

# Economic Order Level Inventory Model with Exponential Increasing Demand

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## ABSTRACT

This paper presented an economic order level inventory model with exponential decreasing demand for deteriorating items. The shortage of items has been considered and optimized. A mathematical inventory model is suggested in the content for a business cycle which starts with shortage and able to place the order according to the demand and customer's response. This model is applicable in various industries and may significantly increase their productivity and efficiency; this illustrates the effectiveness of model.

## 1. Introduction

Due to a big mass of costumers fallow through advertisement which may be through electronic media, news papers, internet or companion so, for rebate in price, management go with advertisement or displaying the goods in super market. For deckling market we considered decay in demand linearly then management manipulates the selling price dynamically and then readjusted demand will be time dependent price sensitive which is more than existing. Also manipulation the selling price dynamically helps to compete to other in deckling market.

Manufacturing Company often hit a problem with the EOQ. When their batch set-up costs are high, the EOQ (can suggest) very large batches- which complicate production/ buying scheduling, give long lead times to costumer need excess storage capacity, and leave too much capital tied-up in stocks. These problems can be avoided by putting an artificially high value on the holding cost but it illustrates is weakness of calculation. In above contribution a motivation is derive to develop an inventory model of different rate of demand considering shortage and constant deterioration rate.

Fergany (2016) proposed a probabilistic multi-item, single source inventory model with varying mixture shortage cost under two restrictions. Chan *et al.* (2003) designed an EPQ model to provide a framework to integrate lower pricing, rework and reject situations by assuming that the items of imperfect quality, not necessarily defective, could be used in another production situation or sold to a particular purchaser at lower price. Roy (2008) measured the effect of price sensitive demand with static price in a cycle but differs in different cycle, he also considered variable holding cost and shortage is completely backlogged. Sen and Chakrabarty (2007) considered a model for constant rate of demand and varying rate of production in which deterioration follows a special form of weibull density function of time allowing shortages.

Many authors have presented various EOQ models in the area of deteriorating items. The first attempts to describe

optimal policy for deteriorating items were made by Ghare P.M. and Schrader, G.P [5]. Aggrawal S.P. and Jain Veena[8] developed optimal inventory management for exponentially increasing demand with deterioration. This paper is wxtentio of paper Shukla *et al.* (2010). Naresh Kumar and Anil Kumar Sharma [4] presented a deterministic production inventory model for deteriorating items with an exponential declining demand.

In the EOQ setup, authors have attempted to determine the optimum ordering policy for deteriorating items under varying situations with different rate of demand. Few items in the market are of high need like sugar, wheat, oil whose shortage damage the customer arrival pattern in the shop. This paper is extension of model designed by Shukla *el al.* (2010). This motivates to shopkeepers (especially retailers) to order for excess quantity of items in spite of deterioration. Moreover, deterioration is manageable phenomenon for many items by virtue of advance storage scientific technology. Some EOQ models in literature of before decade of ethics do not consider the deterioration factor of items but later researches incorporated this parameter in model as a source of esteem importance [see [12], [2], [3], [6], [7], [10] [11].

## 2. Assumption and Notations

The model was developed under the following assumptions:

- Initial demand is  $\mu > 0$  at  $t=0$ ,
- Suppose demand rate  $R(t) = \mu a^{c \sqrt{t}}$  where 'c' is

any real number may be positive or negative subjected to depend on situation, t is time varies from 0 to T where T is prescribed time,

- Suppose initial stock is Q at time  $t=0$ ,
- Lead time is zero,
- Shortage is completely backlogged,
- $C_0$  is cost unit per unit time,
- Holding cost is  $C_1$  unit per unit time,

- Shortage cost is  $C_2$  unit per unit time,
- $\theta$  is on hand inventory random deterioration rate per unit time which depends upon time and higher power of  $\theta$  is negligible,
- There is no replenishment allowed of deteriorated units,
- $K(t^*)$  is optimum cost in system.

### 3. The Proposed Model

Let  $Q + S_1$  be the quantity enter in the system for each of the beginning of business schedule were  $S_1$  is shortage of previous cycle (if any) and ' $Q$ ' is quantity required to consume for the schedule time  $T$ . Let ' $\mu$ ' is the demand at beginning. Due to demand  $R(t) = \mu a^{c t/T}$  the stock ' $Q$ ' reduces gradually till time  $t_1$  and reaches to zero. Thereafter shortage occurs till time  $T$ . Suppose  $I(t)$  be the current stock level at any time  $t$  must satisfy the differential equation in two phases in the schedule time  $T$ . demand rate  $R(t) = \mu a^{c t/T}$  posses the negative derivative through out its domain.

$$\frac{d}{dt} I(t) + \theta I(t) = -\mu a^{c t/T} : 0 \leq t \leq t_1 \tag{1}$$

$$\frac{d}{dt} I(t) = -\mu a^{c t/T} : t_1 \leq t \leq T \tag{2}$$

Total quantity of deteriorated units  $d = Q - \int_0^{t_1} R(t) dt$

$$d = \mu k \left[ a^{c t_1/T} \left\{ \theta t_1 - \theta k \right\} + \mu \theta k^2 \right] \tag{3}$$

Total units of items in inventory during  $(0, t_1)$  is

$$q = \int_0^{t_1} I(t) dt$$

$$q = \mu k a^{c t_1/T} \left\{ \frac{\theta t_1^2}{2} + t_1(1 - \theta k) + \theta k^2 - k \right\} + \mu k^2(1 - \theta k) \tag{4}$$

Total number of shortage units in system,  $S = \int_{t_1}^T I(t) dt$

We substitute the value  $I(t)$  from equation (8)

$$s = \mu k \left[ a^{c t_1/T} \left\{ t_1 - T - k \right\} + k a^c \right] \tag{5}$$

Total cost per unit time

$$K(t_1) = \frac{\mu k C_0}{T} \left[ a^{c t_1/T} \left\{ 1 + \theta t_1 - \theta k \right\} - (1 - \theta k) \right] + \frac{\mu k C_1}{T} \left[ a^{c t_1/T} \left\{ \frac{\theta t_1^2}{2} + t_1(1 - \theta k) - k(1 + \theta k) \right\} + k(1 - \theta k) \right] + \frac{\mu k C_2}{T} \left[ a^{c t_1/T} \left\{ t_1 - T - k \right\} + k a^c \right] \tag{6}$$

$$\Rightarrow \frac{d}{dt_1} K(t_1) = \frac{\mu k}{T} a^{c t_1/T} \left[ \frac{C_1 \theta t_1^2}{2} + (\theta b + C_1 + C_2)t_1 + C_0 - TC_2 \right] = 0$$

$$C_1 \theta t_1^2 + 2(C_0 \theta + C_1 + C_2)t_1 + 2C_0 - 2TC_2 = 0$$

Above equation is quadratic equation of  $t_1$  and we can solve it say which is  $t_1 = t_1^* > 0$

For positive value of  $t_1^*$ ,  $C_1 < C_2$  and  $b \geq 0$ ,  $T \geq 0$ ,  $b \geq 0$ ,  $k \geq 0$ ,  $D \geq 0$

We get  $\frac{d^2}{dt^2} K(t_1) = \frac{\mu a^{c t_1/T}}{kT} \left[ C_1 \theta t_1^2 + (C_0 \theta + C_1 + C_2) \right] + \frac{\mu a^{c t_1/T}}{kT} \left[ k C_1 \theta t_1 - C_2 T + C_0 + k C_1 + k C_2 \right] \geq 0 \tag{7}$

So Average cost  $K(t_1)$  is minimum for said value  $t_1 = t_1^*$

### 4. Conclusion

Proposed model helps to study of different models at a time depending upon different demand rates. Mathematical formulation of economic order quantity model is developed and presented, optimum cost and time was calculated. This model is applicable in various industries and this may significantly increase their productivity and efficiency, this illustrates the effectiveness of model.

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