure 1: Experimental set up

Figure 1 shows the experimental set up used by Malatkar (2003) to measure the time response of the cantilever beam when subjected to harmonic excitation at the base. The beam is made of steel with Young's modulus of *165.5 GPa*, density of $7400\ kg/m^3$ and dimensions as indicated in Figure 1. The shaker excites the base of the beam in the y direction (Figure 4.1). An accelerometer is attached at the same point to monitor the input excitation to the beam. A strain gage is mounted approximately *35 mm* from the base. At this location the strains are maximum and also easy to measure. The strain read at the base is used to obtain the frequency response.
The harmonic excitation applied by the shaker to the base of the beam is given by

$$F = \rho A a_b \cos\Omega t \qquad (1.1)$$

where ρ, A, a_b, and Ω are the density, cross sectional area, maximum amplitude of acceleration at the base, and excitation frequency, respectively. Malatkar (2003) used an excitation frequency of *17.547 Hz* with maximum amplitude of acceleration equal to *2.97g*, where *g* is the acceleration due to gravity. Transients were allowed to die out before the response was recorded.

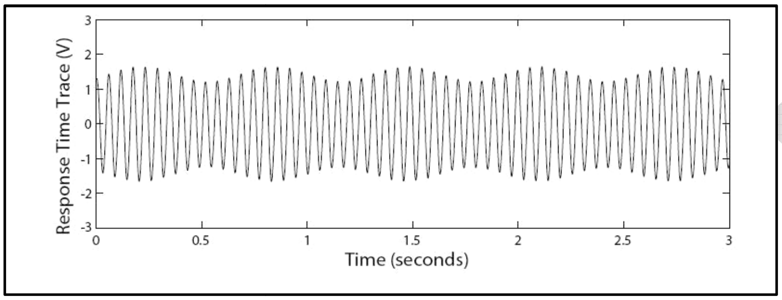

Figure 2: Time response for Ω= 17.547 Hz, a_b= 2.97g

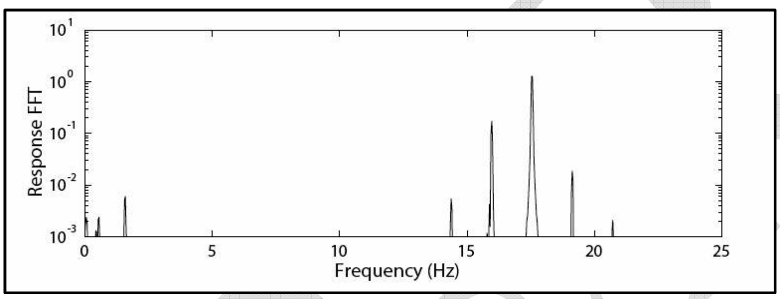

Figure 3: FFT for Ω= 17.547 Hz, a_b= 2.97g

Figure 2 shows the time response of the vertical cantilever beam (Figure 4.1) when Ω= *17.547 Hz* and a_b= *2.97g*. This excitation frequency is close to the third natural frequency of the beam (Table 1). The time response of the beam consists of a high frequency component modulated by a low frequency component.

Mode number (n)	$f_n(Hz)$	ξ_n
1	**0.574**	0.009
2	5.727	0.00185
3	**16.55**	0.00225
4	**32.67**	0.005

Table 1: Experimental natural frequencies and damping ratios [9]

The response in Figure 2 is not the actual displacement, but the strain reading in Volts. However, the frequency response obtained with the strain reading will be the same as the frequency response obtained using the actual displacement. Hence, the strain results can be used to determine the frequency response of the beam at the base. The FFT shown in Figure 4.3 is used to determine the actual frequency components present in the time trace (Figure 2). The high frequency component is centered at *17.547 Hz*, i.e., the excitation frequency. The asymmetric sideband structure around the high frequency peak indicates the third mode frequency component is modulated (Malatkar, 2003). The modulation frequency can be calculated from the sideband spacing, and it is found to be *1.58 Hz* for this case. Figure 3 also indicates the presence of a low frequency component in the response. This low frequency component is centered at *1.58 Hz*, i.e., the modulation frequency of the high frequency component. Anderson, et al. (1992) found the low frequency component in the response of a beam excited close to a high frequency mode, is equal to the modulation frequency of the high frequency component.

Numerical Steady State Response

The numerical results for the response of a point at the base of the beam are presented next. The results were obtained with the program *NLB*, included in Appendix A. The beam dimensions and material properties used are the same as in section 1 The cantilever beam was meshed with 20 elements. Thus, the length of each element is *33.1 mm*. Moreover, each element was divided into *10* subdivisions, resulting in a *x* of *3.1mm* (see Appendix D, Figure D.1). A time step of *0.001 sec* was used for the simulation.

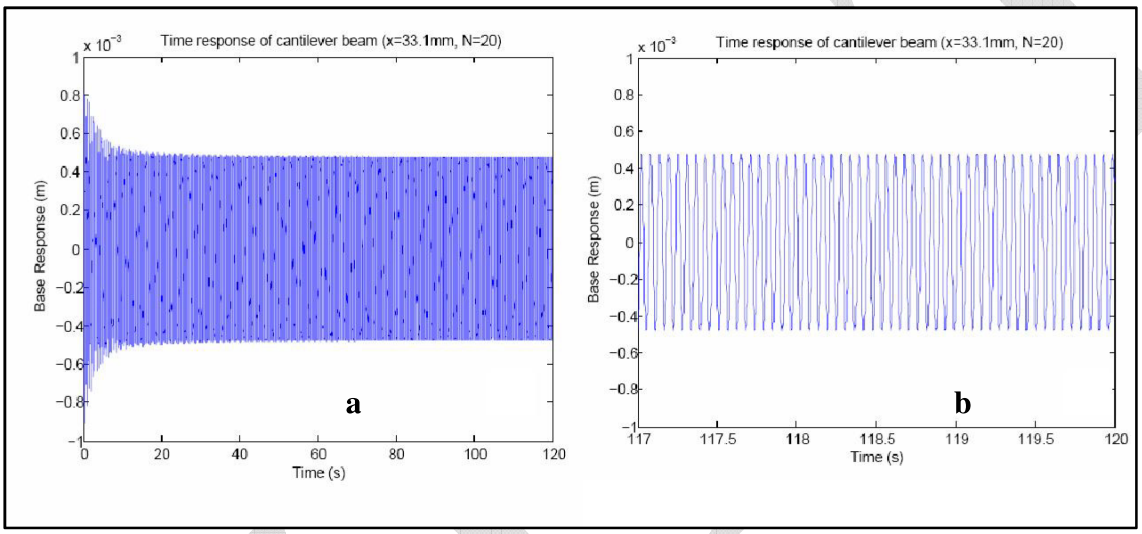

Figure 4: Numerical time trace for x= 33.1 mm.

Figure 4 shows the time trace for a point *33.1 mm* from the base of the beam. For this simulation the excitation frequency and maximum acceleration at the base are *17.547 Hz* and *2.97g*, respectively. Most of the transient response dies out in the first *20* seconds of simulation. The transient response continues to decay at a slower rate from *t=20 sec* to *t= 80 sec* (Figure 4). For time greater than *80* seconds, the response is considered to be the steady state response. Figure 4-b zooms into the last three seconds of the time response in Figure 4-a. The presence of a high frequency component is clear in Figure 4-b.

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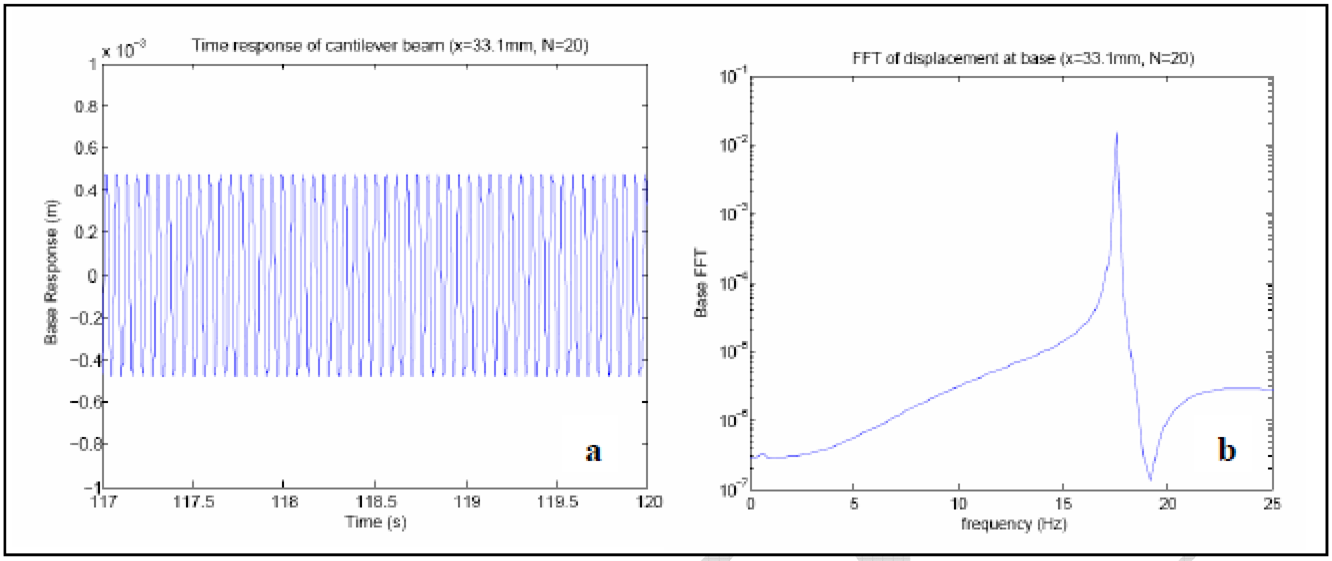

Figure 5: Base response and FFT for Ω= 17.547 Hz, and a_b= 2.97g

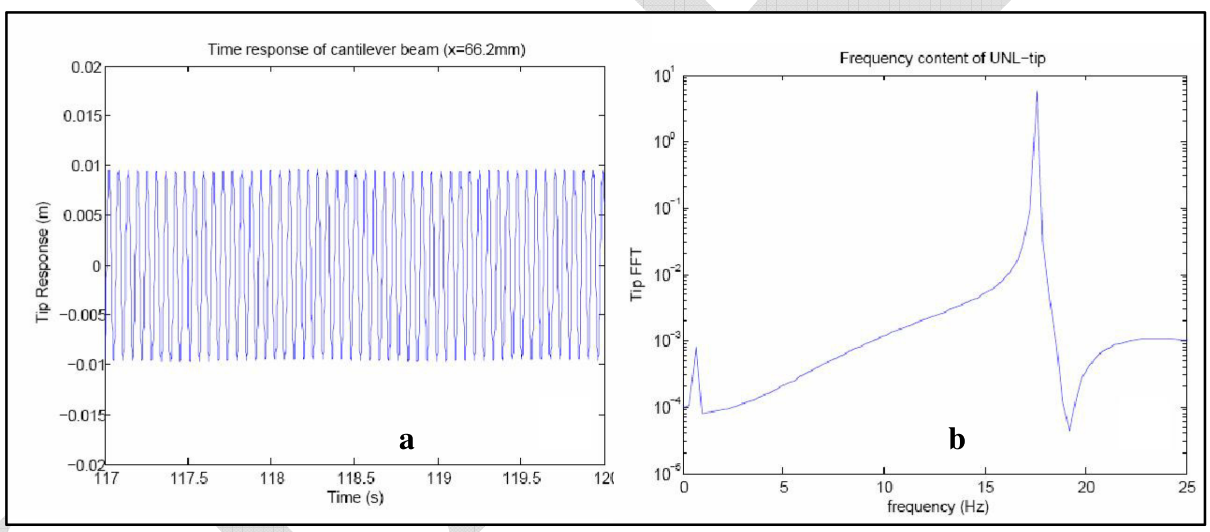

Figure 6: Tip response and FFT for Ω= 17.547 Hz, and a_b= 2.97g

The FFT (Figure 5-b) for the last three seconds of simulation (Figure 5-a) shows two frequency components: one at *17.547 Hz* and another at *0.651 Hz*. Only the high frequency component is in agreement with the experimental results presented in section above. However, there is no side band structure around the high frequency peak. Therefore, the high frequency component is not modulated. Figure 6 shows the response and FFT for the tip of the beam. The high and low frequency components are centered at the same frequencies as the results at the base (Figure 5). However, the amplitude of the low frequency component at the tip of the beam (Figure 6-b) is significantly larger compared to the low frequency component at the base (Figure 5-b). The FFT's for the numerical results were calculated using Matlab®. For this calculation, 512 points were sampled from the last three seconds of the time trace (Figures 5-a and 6-a). The number of sampled points is a power of 2, thus reducing the computation time for the FFT (Baher).The numerical results show a high frequency component that is not modulated, and a low frequency component. These results do not agree with the experimental results

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obtained by Malatkar (2003). This discrepancy with the experimental results could be attributed to the presence of numerical error in the calculation of the nonlinear stiffness matrices kc_{ij} and ki_{ij} . Therefore, the numerical error in these matrices is investigated next.

The nonlinear stiffness matrices are defined in terms of the functions f_1 and f_2 given by

$$f_1 = (v'v'')', \quad f_2 = \frac{\partial^2}{\partial t^2}\int_l^s\int_0^s v'^2\, ds ds \qquad (1.2)$$

In order to investigate the numerical error in the calculation of f_1 and f_2, these quantities are calculated for a displacement of the form

$$v(x,t) = A_1[\sinh \beta_1 x - \sin \beta_1 x - \varepsilon_1(\cosh \beta_1 x - \cos \beta_1 x)]\sin \Omega t \qquad (1.3)$$

The bracketed term in (1.3) is simply the first linear mode shape for a cantilever beam. The constants A_1, β_1 and ε_1 are included in Table B.2. Substituting (1.3) into (1.2) yields the analytical f_1 and f_2, which will be compared to the numerical f_1 and f_2, calculated.

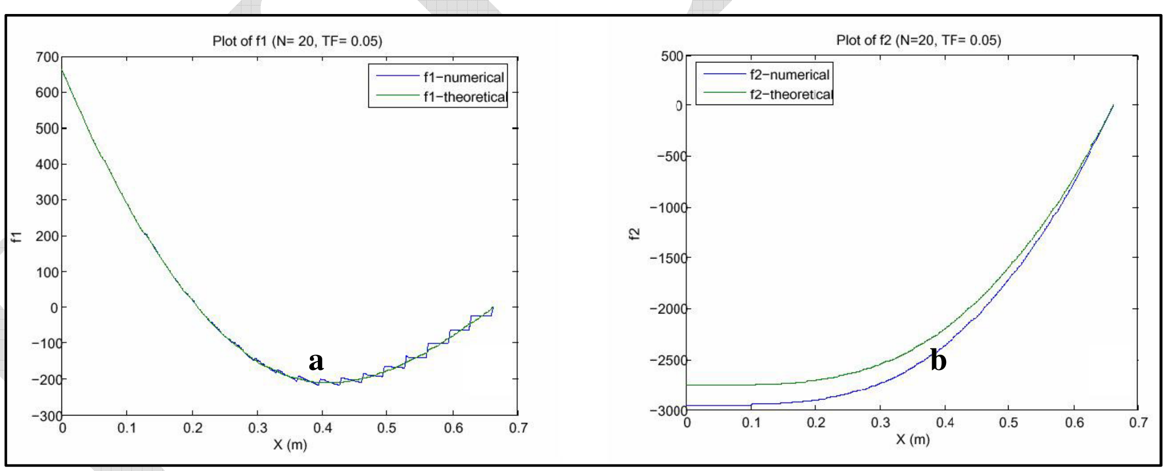

Figure 7: Analytical and numerical f_1 and f_2

Figure 7 shows the analytical f_1 and f_2 compared against their numerical counterparts. The numerical f_1 and f_2 were calculated using 20 elements. There is numerical error in the calculation of f_1 (Figure 7-a). However, this numerical error is noticeable for *x > 300 mm*. At *x= 33.1 mm*, the point of interest for the time response, there is no significant error between the numerical and analytical f_1. Therefore, the numerical error in the calculation of f_1 does not affect the time response of the beam at the base. There is a significant amount of numerical error in f_2

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(Figure 7-b). This error is maximum at the base of the beam. Moreover the inertia nonlinearity, represented by f_2 is known to dominate the response of the high frequency modes (Nayfeh and Pai, 2004). Hence, the significant amount of numerical error on this term contributes to the discrepancy between experimental and numerical results.

Transient Response

This section is devoted to the study of the transient response of the cantilever beam. The finite element model developed in ANSYS® is presented first. The results from this model are compared to results from the *NLB* program.

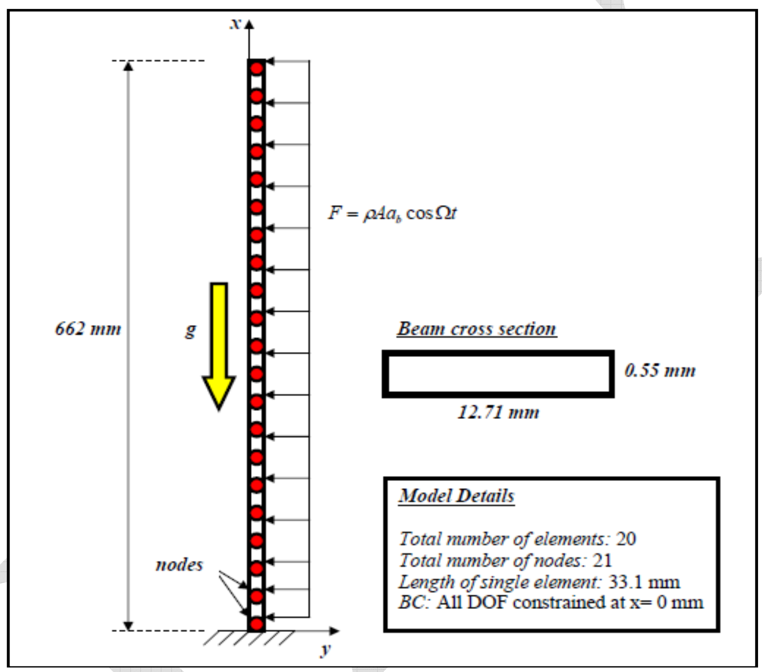

Figure 8: Mesh and boundary conditions for ANSYS® model

Figure 8 shows the geometry used in the finite element model. The vertical beam has the same dimensions of the beam used by Malatkar (2003) to measure the experimental steady state response. Moreover, the beam is made of isotropic steel with properties as listed in section 1. The beam is meshed with *20* beam3 [10] elements. Each element is *33.1 mm* long and has section properties calculated using the dimensions in Figure 8.

All degrees of freedom are constrained at the base of the beam (Figure 8). This is consistent with the boundary conditions of the problem. The effect of gravity is included in the simulation as shown in Figure 8. The forcing function given by (1.1) is distributed evenly throughout all *21* nodes. The transient response of the beam is calculated for the first three seconds. The forcing function given in (1.1) is approximated as a series of straight lines. Figure 9 shows the approximation of one cycle of the forcing function. The cycle of the forcing function is divided into *14* load steps. This is a reasonable approximation of the exact function (Figure 9). A total of *739* load steps is required to obtain the transient response for the first three seconds.

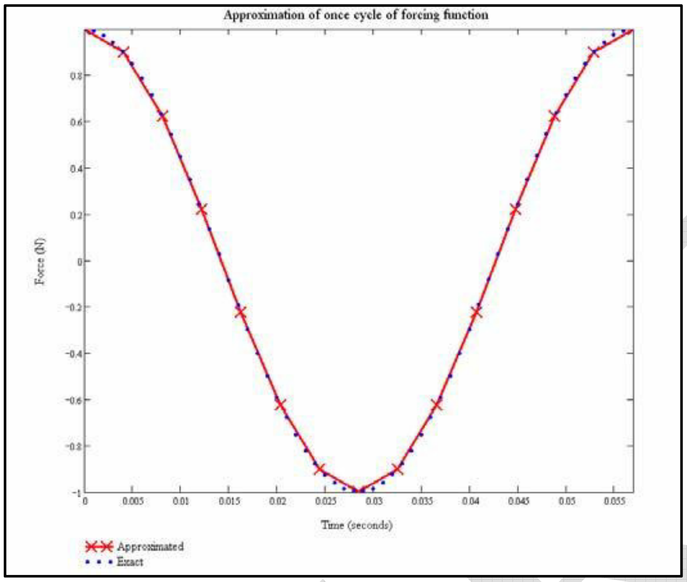

Figure 9: Approximation of one cycle of the forcing function

Figure 10 shows the response and FFT obtained in ANSYS®. The response plot (Figure 10-a) suggests the presence of multiple frequencies. The FFT [11] (Figure 10-b) reveals four frequencies dominating the time response.

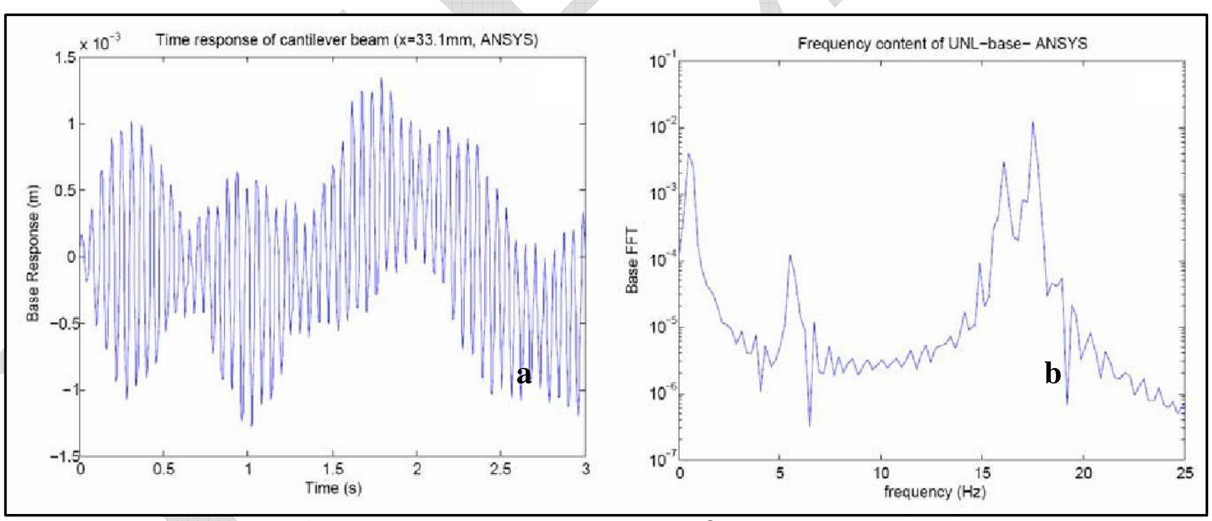

Figure 10: Response and FFT from ANSYS®

The four peaks in the FFT (Figure 10-b) occur at *0.4798 Hz*, *5.518 Hz*, *16.07 Hz* and *17.51 Hz.*

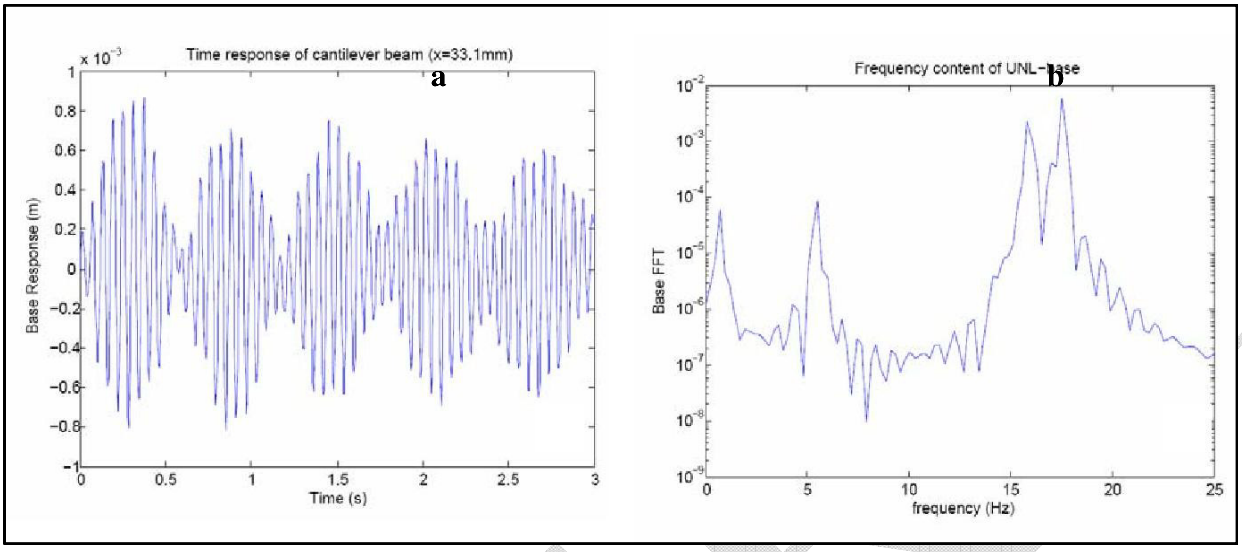

Figure 11: Response and FFT from NLB

Figure 11 shows results obtained with *NLB*. The four frequencies that dominate the response are centered at *0.7197 Hz*, *5.518 Hz*, *15.83 Hz*, and *17.51 Hz*. A combined plot of the FFT obtained with ANSYS® (Figure 12) and the one obtained with *NLB* shows both FFT's are in agreement for the most part. The first frequency obtained with *NLB* is *50%* higher compared to ANSYS®. The remaining three frequencies are extremely close to the numerical results from *NLB*. Also, the first mode has a more significant participation in the time response obtained with ANSYS® (Figure 10), compared to the response obtained with *NLB* (Figure 11).

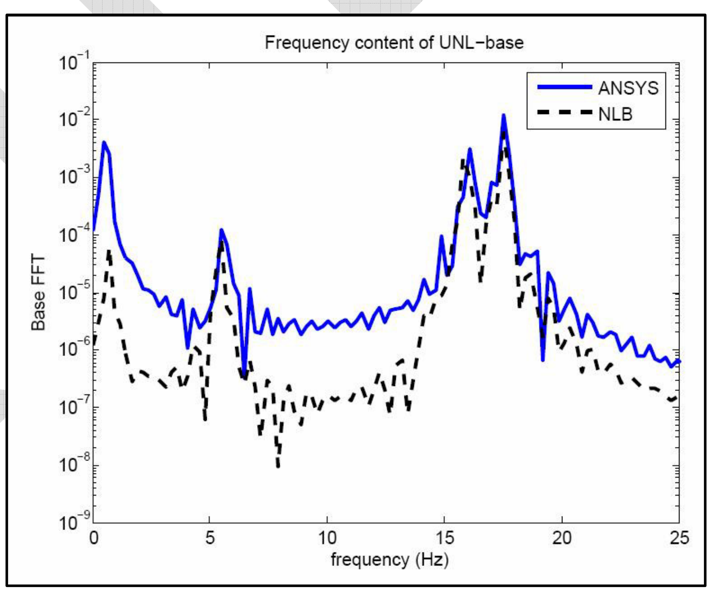

Figure 12: Combined plot of FFT's

Conclusion

The finite element model for the nonlinear transverse vibration of the beam was implemented in the program *NLB* [12]. This program was used to calculate the steady state response of the beam. The numerical results from the program were compared to the experimental results obtained by Malatkar (2003). The *NLB* program was also used to calculate the transient response of the beam, which was compared with the response obtained with ANSYS®. The steady state response for the vertical cantilever beam was measured by Malatkar (2003) for an excitation frequency of *17.547 Hz*, which is close to the third natural frequency of the beam. The FFT of the experimental results shows a modulated high frequency component centered at *17.547 Hz*, and a low frequency component centered at *1.58 Hz*. Moreover, the modulation frequency for the high frequency component is equal to the low frequency component, i.e., *1.58 Hz*. The FFT for the steady state response computed with the program *NLB* shows a high frequency component centered at *17.547 Hz*. Unlike the experimental results from Malatkar (2003), this high frequency component is not modulated. The absence of modulation in the high frequency component is attributed to the presence of numerical error in the computation of the nonlinear inertia term. Nayfeh and Pai (2004) showed that the nonlinear inertia term dominates the response of the third mode in a highly flexible beam. The FFT for the numerical results also shows a low frequency component centered at *0.651 Hz*. This result is not in agreement with the experimental results. The transient response for the first three seconds, obtained with ANSYS® , is dominated by four frequency components centered at *0.4798 Hz, 5.518 Hz, 16.07 Hz,* and *17.51 Hz.* The transient response determined with *NLB* is dominated by frequency components centered at *0.7197 Hz, 5.518 Hz, 15.83 Hz,* and *17.51 Hz.* These results are in agreement with ANSYS® except for the first frequency component, which is *50%* higher with *NLB*. In summary, the numerical results obtained with *NLB* differ from the experimental results (Malatkar, 2003) due to the presence of numerical error in the former. The transient response calculated with *NLB* agrees with the response calculated with ANSYS® for the most part.

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