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Geothermal Analysis of Uturu- Nigeria

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***Abstract:** This paper presents the geothermal analysis of Uturu-- Nigeria using the Rayleigh-Ritz-Chebyshev collocation method. To develop the optimal geothermal design parameters for the Uturu-okigwe axis, the developed model was used to analyze the geothermal performance and investigate the effects of geothermal parameters using a longitudinal fin. The results, however, showed that whenever there is an increase in convective, radioactive, and magnetic parameters, the rate of heat transferred from the geothermal longitudinal fin increases. The result also showed that the Rayleigh-Ritz-Chebyshev spectral collocation method provided the best fit for analyzing the geothermal parameters of Uturu.*

***Keywords:** Rayleigh-Ritz-Chebyshev Collocation method, Runge-Kutta Method, Geothermal Longitudinal Fin, Optimal Thermal Design Parameters, Model.*

Nomenclature:

ar – The aspect ratio
M –fin parameter
b -fin length
m^2 -Thermo-geometric fin parameter
Ac- Cross-sectional area
Nr- Radiative parameter
Ap -Profile area of the fins
P - the fin perimeter
Bi -Biot number
T -Temperature
h- Coefficient of heat transferred
T∞- Ambient temperature
k - Fin material thermal conductivity
Tb -Temperature at the base of the longitudinal fin
ka-fin material at ambient temperature thermal conductivity
X- Dimensionless fin length
kb -Thermal conductivity
q -heat transfer rate
K - Fin material
of -Dimensionless heat transfer
β -Thermal conductivity parameter
ϴb- longitudinal fin
δ – fin thickness,
M η- Efficiency
θ -Dimensionless temperature
ε -Effectiveness

I. INTRODUCTION

The geothermal analysis of longitudinal fins both in convective and radioactive medium were subjected to intensive research over the past few decades (George & Raed, 2018, [1]). The need to investigate the geothermal performance of longitudinal fins ensured that numerical and analytical methods were developed using different techniques to proffer solutions to nonlinear equations. For two infinite parallel vertical plate fields, Sharifzadeh et al., (2015), [2] employed collocation and Galerkin's method to analyze incompressible fluid of third grade. Sobamowo et al., (2017), [3] conducted the thermal analysis of a natural convection porous fin with temperature-dependent thermal conductivity and internal heat generated using Galerkin's method. Rajul et al., (2017) [4] adopted the local Petrov- Galekin (MLPG) method to analyze transient heat transfer for longitudinal fins. Indra & Barry, (2010), [5] used the same method to confirm that thermal conductivity varies with material condition. Oguntala et al., (2019), [6] investigated the inclination of the thermal performance of heat sinks using the pseudo-spectral collocation method. Khani et al., (2016), [7] analyzed the thermal analysis of a fully wet porous using spectral collocation method. Marcos & José Luiz (2010), [8] adopted the generalized integral transform technique to investigate the effects of thermodynamic properties on longitudinal fins.

Atefeh et al., (2020), [9] investigated the performance of ground heat exchangers equipped with internal and external fins. Sobamowo (2016), [10] used the finite difference method in MATLAB to analyze the heat transferred and heat generated in a longitudinal fin. To analyze the distribution of a straight fin, Tabet et al., (2015), [11] adopted PAD Approximation (PADM) and Adomian Decomposition Method (ADM) to solve the boundary problems. Demba et al., (2016), [12] used the Runge–Kutta–Nystrom (RKN) method to analyze the performance of the longitudinal fins. Saeid (2006), [13] adopted the Homotropy Analysis Method (HAM) as the analytical tool for nonlinear problems [14]. Several pieces of literature in the past have adopted approximate analytical methods to determine the optimal values of longitudinal fins [15]. However, the search for a value that would satisfy the boundary and determine the auxiliary parameters orchestrated this research [16]. Therefore, in the present study, the geothermal analysis of Uturu-- Nigeria using the Rayleigh-Ritz-Chebyshev Collocation Method was developed to carry out the performance evaluation of Uturu to determine if it is a geothermal viable point [17].

II. PROBLEM FORMULATION

The development and analysis of the geothermal model are based on energy balance analysis in the longitudinal fin. This analysis considers some assumptions.

1. The geothermal heat flow in the fin and the temperature remains constant with time.
2. The temperature of the geothermal medium

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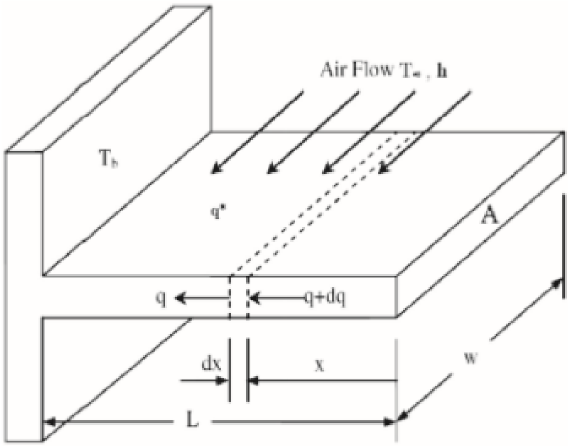
surrounding the film is uniform.
3. No known resistance at the point where the base of the fins joins the prime surface.
4. The temperature at the base of the fins is uniform.



[Fig.1: Schematics of the Longitudinal Straight Fin.] [10]

The differential equation for Rayleigh-Ritz is given by the equation below:

$$\frac{d}{du}\{k_\alpha[1+n(T-T_\alpha)]A_{cr}\frac{dT}{du}+\frac{4\sigma A_{Cr}}{3Br}\frac{dT^4}{du}\}= hp(T-T_\infty)+\sigma \in p(T^4-T_\alpha^4)\,du \quad (1)$$

Simplifying further gives the following governing differential equation

$$\frac{d}{du}[1+n(T-T_\alpha)]\frac{dT}{du}]+\frac{4\sigma}{3B_Rk_\alpha}\frac{d}{du}(\frac{dT^4}{du})-\frac{h}{k_\alpha t}(T-T_\alpha)-\frac{\sigma\in}{k_\alpha t}(T^4-T_\alpha^4)=0 \quad (2)$$

The boundary initial conditions are:

U=0, $\frac{dT}{du}$=0

U=0, T=T_b (3)

Let,

$$\frac{JexJe}{\sigma}=\sigma B_e^2u^2 \quad (4)$$

Substituting equation (4) in (2)

$$\frac{d}{du}[1+n(T-T_\alpha)]\frac{dT}{dx}]+\frac{4\sigma}{3B_Rk_\alpha}\frac{d}{du}(\frac{dT^4}{du})-\frac{h}{k_\alpha t}(T-T\alpha)-\frac{\sigma\in}{k_\alpha t}(T^4-T_\alpha^4)=0 \quad (5)$$

T^4 is expressed as a linear function of temperature.

$$T^4=4T_\alpha^3T-3T_\alpha^4 \quad (6)$$

Substitution (6) into (7)

Introducing dimensionless parameters of equation (3)

$$\frac{d}{du}[1+n(T-T_\alpha)]\frac{dT}{dx}]+\frac{16\sigma}{3B_Rk_\alpha}\frac{d}{du}(\frac{d^2T}{du^2})-\frac{h}{k_\alpha t}(T-T\alpha)-\frac{4\sigma\in pT_\alpha^3}{k_\alpha t}(T-T_\alpha)=0 \quad (7)$$

Introduction dimensionless parameters of equation (8) into equation (9)

$$u=\frac{u}{b},\theta=\frac{T-\Gamma_\alpha}{Tb-T\alpha},B=n(T_b\text{-}T_\alpha),M^2=\frac{pbh}{A_bk_\alpha},R_d=\frac{4\sigma stT_\alpha^3}{3BRK\alpha},N_r=\frac{4\sigma st\varepsilon bT_\alpha^3}{K\alpha} \quad (8)$$

The dimensionless form of (7)

$$(1+4R_d)\frac{d^2\theta}{du^2}+B\theta\frac{d^2\theta}{du^2}+B\left(\frac{d\theta}{du}\right)^2m^2\theta-N_r\theta=0 \quad (9)$$

Therefore,

$$\frac{d^2\theta}{du^2}+\frac{B}{(1+4R_d)}\theta\frac{d^2\theta}{du^2}+\left(\frac{B}{1+4R_d}\right)\left(\frac{d\theta}{du}\right)^2-\frac{M^2}{(1+4R_d)}\theta\frac{-Nr}{(1+4R_d)}\theta \quad (10)$$

This is also same as

$$\frac{d^2\theta}{du^2}+\hat{B}\theta\frac{d^2\theta}{dx^2}+\hat{B}\left(\frac{d\theta}{du}\right)^2-(M^x)_\theta^2-\widehat{N}_r\theta=0 \quad (11)$$

Where,

$$\hat{B}=\frac{B}{(1+4R_d)}[\widetilde{m}]^2=\frac{M^2}{(1+4R_d)},\widetilde{N}_r=\frac{N_r}{(1+4R_d)} \quad (12)$$

The dimensionless boundary conditions are written as;

U=0, $\frac{d\theta}{du}=0$

$U=1,\theta=1$ (13)

Solution Procedure

Rayleigh's Rits makes use of the Chebyshev collection spectral method. Chebyshev polynomial of degree N defined on the interval [1,-1], can be used for nodes [1,-1] namely;

U_i=cos $\left(\left(\frac{i\pi}{n}\right),i=0,1,2,3,4\ldots n\right.$ (14)

The derivatives are given by;

$$f''(u_i)=\sum_{i=0}^{n}d^nk_if(u_i),n=1,2\ldots \quad (15)$$

Let $d^n{}_{KJ}$ be the matrix of order n

When n=1,

$$d^1{}_{Ki}=\frac{4y_i}{n}\sum_{n=0,i=0}^{n}1\sum_{n+1=0dd}^{n}1\frac{ny_n}{C_L}T_l^n(u_k)T_n(u_i),k,i=0,1,\ldots n \quad (16)$$

n=2

$$d^2k_i=\frac{4y_i}{n}\sum_{n=0,i=0}^{n}1\sum_{n+1=0dd}^{n}1\frac{ny_n(n^2-L^2)}{c_1}T_L^n(u_k)(Tn)(u_i),\text{k,i=0,}1\ldots n \quad (17)$$

$(Tn)(u_i)$ are the Chebyshev polynomial and the coefficients y_i and c_i are defined as follows;

Y_i=between 0.5 to 1 i=o or n and i=1,2…n-1 (18)

C_i=between 1 to 2 i=o or n and i=1,2…n-1 (19)

It is imperative to make a suitable linear transformation and then transform the physical domain [-1, 1], there is a need to sample the unknown functions w at Chebyshev points to obtain the data vectors

W=[w(U_o), w(U_1),w(U_2),…w(U_N)]^2 (20)

The Chebyshev polynomial p of degree n will interpolate the data (ie p(ui)=wi,i=0,1..n). Obtaining the spectral derivative vector w by differentiating p and then evaluating at grid points

(ie,wi=p'(ui)=wi,i=0,1..n).This will transform the nonlinear differential equation into a system that will be solved using Newton's iteration method.

$$\frac{d^2\tilde{\theta}}{du^2}+B\tilde{\theta}\frac{d^2\tilde{\theta}}{du^2}+\left(\frac{d\tilde{\theta}}{du}\right)^2-m^2\tilde{\theta}-s_H\tilde{\theta}^2+s_HQY\tilde{\theta}+s_H\tilde{Q}=0 \quad (21)$$

The boundary conditions,

$\tilde{\theta}'(-1)=0$

$\tilde{\theta}'(1)=1$

The boundary conditions and equations are transformed into a system of algebraic equations after applying the Chebyshev method.

$$\sum_{i=0}^{n}d^2k_i\theta(u_i)+B\sum_{i=0}^{n}\tilde{\theta}(u_i)d_{k_i}^2\hat{\theta}(u_i)+B\sum_{i=0}^{n}\tilde{\theta}(u_i)d_{k_i}^1$$

$$\sum_{i=0}^{n}\tilde{\theta}(u_i)-m^2\theta\widetilde{(ui)}-s_H[\theta\widetilde{(ui)}]^2+(ui)+s_H\tilde{Q}=0 \quad (22)$$

The boundary conditions are;

$\sum_{i=0}^{n}dk_i\theta(u_i)=0,\theta(ui)$=1 (23)

This system is solved using Newton's method to extract the temperature distribution in the fins.

The fin base heat flux is given by;

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$$Q_b = \frac{q_l}{k_\alpha A_C(Tb-T_\alpha)} = [1+B\theta]\frac{d\theta}{du}]_{x=1} \quad (24)$$

Heat transfer in geothermal porous media;

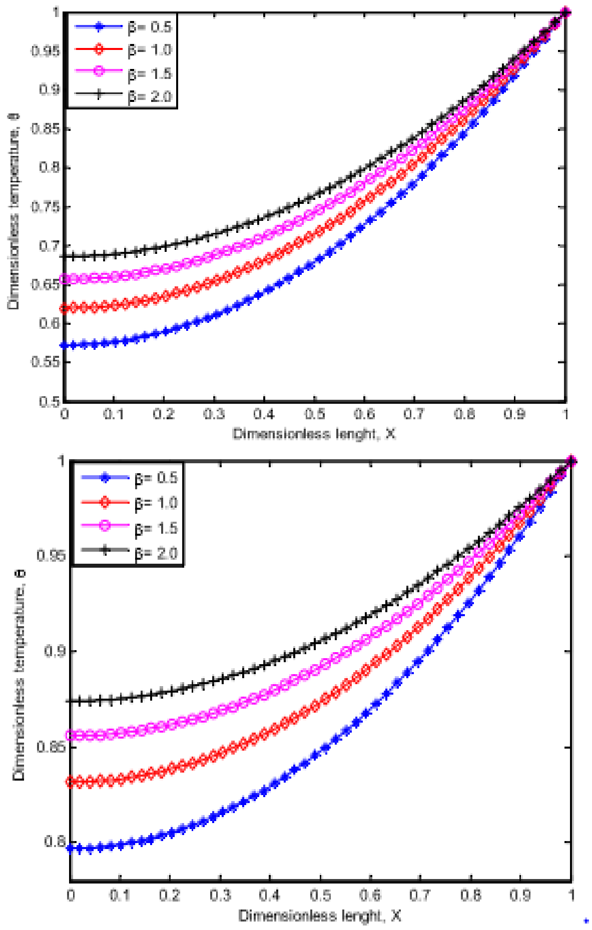
$$\frac{q_b}{q_s}=\frac{k_{eff}(T)A_b\left(\frac{d\Gamma}{du}\right)_{u=0}}{hAs(Tb-T_\alpha)} \quad (25)$$

In dimensionless form, it is given as;

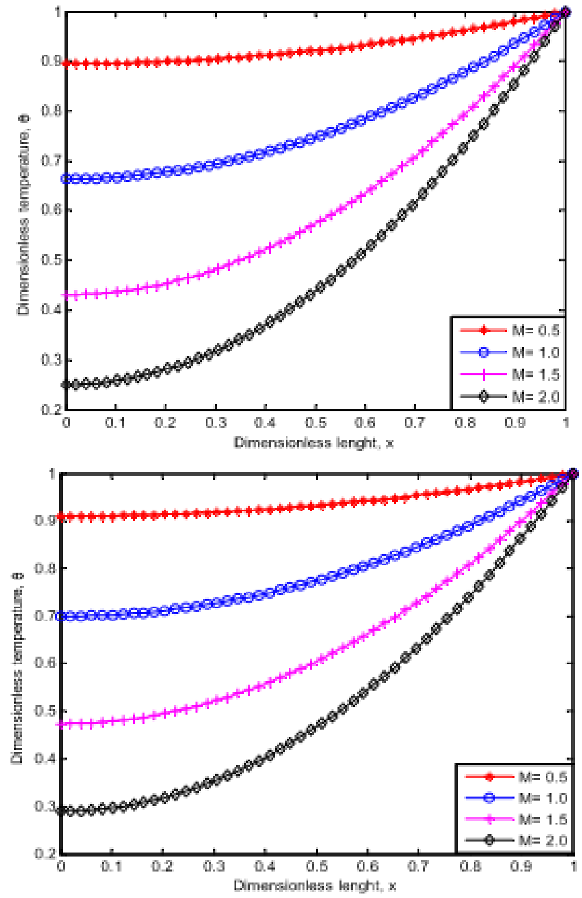
$$\frac{q_b}{q_s}=\frac{A_r}{Nu}\left[(1+B\theta)\frac{d\theta}{du}\right] u=\phi \quad (26)$$

III. RESULTS AND DISCUSSION

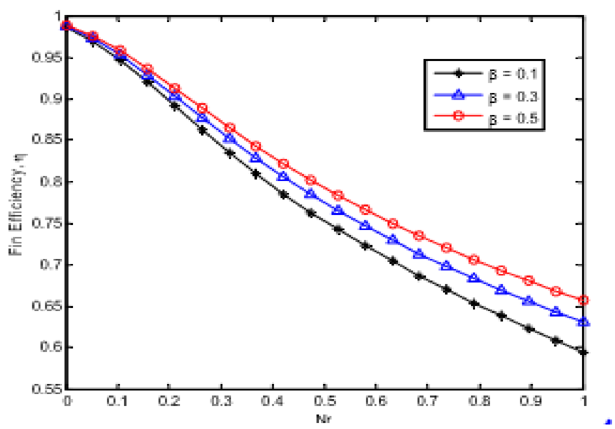
Figures 2, 3, and 4 depict the effects of geothermal parameters on longitudinal fins. As the geothermal parameters increase, the rate of heat transferred through the longitudinal fins increases while the temperature in the longitudinal fins drops. Also, the results show that the ratio of the convective heat transferred to the conductive heat transferred has many effects on the temperature distribution along the geothermal longitudinal fins, the rate of heat transfer at the base of the longitudinal fins, and the efficiency of the geothermal longitudinal fins. As the coefficient of heat transferred h increases, the ratio of the coefficient of heat transferred and the thermal conductivity of the longitudinal fins' material increases at the base of the geothermal fins, and eventually, the temperature distribution along the longitudinal fins, especially at the tip of the geothermal longitudinal fin increases. This shows that the efficiency of the geothermal longitudinal fins is achieved at high values of the geothermal region.



[Fig.2: Effects of non-linear Parameter β on fins when M=1.50, N=1.75, M=0.50, N=1.25]



[Fig.3: Effects of M on fins When β=0.1, N=0, β=0.3, N=0]



[Fig.4: Effects of Nr on the Temperature Distribution in the Fin]

In addition, the figures also depict the effects of nonlinear parameters on the geothermal performance of the longitudinal fins. The results justified that parameters such as the nonlinear thermal conductivity parameter, thermo-geometric term, and the radiation number have direct effects on the rate of heat transfer at the base of the geothermal longitudinal fin. The results also showed that varying the geothermal parameters will agree with the numerical method (NM) results using Runge-Kutta with shooting.

IV. CONCLUSION

This paper analyzed the geothermal heat transferred in a convective longitudinal fin with temperature-dependence

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thermal conductivity using the Rayleigh Rits-Chebyshev spectral collocation method. The developed heat model was used to analyze the geothermal performance, establish optimal thermal design parameters, and further investigate the effects of thermo-geometric and thermal conductivity (non-linear) parameters on the geothermal performance of the longitudinal fin.

DECLARATION STATEMENT

I must verify the accuracy of the following information as the article's author.

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- **Funding Support:** This article has not been sponsored or funded by any organization or agency. The independence of this research is a crucial factor in affirming its impartiality, as it has been conducted without any external sway.
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- **Data Access Statement and Material Availability:** The adequate resources of this article are publicly accessible.
- **Authors Contributions:** The authorship of this article is contributed solely.

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