REVIEWS

Elements of Statistics. By A. L. BOWLEY. (London : P. S. King & Son. 1901.)

IT is somewhat remarkable that up to the present time there should be no book on the Elements of Statistics written in the English language. England possesses several statisticians of great eminence. The Statistical Society celebrated its jubilee a good many years ago, and popular magazines often contain most admirable statistical articles. What, then, is the reason why we have no elementary text-books on the subject? Probably because elementary statistics was not until recently taught as a subject at the universities ; or if a course of lectures on that subject was advertised, students were not pressed to go, and were encouraged in the belief that a little common sense could easily take the place of regular training in dealing with tables of numbers. The London School of Economics has, however, since its foundation had systematic courses of lectures on the elements of statistics, and the School therefore is entitled to share with Mr. Bowley the congratulations which are due to him for supplying a long-felt want. Without further ado it may be plainly stated that, whatever its shortcomings, which are dealt with later, this book is the best book on the Elements of Statistics written in English, French, German, or Italian.

The first chapter deals with the scope and meaning of statistics, a topic which has been debated for more than a century. Mr. Bowley does not distinguish clearly between the theory of statistics and the theory of errors of observation, and many of his statements, such as " Statistics may rightly be called the science of averages," or, " The chemist experimenting in his laboratory is like the statistician ; the chemist theorising in his study is like the economist," would form a pretty subject for a three hours essay. But discussions as to the scope and meaning of science, though of great value for many purposes, are of little value in teaching us the elements of that science. Chapters II. and III. deal with the general method of statistical investigation and illustrations of that method. The population census is so important that it is right to give great attention to it, and it would have been well if Mr. Bowley had given more prominence to the very important

fact that the official figures of the population at each age, and during inter-censal periods, are obtained by an elaborate process of calculation based upon assumptions of doubtful validity. A description of the work of the Labour Department occupies more than twenty pages, and might well have been omitted. The many examples of the epistolary style of a Government department are scarcely of permanent value. On the other hand, the section. on the Statistics of Foreign Trade assists in making intelligible the Export and Import figures published by the Board of Trade.

With Chapter IV., which deals with Tabulation, the real business of the book begins, but tabulation depends so much upon the particular figures and so little upon general principles that it is a difficult and dull subject to discuss. In fact, it is hard to deal with it adequately until the theory of means and averages and of the distribution of figures about means has been explained. Mr. Bowley feels this, and uses words such as median and quartile, referring the reader to a later page for their meaning ; but this procedure is rather confusing, and when one looks on to page 124 to find the definition of the median we find the following passage : " The median with its dependents, the quartile, decile and percentile, has already been used on page 92. Arrange the items of the group in ascending order of magnitude ; the item half way up the list is the median ; those one-quarter and three-quarters up are the quartiles ; those one, two, ... nine-tenths up are the deciles ; those one, two, ... ninety-nine hundredths up are the percentiles. The median is the most useful of the averages." This is not a very clear definition of the median. It is easy to see from the instances given what the median means, but all statistical terms should be clearly and succinctly defined before illustrations of them are given. Averages form the subject of Chapter V. It is a great pity that more use was not made of Fechner's article *Ueber der Ausgangswerth der kleinsten Abweichungssumme*, which is a most valuable piece of work and too little known. After arithmetic averages, weighted averages are dealt with, but not so fully as they deserve. Professor Edgeworth's elaborate discussions of systems of weighting are in general too difficult for the elementary student to follow, but a more elementary account of the kinds of weighting which are suitable for different purposes would have been extremely valuable, and the need for weighting in some cases is obvious. On p. 112 we have the statement, " But it is at once clear that we cannot allow the commodities given to have equal influences on the result ; wheat is of greater importance than sugar and meat than silver." What does importance mean in this context ? That is the whole difficulty, and it should have been properly discussed. This occurs in a passage dealing with index numbers which are not defined, and on referring to Chapter IX., which deals specifically with them, we are told that " they are used to measure the change in some quantity which we cannot observe directly," and that " The general method of forming an index-number is as follows

. . . . ; " but again no proper definition is given. No doubt it is extremely difficult to define "index-number," but it is a defect that the attempt is not made. But to return to the chapter on averages. Quetelet's average man is mentioned, but no hint is given that Quetelet's views have been attacked on the ground that the magnitudes of different parts of the body are correlated. Westergaard's illustration of the right-angled triangle (*Grundzüge*, p. 189) should have been enough to condemn Quetelet's average man to oblivion.[1] Under the heading geometric mean we are told that it was used by Jevons, but that he did not justify or explain its use. This is not strictly accurate. It is true that the section " Of the Meaning of an Average Rise of Prices " is slight and obscure, but this was criticised by Professor Laspeyres (*Jahrbuch von Nationaloekonomie*, Bd. iii., 81), and Jevons replied in an article in the *Journal of the Statistical Society* (June, 1865), in which he also discusses the harmonic mean, and gives as his reasons for adopting the geometric mean, " (1) because it lies between the other two ; (2) it presents facilities for the calculation and cor-rection of results by the continued use of logarithims, without which the inquiry could hardly be undertaken ; (3) it seems likely to give in the most accurate manner such general change in prices as is due to a change on the part of gold." And Mr. Bowley seems to prefer the geometric mean, at any rate when it differs much from the arithmetic (p. 129). The statement on page 130 that "the arithmetic average, median, and mode are close together if the group is symmetrical" is open to criticism. If the group is symmetrical, and consists of an odd number of instances, do not the arithmetic average and median coin-cide? And cannot there be two modes equidistant from the median in a symmetrical group? However, in any concrete case this statement is justified.

Chapter VI., which contains examples of the use of averages in tabulation will be instructive to the beginner. The important idea of dispersion is introduced in this chapter, and again we find an in-adequate definition. The writer states that " It is important to choose a simple measure of the dispersion of a group that can be easily appreci-ated and calculated," before he has explained the meaning of disper-sion, which he states later is measured by half the distance between the quantities expressed as a fraction of the arithmetic averages. It would have been better to have given this definition first and then to have discussed the properties of this fraction or to have explained the idea of dispersion,and then to have shown that this fraction is a good measure of it. The chapter on graphic method, which follows, contains a slip which it is important to point out, because it is a mistake which has often caused great confusion. A table is given showing the estimated number per 1,000 of the population at and above each

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[1] It must, however, be admitted that Professor Edgeworth does not altogether agree with the force of Westergaard's argument. See *Journal of the Statistical Society*, December, 1893.

age, and then the diagram which represents these is headed "numbers surviving at each age i n a generation of 1,000." But this is to confuse an age distribution table with a life table, and they are by no means the same in an increasing population. Apart from this, the chapter is a very useful one, and the exposition of an erroneous use of statistics (aptly illustrated by a dispute between the *Saturday Review* and Sir Robert Giffen) is most instructive, and should prove a useful warning to amateur statisticians. In discussing the method of smoothing by averages more space should be given to showing how the period can be ascertained, and how it is changing, because a smoothing based on an erroneous measure of the period may be worse than useless. The section of this chapter on comparisons of series of figures contains a good deal of useful advice, but it is unfortunate that Mr. Bowley often refers to "fluctuations" without defining their meaning. It is particularly important to do this; partly because statisticians have experienced great difficulty in finding a measure of these when they have considered such questions as "Did the new Bourse law in Berlin diminish or increase the fluctuations in the prices of grain?" and partly also because the word "fluctuation" has a definite technical meaning in statistical language. In the section on periodic figures it should be stated how the total number of members of the Ironfounders in each month are estimated, because if only the annual figures are available, a method of estimating the figures which causes a discontinuity at the end of each year might affect the December or January numbers of unemployed when expressed as percentage of the total number of members. The chapter on accuracy is useful because it contains part of Mr. Bowley's work (already published in the *Journal of the Statistical Society*) on the accuracy of an average, but the statement (p. 210) that "the fact that people frequently return their ages at the nearest round number causes unbiased error, and on the whole does not affect the average" is not quite true, because usually there are more persons whose age is just below a round number than just above it. Index numbers, are treated rather meagrely in Chapter X., because the scheme of the book is that special questions should not be discussed. The first part of the book ends with a chapter on interpolation which is very useful, because in most text books this subject is not adequately explained. Interpolation is continually needed by the statistician and the theory of it is very imperfectly understood. If most people realised how much it has to be used and how difficult and laborious it is they would form a better judgment of the value of estimated figures.

Part I. of the book is for beginners; very little mathematics is used; but in Part II. we have the elements of the more important branches of statistical theory which demand mathematical treatment. Mr. Bowley has very cleverly avoided the use of more than a very small amount of mathematics. In his object of making "clear the groundwork of the subject, so that it will be the easier for students to follow

modern writers on statistics," he has been successful.   There will no
longer be any excuse for persons of some reputation as statisticians
to show themselves ignorant of modern methods.   Such an intro-
duction has been wanted for several years.   To discuss it in detail
would take too much space, Mr. Bowley's chief defect is that he is
liable to state dogmatically what is often only a matter of opinion.
For example, he adopts Dr. Venn's view of the meaning of chance,
which is one that has been greatly doubted.   But allowance must
be made for the difficulty of the task.   The student does not want a
discussion of the philosophy of the theory of probability, but is content
to accept any reasonable view as a basis for his deductions.

It is easy to find faults in any book on statistics.   The chief fault
of this one is a certain obscurity of style and looseness of writing which
make it difficult to follow, but in comparison with previous books
covering the same sort of ground, this book stands in the front rank.
A certain amount of re-writing and a little compression would be a
great improvement.   But if the world is to wait for an ideal book on
statistics it will have to wait a long time, in the meantime when any
one wants to know of an elementary book on statistics he may safely
be referred to Mr. Bowley's.   C. P. SANGER

Essays on the Monetary History of the United States.   By
    CHARLES J. BULLOCK, Ph.D.   (New York : The Macmillan
    Co.   1900.)

OF the three essays contained in this volume the first may be de-
scribed as the general illustration of a thesis, two special examples of
which are noticed with particular detail in the second and the third.   The
thesis is this :   " All the varied currency experiments with which " the
Americans " have been vexed for nearly three centuries have been, first
and fundamentally, efforts to secure a cheap medium of exchange."   The
general illustration is furnished by a " systematic effort to supply a unitary
interpretation of the leading facts in the history of American currency " ;
and for this endeavour Professor Bullock claims the merit of originality,
though he does not contend that the thesis itself is novel.   The special
examples are supplied by more detailed accounts of the paper cur-
rencies of North Carolina and New Hampshire respectively.   In the first
essay the author passes in review successive periods of American history,
noting the various currency experiments, which were tried, and found
wanting, or were discovered to involve danger or disaster.   From the
original wampum and barter currency, with its harassing uncertainty,
and early metallic currencies of silver and gold, with fluctuations
in rating, manipulated of deliberate purpose, we proceed to successive
periods of paper money.   Of these the " colonial issues " were only re-
strained by the authority of the home Ministry exerted through the
medium, not always effectual, of the Governor.   The " Continental paper