ART. IX. — *On the Theoretical Temperature of the Sun; under the Hypothesis of a Gaseous Mass maintaining its Volume by its Internal Heat, and depending on the Laws of Gases as known to Terrestrial Experiment;* by J. HOMER LANE, Washington, D. C.

[Read before the National Academy of Sciences at the session of April 13–16, 1869.]

MANY years have passed since the suggestion was thrown out by Helmholtz, and afterwards by others, that the present volume of the sun is maintained by his internal heat, and may become less in time. Upon this hypothesis it was proposed to account for the renewal of the heat radiated from the sun, by means of the mechanical power of the sun's mass descending toward his center. Calculations made by Prof. Pierce, and I believe by others, have shown that this provides a supply of heat far greater than it is possible to attribute to the meteoric theory of Prof. Wm. Thomson, which, I understand, has been abandoned by Prof. Thomson himself as not reconcilable with astronomical facts. Some years ago the question occurred to me in connection with this theory of Helmholtz whether the entire mass of the sun might not be a mixture of transparent gases, and whether Herschel's clouds might not arise from the precipitation of some of these gases, say carbon, near the surface, with their revaporization when fallen or carried into the hotter subjacent layers of atmosphere beneath; the circulation necessary for the play of this Espian theory being of course maintained by the constant disturbance of equilibrium due to the loss of heat by radiation from the precipitated clouds. Prof. Espy's theory of storms I first became acquainted with more than twenty years ago from lectures delivered by himself, and, original as I suppose it to be, and well supported as it is in the phenomena of terrestrial meteorology, I have long thought that Prof. Espy's labors deserve a more general recognition than they have received abroad. It is not surprising, therefore, in a time when the constitution of the sun was exciting so much discussion, that the above suggestions should have occurred to myself before I became aware of the very similar, and in the main identical, views of Prof. Faye, put forth in the Comptes Rendus. I sought to determine how far such a supposed constitution of the sun could be made to connect with the laws of the gases as known to us in terrestrial experiments at common temperatures. Some calculations based upon conjectures of the highest temperature and least density thought supposable at the sun's photosphere led me to the conclusion that it was extremely difficult, if not impossible, to make out the connection in a credible manner. Nevertheless, I mentioned my ideas to Prof. Henry, Secretary of the Smithsonian Institution, when he immediately referred me to a number of the Comptes Rendus, then recently received, containing Faye's exposition of his theory. Of course nothing is further from my purpose than to make any kind of claim to any thing in that publication. After becoming acquainted with his labors I still regarded the theory as seriously lacking, in its physical or mechanical aspect, the direct support of confirmatory observations, and even as being subject to grave difficulty in that direction. In this attitude I allowed the subject to rest until my friend Dr. Craig, in charge of the Chemical Laboratory of the Surgeon General's office, without any knowledge of Faye's memoir, or of my own suggestions previously made to Prof. Henry and another scientific friend, fell upon the same ideas of the sun's constitution, availing himself, precisely as I had done, of Espy's theory of storms. Dr. Craig's ideas were communicated to a company of scientific gentlemen early last spring, and soon after, Prof. Newcomb, of the U. S. Naval Observatory, entered into a general survey of the nebular hypothesis. These communications of Dr. Craig and Prof. Newcomb led me to enter into a renewed examination of the mechanical embarrassment under which I had believed the theory to labor. Not any longer relying on my first rough estimate based on assumed high temperatures at the photosphere, the question was now inverted. Assuming the gaseous constitution, and assuming the laws expressed in Poisson's formulæ, known to govern the constitution of gases at common temperatures and densities, what shall we find to be the temperatures and densities corresponding to the observed volume of the sun supposing it were composed of some known gas such as hydrogen, or supposing it to be composed of such a mixture of gases as would be represented by common air. Pure hydrogen will, of course, give us the lowest temperature of all known substances, under the general hypothesis.

The question was resolved, and the results were communicated in graphical and numerical form in May or June last to two or three scientific friends, but their publication has been delayed by an unavoidable absence of several months from home.

Premising that the unit of density shall correspond to a unit of mass in the cube of the unit of length, the unit of force to the force of terrestrial gravity in the unit of mass, and the unit of pressure or elasticity in the gas to the unit of force on a surface equal to the square of the unit of length:

Let r=the distance of an element of the sun's mass from the sun's center,

t=the temperature of the element,

σt=its atmospheric subtangent, referred to the force of gravity at the earth's surface, or height of the column of homogeneous gas, whose terrestrial gravitating force would equal its elasticity,

ϱ=its density, or mass of its unit volume,

=force of terrestrial gravity in its unit volume,

$\varrho \sigma t$=its elasticity, or elastic force per unit surface,

R=the earth's radius,

M=the earth's mass,

m=the mass of the part of the sun's body contained in the concentric sphere whose radius is r,

$\frac{M r^2}{m R^2} \sigma t$= the subtangent of the gas under its actual gravitating force in the sun.

The condition of equilibrium between the gravitating force of a thin horizontal layer of gas whose thickness is dr, and the difference of elastic force between its lower and upper surfaces, is expressed by the equation,

$$d \cdot \varrho \sigma t = -\frac{m R^2}{M r^2} \varrho \, d r.$$

Under the hypothesis that the law of Mariotte and the law of Poisson prevail throughout the whole mass, and that this mass is in convective equilibrium, we have

$$\sigma = \text{a constant}, \qquad (1)$$
$$t = t_1 \varrho^{k-1},*$$

t_1 representing the value of t in the part of the mass where the density is a unit.

The theoretical difficulties which, if the supply of solar heat

* k represents the ratio of the specific heat of a gas under constant pressure to its specific heat under constant volume.

is to be kept up by the potential due to the mutual approach of the parts of the sun's mass consequent on the loss of heat by radiation, come in when we suppose a material departure from these laws of Mariotte and of Poisson at the extreme temperatures and pressures in the sun's body, or how far such difficulties intervene, will be considered further on.

By means of the constant value of σ, and the value of t given in (1), the above differential equation is transformed into

$$k\,\sigma\,t_1\,\varrho^{k-2}d\varrho=-\frac{m}{M}\,\frac{R^2}{r^2}d\,r,$$

the integral of which gives

$$1-\left(\frac{\varrho}{\varrho_0}\right)^{k-1}=\frac{k-1}{k}\,\frac{R^2}{\sigma M t_1\,\varrho_0{}^{k-1}}\int_0^r\frac{mdr}{r_2},\qquad(2)$$

in which ϱ_0 is the value of ϱ at the sun's center.

We have also

$$m=4\pi\int_0^r\varrho r^2\,dr=4\pi\varrho_0\int_0^r\frac{\varrho}{\varrho_0}r^2\,dr.\qquad(3)$$

If now we put

$$r=\left(\frac{k\sigma Mt_1}{4(k-1)R^2\pi\varrho_0{}^{2-k}}\right)^{\frac{1}{2}}x,\qquad(4)$$

we shall have

$$m=4\pi\varrho_0\left(\frac{k\sigma Mt_1}{4(k-1)R^2\pi\varrho_0{}^{2-k}}\right)^{\frac{3}{2}}\mu,\qquad(5)$$

in which

$$\mu=\int_0^x\frac{\varrho}{\varrho_0}x^2\,dx,\qquad(6)$$

and equation (2) becomes

$$1-\left(\frac{\varrho}{\varrho_0}\right)^{k-1}=\int\frac{\mu dx}{x^2}.\qquad(7)$$

In equations (6) and (7) it is plain that upon the value of k alone depends: first the form of the curve that expresses the value of $\frac{\varrho}{\varrho_0}$ for each value of x; secondly, the value of the upper limit of x corresponding to $\frac{\varrho}{\varrho_0}=0$; and thirdly, the corresponding value of μ. These limiting, or terminal, values of x and μ, cannot be found except by calculating the curve, for equations (6) and (7) seem incapable of being reduced to a complete general integral. But when these values have been found for any proposed value of k, they may be introduced once for all into equations (4) and (5), from which the values of ϱ_0, and of $\sigma\,t_1$, are at once deduced.

I have made these calculations for two different assumed values of k, viz., $k=1{\cdot}4$, which is near the experimental value it has in common air, and $k=1\frac{2}{3}$, which is the maximum possible value it can have in the light of Clausius' theory of the constitution of the gases. The calculation of the curve of $\frac{\varrho}{\varrho_0}$, or of $\left(\frac{\varrho}{\varrho_0}\right)^{k-1}$, begins at the sun's center where $x=0$. For the small values of x, integration by series enables us readily to deduce from equations (6) and (7) the following approximate numerical equations:

For $k=1{\cdot}4$,

$$\mu=\tfrac{1}{3}x^3-\tfrac{1}{12}x^5+\tfrac{5}{336}x^7-\tfrac{125}{54432}x^9+\ \&c. \qquad (8)$$

$$1-\left(\frac{1}{\varrho_0}\right)^{\cdot 4}=\tfrac{1}{6}x^2-\tfrac{1}{48}x^4+\tfrac{5}{2016}x^6-\tfrac{125}{435456}x^8+\ \&c. \qquad (9)$$

For $k=1\frac{2}{3}$,

$$\mu=\tfrac{1}{3}x^3-\tfrac{1}{20}x^5+\tfrac{1}{240}x^7-\tfrac{1}{3888}x^9+\ \&c. \qquad (10)$$

$$1-\left(\frac{\varrho}{\varrho_0}\right)^{\frac{2}{3}}=\tfrac{1}{6}x^2-\tfrac{1}{80}x^4+\tfrac{1}{1440}x^6-\tfrac{1}{31104}x^8+\ \&c. \qquad (11)$$

For larger values of x, until $\left(\frac{\varrho}{\varrho_0}\right)^{k-1}$ becomes sufficiently small as there is no need of great precision in these calculations, I have merely developed the values of μ and $\left(\frac{\varrho}{\varrho_0}\right)^{k-1}$ corresponding to $x=1{\cdot}1$, $x=1{\cdot}2$, $x=1{\cdot}3$, &c., by means of differences taken from the differential co-efficients at the middle of each increment of x, and for the same reason have thought it sufficient to begin with $x=1$, in equations (8) and (9) or (10) and (11). After arriving at a sufficiently small value of $\left(\frac{\varrho}{\varrho_0}\right)^{k-1}$ the calculation is finished by aid of the following approximate equations also derived by integration fron (6) and (7).

$$\mu'-\mu=\frac{k-1}{k}\,\mu'^{\frac{1}{k-1}}\,x'^{\,2-\frac{2}{k-1}}\,(x'-x)^{1+\frac{1}{k-1}}\,(1+X) \qquad (12)$$

$$\left(\frac{\varrho}{\varrho_0}\right)^{k-1}=\frac{\mu'(x'-x)}{x'x}-\frac{(k-1)^2}{k(2k-1)}\mu'^{\frac{1}{k-1}}\,x'^{-\frac{2}{k-1}}\,(x'-x)^{2+\frac{1}{k-1}}$$
$$-\frac{(k-1)(2k^2-3k+2)}{k(2k-1)(3k-2)}\,\mu'^{\frac{1}{k-1}}\,x'^{-1-\frac{2}{k-1}}\,(x'-x)^{3+\frac{1}{k-1}} \qquad (13)$$

In these equations x' and μ' are the values of x and μ corresponding to $\frac{\varrho}{\varrho_0}=0$, or the upper limit of the supposed solar atmosphere, and

$$X=-\frac{k(2k-3)}{(k-1)(2k-1)}\frac{x'-x}{x'}+\frac{k(k-2)(2k-3)}{2(k-1)^2(3k-2)}\frac{(x'-x)^2}{x'^2}+\ \&c.$$

$$-\frac{k-1}{2k(2k-1)}\mu'^{-1+\frac{1}{k-1}}x'^{2-\frac{2}{k-1}}(x'-x)^{1+\frac{1}{k-1}}+\ \&c.$$

With the values of x' and μ' determined, using r' and m' to express in like manner the corresponding values of r and m at the upper limit of the theoretical atmosphere, we find from equations (4) and (5)

$$\varrho_0=\frac{m'x'^3}{4\pi\mu' r'^3}, \qquad (14)$$

$$\sigma t_1=\frac{4\pi(k-1)R^2r'^2\varrho_0^{2-k}}{kMx'^2},$$

and by equation (1), $$\sigma t=\frac{4\pi(k-1)R^2r'^2\varrho_0}{kMx'^2}\left(\frac{\varrho}{\varrho_0}\right)^{k-1}, \qquad (15)$$

$$=\frac{k-1}{k}\frac{m'R^2x'}{\mu' Mr'}\left(\frac{\varrho}{\varrho_0}\right)^{k-1} \qquad (16)$$

A glance at equation (7) will show that $\frac{\mu'(x'-x)}{x'x}$, equation (13), or $\frac{\mu'}{x'}\frac{r'-r}{r}$ may be taken equal to $\left(\frac{\varrho}{\varrho_0}\right)^{k-1}$ throughout the considerable upper part of the volume of the hypothetic gaseous body in which $1-\frac{\mu}{\mu'}$, or $1-\frac{m}{m'}$, is sufficiently small to be neglected. This substitution in the last equation gives

$$\sigma t=\frac{k-1}{k}\frac{m'R^2}{Mrr'}(r'-r),\text{ nearly}, \qquad (17)$$

and also $$\varrho=\left(\frac{\mu'}{x'}\right)^{\frac{1}{k-1}}\varrho_0\left(\frac{r'-r}{r}\right)^{\frac{1}{k-1}}\text{ nearly},$$

$$=\frac{1}{4\pi}\mu'^{-1+\frac{1}{k-1}}x'^{3-\frac{1}{k-1}}\frac{m'}{r'^3}\left(\frac{r'-r}{r}\right)^{\frac{1}{k-1}} \qquad (18)$$

Now the mechanical equivalent of the heat in the mass ϱ of a cubic unit in volume of any perfect gas whose atmospheric subtangent is σt, is $\frac{1}{k-1}\varrho\cdot\sigma t$, and the mechanical equivalent of the heat that it would give out, in being cooled down under constant pressure to absolute zero, is $\frac{k}{k-1}\varrho\cdot\sigma t$. If the density ϱ is taken in units of the density of water, and the unit of length be the foot, this expression is multiplied by $62\frac{1}{2}$ to give for the mechanical equivalent in foot pounds

$$62\tfrac{1}{2}\frac{k}{k-1}\,\varrho\cdot\sigma t=\frac{62\frac{1}{2}}{4\pi}\,\mu'^{\,-1+\frac{1}{k-1}}\,x'^{\,3-\frac{1}{k-1}}\,\frac{m'^2R^2}{Mr'^4}\left(\frac{r'-r}{r}\right)^{1+\frac{1}{k-1}} \qquad (19)$$

The mechanical equivalent $\frac{1}{k-1}\,\varrho\cdot\sigma t$, of the heat in the mass ϱ, viewed in the light of Clausius' mechanical theory of the gases, includes the motions of the separate atoms of each supposed compound molecule relatively to each other, as well as the motion of translation which each compound molecule makes in a straight path through free space till it impinges upon another compound molecule. If we wish to find the mechanical equivalent which would be due to this motion of translation alone, we must put $k=1\frac{2}{3}$ in the factor $\frac{1}{k-1}$ by which $\varrho\cdot\sigma t$ is multiplied, and this gives $\frac{3}{2}\varrho\cdot\sigma t$. To find from this the mean of the squares of the velocities of translation of the compound molecules, we divide by the mass ϱ, and, if the foot be the unit of length, multiply by 64·3, whence we have for the velocity found by taking the square root of this mean of the squares

$$8\cdot02\sqrt{\tfrac{3}{2}\sigma t}=8\cdot02\left(\frac{3}{2}\,\frac{k-1}{k}\,\frac{m'R^2x'}{\mu'Mr'}\right)^{\frac{1}{2}}\left(\frac{\varrho}{\varrho_0}\right)^{\frac{k-1}{2}} \qquad (20)$$

Determination of the curve of density for $k=1\cdot4$.—Beginning with $x=1$, in equations (8) and (9), we find $\mu=\cdot2626$ and $\left(\frac{\varrho}{\varrho_0}\right)^{\frac{4}{10}}=\cdot8520$. Developing the values of μ and $\left(\frac{\varrho}{\varrho_0}\right)^{\frac{4}{10}}$ for $x=1\cdot1$, $x=1\cdot2$, &c., by means of differences we arrive at the values $\mu=2\cdot145$ and $\left(\frac{\varrho}{\varrho^0}\right)^{\frac{4}{10}}=\cdot1378$ when $x=4\cdot0$. Putting these values into equations (12) and (13) we find

$$x'=5\cdot355,\quad \mu'=2\cdot188.$$

If we now allow $\frac{1}{22}$d of the radius of the photosphere, or about 20,000 miles, for the height of the theoretic upper limit of the solar atmosphere above the photosphere, and if we take the mean specific gravity of the earth's mass at $5\frac{1}{2}$, and the mean specific gravity of the sun within the photosphere at $\frac{1}{4}$ that of the earth, as it is known to be, these values of x' and μ' give us in equation (14)

$$\varrho_0=28\cdot16,$$

so that the density of the sun's mass at the center would be nearly one-third greater than that of the metal platinum.

Curve of density for $k=1\frac{2}{3}$.—For this value of k the numerical coefficients in equations (8) and (9) are replaced by those in (10) and (11). Otherwise, the same process employed with the value $k = 1{\cdot}4$, gives, starting with $x = 1$, $\mu = {\cdot}2875$ and $\left(\frac{\varrho}{\varrho_0}\right)^{\frac{2}{8}} = {\cdot}8452$, and developing for $x=1{\cdot}1$, $x=1{\cdot}2$, &c., brings us to $\mu=2{\cdot}557$ and $\left(\frac{\varrho}{\varrho_0}\right)^{\frac{2}{8}} = {\cdot}1591$, for $x=3{\cdot}0$, and finally gives us

$$x'=3{\cdot}656, \quad \mu'=2{\cdot}741,$$

and if we now assume the same height as before for the theoretic upper limit of the sun's atmosphere, instead of $\varrho_0=28{\cdot}16$, we find

$$\varrho_0=7{\cdot}11.$$

The new curve of density is found in the same way as the first, and is presented to the eye in the diagram in comparison with it. In the upper part of both curves the scale of density is increased ten fold, and it is, in part only, evident to the eye how immensely different, for the two values of k, becomes the density in the upper parts of the sun's mass. It appears to the eye only in part because the ratio of the two densities multiplies itself rapidly in approaching the upper limit of the atmosphere.

The above was communicated in writing as here given, to the Academy at its late session.* The draft of the following, and a part of the details of its substance, have been prepared since.

Equation (20) gives in feet the square root of the mean square of velocity of translation of molecules ($8{\cdot}02\sqrt{\frac{3}{2}\sigma t}$). At the sun's center we find this would be 331 miles per second for the curve of density corresponding to $k=1\frac{2}{3}$, and 380 miles per second for the curve of density corresponding to $k=1{\cdot}4$.

In 1838 Pouillet, following the law of heat radiation given by Dulong and Petit, estimated the temperature of the radiating surface of the sun, from observations by himself of the quantity of heat it emits, at from 1461° C. to 1761° C. Herschel, from Pouillet's observations, and his own made at the Cape of Good Hope about the same time, adopts, after allowing one-third for the absorption of our atmosphere, forty feet as the thickness of ice that would be melted per minute at the sun's sur-

* I desire here to state that the formulæ which show the relation between the temperature, the pressure, the density, and the depth below the upper limit of the atmosphere, so far as they apply to the upper part of the sun's body, were independently pointed out by Prof. Peirce, in a very interesting paper which that distinguished physicist read before the Academy at the same session, and prior to the presentation of this paper. Also to recall a fact which I first learned from Prof. Peirce's mention of it to the Academy, viz. that Prof. Henry long ago threw out the idea of the atmospheric condition to which Prof. Thomson has more recently given the term convective equilibrium, viz., such that any portion of the air, on being conveyed into any new layer above or below, would find itself reduced, by its expansion or compression, to the temperature of the new layer.

face. The temperature of the radiating surface calculated from this datum by the formula of Dulong and Petit, and with the co-efficient of radiation found by Prof. W. Hopkins for sandstone, the smallest co-efficient he found, is 1550° C. or 2820° Fah. But then the solar radiation is many thousands of times greater than the greatest in Dulong and Petit's experiments, so that these calculations of the temperature of the sun's photosphere have little weight notwithstanding the simplicity and accuracy with which the formula represents the experiments from which it was derived. Nothing authorizes us to accept the formula as more than an empirical one. It seems desirable that experiments similar to those of Dulong and Petit should be made on the rate of cooling of intensely heated bodies, such as balls of platinum not too large. By placing the heated ball in the center of a hollow spherical jacket of water, either flowing or in an unchanged mass, the quantities of heat radiated in successive equal spaces of time will be determined, and the corresponding differences of temperature in the heated ball can at least be estimated with whatever probability we may rely on our knowledge of the specific heat of its material. At present the best means we have of forming any judgment of the probable temperature of the source of the sun's radiation, is perhaps to be found in a comparison between the effects of the hydro-oxygen blowpipe, and the recorded effects of Parker's great burning lens. I am not aware that this method has before been resorted to.

If the angle of aperture at the focus of a burning lens, or combination of lenses, be called $2a$, the radiation received by a small flat surface at the focus will be $\sin^2 a$, if a unit be taken to represent the radiation the same small flat surface would receive just at the sun's surface. Parker's lens, with the small lens added, had, at the focus so formed, an angle of aperture of about 47°. A small flat surface at its focus would therefore receive about one-sixth the radiation that it would just at the sun, making no allowance for absorption by the atmospheres of the earth and sun and rays lost in transmission through the lenses. Pouillet, from the experiments already alluded to made by himself, found his atmosphere in fine weather transmitted, of the sun's heat rays, about the fraction $\frac{4}{5}$ raised to a power whose exponent is the secant of the sun's zenith distance. This, of course, leaves out of view the heat rays of low intensity which are totally absorbed by the atmosphere. He also concluded from comparison with other experiments of his own with a moderately large burning glass, that that glass transmitted $\frac{7}{8}$ of the heat rays incident on it. If we assume the same fraction for each of the two lenses of Parker's combination, and assume further that the sun's zenith distance did not exceed 48° in the experiments made with it, we find for the fractional multiplier expressing the part of the sun's heat radiation which arrived at the focus unintercepted, $(\frac{4}{5})^{1\ 1\cdot2}(\frac{7}{8})^2 = \cdot55$. Hence the radiation actually received by a small flat surface at the focus was ·09, or about one-eleventh, of what it would receive just at the sun. The heat so received by any body so placed in the focus, must, after the body has acquired its highest temperature, be emitted from it at the same rate. The heat so emitted will consist: first, of heat radiated into that part of space toward which the radiating surface of the body looks; secondly, of heat carried of by convection of the air; thirdly, of heat conducted away by the body supporting the body subjected to experiment; fourthly, of heat rays, if any, reflected, and not absorbed, by the body subjected to experiment. Assuming it as a reasonable conjecture that full half of all this* consists of heat *radiated* into the single *hemisphere* looking upon a flat surface, we may conclude that the body, at its highest acquired temperature, radiated not less than $\frac{1}{20}$th as much heat as is radiated by an equal extent of surface of the sun's photosphere, over and above such part of that radiation as may be intercepted by the sun's atmosphere, and such rays of low intensity as are *totally* absorbed by our own atmosphere, the whole of which apparently cannot be great. No allowance seems necessary for the chromatic and spherical dispersion of the lenses, since the diameter of the focus is stated at half an inch, while the true diameter of the sun's image would be not less than one-third of an inch.

Now we are not without the means of forming a probable approximate estimate of this temperature at which the radiation becomes $\frac{1}{20}$th, more or less, of that of the sun's photosphere. We are told that in the focus of Parker's compound lens 10 grains of very pure lime ("white rhomboidal spar") were melted in 60 seconds. We may presume that in that length of time the temperature of the lime, after parting with its carbonic acid, made a near approximation to the maximum at which it would be stationary, a presumption confirmed by the period of 75 seconds said to have been occupied in the fusion of 10 grains of carnelian, and by the considerable period of 45 seconds for the fusion of a topaz of only 3 grains, and 25 seconds for an oriental emerald of but 2 grains, and in fact sufficiently

* As to the heat carried off by convection of the air, if its quantity be calculated by the formula given by Dulong and Petit for that purpose, it comes out utterly insignificant in comparison with the heat received from the burning glass. The conjectural allowance of $\frac{4}{9}$ths in all, of this, is likely, therefore, to be much too large. Not much reliance, indeed, can be placed upon the formula here mentioned, at such a temperature as 4000° Fah., yet, as by it the convection is taken proportional to the 1·233 power of the difference of temperature, it seems unlikely that it gives a quantity very many fold less than the truth.

proved, it would seem, by observing that the heat we have estimated to fall at the focus, upon a flat surface, would suffice, if retained, to raise the temperature of a quarter of an inch thick of lime 4000° Fah. in 5 seconds. If, then, we may take the temperature *maintained* at the focus of Parker's lens to have been at the melting point of lime, we may conclude that it is also not far from the temperature given by the hydro-oxygen blowpipe. Dr. Hare, who was the first inventor of this instrument, and the discoverer of its great power, melted down, by its means, in partial fusion, a very small stick of lime cut on a lump of that material, which we understand to have been a very pure specimen. Burning glass and blowpipe seem each to have been near the limit of its power in this apparently common effect. But Deville found the temperature produced by the combination of hydrogen and oxygen under the atmospheric pressure to be 2500° Cent. As the lime in the heated blast would radiate rapidly, its temperature must have been lower than that of combined hydrogen and oxygen, and I have called it 2220° Cent. or 4000° Fah.

The formula of Dulong and Petit, with the co-efficient found by Hopkins, as already mentioned, gives for the quantity of heat radiated in one minute by a square foot of surface of a body whose temperature is $\theta+t$ centigrade, into a chamber whose temperature is θ centigrade, when expressed with the unit employed by Hopkins,

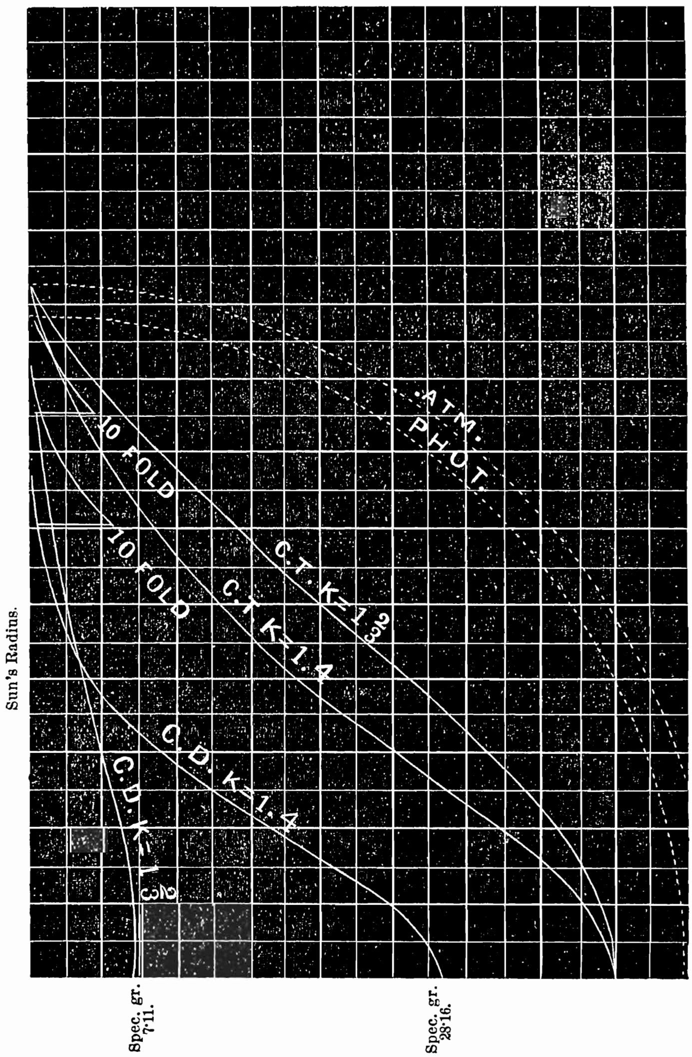
$$8{\cdot}377\,(1{\cdot}0077)^{\theta}\,[(1{\cdot}0077)^{t}-1].$$

It will be convenient, and, in the discussion of the high temperatures with which we are concerned, will involve no sensible error, to use the hypothesis that the space around the radiating body is at the temperature of 0° C. and the formula for the radiation then becomes,

$$8{\cdot}377\,[(1{\cdot}0077)^{t}-1]. \qquad (21)$$

The unit used by Hopkins, in the formula here given, is the quantity of heat that will raise the temperature of 1000 grains of water 1° centigrade. Expressed by the same unit, the quantity adopted by Sir J. Herschel as the amount of the sun's radiation, viz. that which would melt 40 feet thick of ice in a minute (at the sun's surface), is 1,280,000. The $\frac{1}{20}$th of this, or 64,000, expresses, therefore, the quantity which we have estimated the lime under Parker's lens to have radiated, per square foot of its surface, at its estimated temperature of 4000° Fah. If now we calculate its temperature by the above formula, from the estimated radiation, the result is 1166° Cent. or 2130° Fah. This is manifestly much below the real temperature, and so far below that there can be no doubt the formula of Dulong and Petit has failed at the melting point of lime. If instead of the co-efficient 8·377 we had used the larger co-efficient 12·808 which Hopkins gives for unpolished limestone, the formula would have been reduced only 53° Cent. It best suits the direction of our inquiry to use the smallest co-efficient which Hopkins' experiments gave, since we are seeking the *highest* temperature which can be plausibly deduced from the sun's radiation. For ease of expression, the curve which we will imagine for representing the actual relation of radiation to temperature, the horizontal ordinate standing for the temperature and the vertical ordinate for the radiation corresponding thereto, may be called the curve of radiation. The course of this curve from the freezing point of water to a point somewhat below the boiling point of mercury is correctly marked out to us by the formula. Beyond that we have but the rough approximation which we can get by means of the above comparison, to the single point of the curve where the radiation is $\frac{1}{20}$th that of the sun's photosphere. The attempt, from these data, to extend the curve till it reaches the full radiation of that photosphere, must be mainly conjectural. As a basis for the most plausible conjecture I am able to make let us assume: first, that the upward concavity of the curve of radiation, which increases very rapidly with the temperature as far as the curve follows the formula of Dulong and Petit, is at no temperature greater than that formula would give it at the same temperature; secondly, that the curve of radiation is nowhere convex upward. If, then, we set out from these two conjectural assumptions—of the degree of probability of which each one must form his own impression—the greatest temperature the sun's photosphere could have consistently with the radiation of 64,000 at the temperature of 4000° Fah., is found by drawing through the point representing that radiation and that temperature a straight line tangent to the curve of the formula. The line so drawn would cross the real curve of radiation in a greater or less angle at the radiation of 64,000 and temperature of 4000° Fah., and at higher temperatures would fall more or less *below* that curve, and its intersection with the sun's radiation of 1,280,000 would be at a temperature greater than that of the curve, that is to say, greater than the temperature of the sun's photosphere. This greater temperature is 55,450° Fah.

A different train of conjecture led me at first to assume a temperature of 54,000° Fah., and this last number I will here retain since it has been already used as the basis of some of the calculations we now proceed to give. It must be here recollected that we are discussing the question of *clouds* of *solid* or at least *fluid* particles floating in non-radiant gas, and constituting the sun's photosphere. If the amount of *radiation*



Explanation.—ATM., Assumed theoretic upper limit of atmosphere; PHOT., Photosphere; C.T.K.$=1\frac{2}{3}$, Arbitrary Curve of temperature for $k=1\frac{2}{3}$; C.T.K.$=1{\cdot}4$, Arbitrary Curve of temperature for $k=1{\cdot}4$; C.D.K.$=1{\cdot}4$, Absolute Curve of density for $k=1{\cdot}4$; C.D.K.$=1\frac{2}{3}$, Absolute density for $k=1\frac{2}{3}$.

would lead us to limit the temperature of such clouds of solids or fluids, so also it seems difficult to credit the *existence* in the solid or fluid form, at a higher temperature than 54,000° Fah. of any substance that we know of.

If then we suppose a temperature of 54,000° Fah., what would be the density of that layer of the hypothetic gaseous body which has that temperature, and what length of time would be required, at the observed rate of solar radiation, for the emission of all the heat that a foot thick of that layer would give out in cooling down under pressure to absolute zero? The latter question depends on the mechanical equivalent of this heat for a cubic foot of the layer of gas, and the two questions, together with that of the depth at which the layer would be situated below the theoretic upper limit of the atmosphere, are answered by equations (17), (18), and (19), provided we knew the value of k and the value of σ in the body of gas. The less the atomic weight of the gas the greater the value of σ, and the greater the density of the layer of 54000° Fah. and the greater the quantity of heat which a cubic foot of it would give out in cooling down. I therefore base the first calculation on hydrogen as it is known to us. The value of σ is in that case about 800 feet, and the value of k about 1·4, nearly the same as in common air. These values would give for the layer of 54000° Fah. a specific gravity about ·00000095 that of water, or about one 90th that of hydrogen gas at common temperature and pressure, and the mechanical equivalent of the heat that a cubic foot of the layer would give out in cooling down under pressure to absolute zero would be only about 9000 foot pounds, whereas the mechanical equivalent of the heat radiated by one square foot of the sun's surface in one minute is about 254,000,000 foot pounds. The heat emitted each minute would, therefore, be fully half of all that a layer ten miles thick would give out in cooling down to zero, and a circulation that would dispose of volumes of cooled atmosphere at such a rate seems inconceivable.

It may possibly appear to some minds that the difficulty presented by this aspect of the case will vanish if we suppose the photosphere, or its cloudy particles, to be maintained by radiation at a temperature to almost any extent lower than that of convective equilibrium. This would enable us to place the theater of operations in a lower and denser layer of atmosphere, but the supposition seems to me difficult to realize unless, as the hot gases rise from beneath, precipitation could commence at a temperature many times higher than the 54000° Fah. which we have estimated for the upper visible surface of the clouds, and this, as before intimated, seems to me itself extremely improbable.

I may mention here that my friend Dr. Craig, in an unpublished paper, following the hint thrown out by Frankland, is disposed to favor the idea that the sun's radiation may be the radiation of hot gases instead of clouds. At present I shall offer no opinion on that point one way or the other, but will only state it as my impression that if the theory of precipitated clouds, as above presented, is the true one, something quite unlike our present experimental knowledge, or at least much beyond it, is needed to make it intelligible.

The first hypothesis which offers itself in an attempt to make the theory rational is suggested by one point in Clausius' theory of the constitution of the gases, already alluded to. In forming his theory Clausius found that the known specific heats of the gases are all much too great for free simple atoms impinging on one another, and he therefore introduced the hypothesis of compound molecules, each compound molecule being a system of atoms oscillating among each other under forces of mutual attraction. Now if this were accepted as the actual constitution of the gases it is of course easy enough to conceive that in the fierce collisions of these compound molecules with each other at the temperatures supposed to exist in the sun's body, their component atoms might be torn asunder, and might thenceforth move as free simple molecules. In this case, still retaining the hypothesis of Clausius' theory, that the average length of the path described by each between collisions is large compared with the diameter of the sphere of effective attraction or repulsion of atom for atom, the value of k would reach its maximum of $1\frac{2}{3}$. Experiment has not shown us any gas in this condition, and for the present it is hypothetical. Even in hydrogen the value of k does not materially, if any, exceed the value of 1·4 which it has in air. But if it were found that the hydrogen molecule is compound, and that in the body of the sun the heat splits this molecule into two equal simple atoms, and in fact that all the matter in the sun's body is split into simple free atoms equally as small, then, while the value of k would be $1\frac{2}{3}$, the value of σ would be about 1600 feet. If with these values we repeat the calculation of the density of the layer of 54000° Fah. we find its specific gravity to be 0·000363 of that of water, or 4·35 times that of hydrogen gas at common temperature and pressure and in its known condition, or 8·7 times that which the hydrogen in the hypothetic condition would have if it retained that condition at common temperature and pressure. We find also that the mechanical equivalent of all the heat that a cubic foot of the layer would give out in cooling down, under pressure, to zero, would be no less than 13,500,000 foot pounds. Instead, therefore, of a layer ten miles thick, it would now require only a thickness of 38 feet to give out, in cooling down to zero, twice the heat emitted by the sun in one minute. It will be seen, (equations (17) and (19)), that this thickness, retaining the constant value $k=1\frac{2}{3}$, would diminish with the $2\frac{1}{2}$ power of the masses of the atoms into which the sun's body is hypothetically resolved (the reciprocal of the value of σ), and I leave each to form his own impression how far this view leads towards verisimilitude.

It is important to add that the depth of the layer of 54000° Fah. below the theoretic upper limit of atmosphere, when calculated with value $k=1{\cdot}4$, $\sigma=800$ feet, comes out only 1107 miles, and with the values $k=1\frac{2}{3}$ and $\sigma=1600$ feet only 1581 miles. This calculation of the depth, unlike the other results above, may be said to be independent of the question of the constitution of the sun's interior mass. It is alike difficult, on any plausible hypothesis, to reconcile a temperature no higher than 54000° Fah. with any perceptible atmosphere extending many thousand miles above, and yet no less an authority than Prof. Peirce has assigned a hundred thousand miles as the height of the solar atmosphere above the photosphere, at the same time, however, pointing out the enormous temperature which, under convective equilibrium, this would imply at the level of the photosphere. But all are not yet agreed that the appearances seen at such distances from the sun are proof of the existence of a true atmosphere there. It will be seen that the numbers I give above were obtained from a first hypothesis of an atmospheric limit 20,000 miles above the photosphere, but for the purpose of this paper it is of no consequence to repeat the calculation from a different limit.

It is, I believe, recognized on theoretical grounds that in an atmosphere containing a mixture of gases of unequal density the lighter gases might be expected to diffuse in greater proportion into the higher parts of the atmosphere and the heavier gases into the lower parts. But perhaps the supposed circulation which the emission of heat maintains within the photosphere must renew mixture at a rate sufficient to mask the rate which theory would assign for diffusion. I have not attempted a theoretic comparison between these two tendences. It will suffice here to repeat that the above numerical results, so far as they may be thought to give countenance to the theory in its mechanical aspect, require that the entire inner mass of the sun shall have, at a mean, (in the supposed state of dissociation), the very small atomic weight specified. We may notice in this connection the uniform proportion of oxygen and nitrogen gases in our atmosphere at the height of four miles or more at which the analysis has been made. Without having gone into a critical examination of the question, I suppose that at that height the proportion of oxygen which the theory of diffusive equilibrium would assign is notably diminished, and that it would be found that the circulation of the air is sufficiently active to mask the theoretic rate of diffusion.

The second hypothesis which might offer itself in an attempt to make the theory rational, but which a very little reflection is, I think, sufficient to set aside, is that which would modify Clausius' theory of the gases by assuming that in the sun's body the average length of the excursion made by each molecule between two consecutive collisions, becomes very short compared with the radius of the sphere of repulsion of molecule for molecule, and with the average distance of their centers at nearest approach. This way of harmonizing the actual volume of the sun with such a temperature as 54000° Fah. in the photosphere, and with the smallest density which we can credit in the photosphere, would involve the consequence that the existing density of almost the entire mass of the sun is very nearly uniform and at its maximum possible, or at all events that any further sensible amount of collapse must be productive of but a very small amount, comparatively, of renewed supplies of heat, for the obvious reason that this hyphothesis carries with it almost the entire neutralization of the force of gravity by the forces of molecular repulsion. In like manner it involves the consequence that in any such small contraction of the photosphere as can have taken place within the history of total eclipses, it is but a very small fraction of the sun's mass, near its surface, that can have taken part in the collapse to any thing like a proportionate extent. Hence it also extremely restricts the period during which we could suppose the sun to have existed under anything like its present visible magnitude in the past, consistently with the production in the way supposed of the supplies of heat it has been sending out. Another thing involved in this second hypothesis is the fact which Prof. Peirce has pointed out to the Academy, viz.: that the existing molecular repulsion in the sun's body would immensely exceed such as would be indicated by the modulus of elasticity of any form of matter known to us.

In conclusion, I do not mean to say that there is any *invincible logical exclusion* of any law of the action of gases different from what is specified or alluded to above. I only mean that, so far as I can see, any theory of heat which is based *simply* and *solely* upon *molecular attraction* and *repulsion* dependent on molecular distance alone, cannot in its application to the sun, escape from the conditions indicated in this paper. It is certainly not absurd to imagine heat to be an agent of some kind so constituted that it cannot be thus represented by the sole conditions of motion and of molecular attraction and repulsion, but yet so constituted that in its effects upon matter it follows the conditions of mechanical equivalency as defined by Joule. In fact, such exceptional cases as the expansion of water in freezing seem to favor such a view, though the range of that phenomenon is very limited. One way of forming a mechanical representation of such a constitution would be by associating molecular motion with the mechanical powers, either with or without molecular attraction or repulsion; the manner in which the imagined mechanical power (or link) attaches itself to the molecules which it connects—so as to make their motion determine their mutual approach or recession or change of relative direction—being dependent on the existing motions and other conditions in such a way as to produce the observed phenomena. The possibility of such a mechanical *representation* is sufficient to show that such a supposed *constitution* is not *logically excluded*, but to accept such a mechanical representation as a physical fact is quite another matter, and, as it seems to me, a very difficult one. Of course this difficulty does not present itself when we suppose that heat is not motion.