point of view and that throws new light on the philosophical questions which permeate the various mathematical developments. Among the chapters which might appeal especially to such readers we may mention those bearing the following headings: "The axiom of infinity," "Mathematical productivity in the United States," and "Concerning multiple interpretations of postulate systems and the 'existence' of hyperspace."

In Chapter IX. Professor Keyser discusses "Graduate mathematical instruction for graduate students not intending to become mathematicians," arguing that such courses need not presuppose a first course in calculus, but could be based upon the mathematical preparation gained in a year of collegiate study. He would begin such a course "with an exposition of the nature and function of postulate systems and of the great rôle such systems have always played in the science, especially in the illustrious period of Greek mathematics and even more consciously and elaborately in our own time."

The headings of the nine chapters which have not been mentioned in what precedes are as follows: "The human significance of mathematics," "The humanization of the teaching of mathematics," "The walls of the world; or concerning the figure and the dimensions of the universe of space," "Mathematical emancipation; dimensionality and hyperspace," "The universe and beyond; the existence of the hypercosmic," "The permanent basis of a liberal education," "The source and function of a university," "Research in American universities," and "Mathematics."

Some of these titles are the subjects of addresses delivered by Professor Keyser before large audiences, and many of those who recall his stimulating language will doubtless welcome the opportunity to secure a collection covering such a wide scope of interests which are common to all, but which should appeal especially to those devoted to the borderland between philosophy and mathematics. One finds here a mixture of the most modern theories and the emotional descriptions of past generations, a charming flow of language illuminating most recent advances and, above all, an inspiring tableland of thought which is easily accessible to all but which is closely related with fundamental questions of education.

The mathematicians, as a class, are perhaps too much inclined to put off the historic, philosophic and didactic questions for later consideration, following the example of the great mathematical encyclopedias which are in course of publication. As a result the majority of them become so engrossed in the technical developments of their subjects as to find little time for the postponed questions of the most fundamental importance—a fate which seemed to threaten the encyclopedias just mentioned. A work in which some of these fundamental questions are handled in an attractive manner is therefore a valuable and timely addition to the mathematical literature.

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EQUATIONS AS STATEMENTS ABOUT THINGS

In the teaching of elementary physics and mathematics, much trouble is often caused by the fact that students who can readily solve an equation given them are unable to formulate in mathematical terms the data occurring in a practical problem. The purpose of this paper is to report briefly the results of several years' experience with a plan designed to remove as much as possible of this trouble by making the equations show more readily their meanings as shorthand statements of the facts. While there is probably nothing about these ideas that has not been suggested before, such suggestions, when applied at all to teaching, seem to have been rather vague and incomplete, or else applied only to one branch of the subject. In this case the plan to be outlined has been used in a general course of physics and in a course in mechanics, with results much more satisfactory than those obtained by the ordinary method.

To illustrate the difference between the old plan and the new, let us consider a single equation, the falling body law

$$s = \tfrac{1}{2} gt^2.$$

On the old plan, such an equation is merely a set of instructions for the computation of s. If the body has fallen three seconds, the student is expected on the old plan to write

$$s = \tfrac{1}{2} \times 32 \times 3^2 = 144.$$

This process, simple as it appears to the teacher, is not so simple for the student, as it really involves identifying t as the *number of seconds* the body has fallen, g as the *number of ft./sec.*^{2} in the gravity acceleration, performing the computation and then interpreting the result as a *number of feet*. One obvious cause of trouble is the necessity for using certain definite units on each side, with the errors made by the use of the wrong units; and another, perhaps not so obvious, is the fact that the formula itself is not a statement about a real distance of so many feet, a real acceleration of so many ft./sec.^{2} and a real time of so many seconds, but about pure numbers, mere incomplete "so many s," the most abstract things yet invented by man. Under these conditions is it surprising that a freshman fails to formulate his data into mathematical equations?

On the new plan, the equation is taken as a statement about actual concrete things. In this particular case, the computation would take the form,

$$s = \tfrac{1}{2} \times 32 \frac{\text{ft.}}{\text{sec.}^2} \times 3^2 \text{ sec.}^2 = 144 \text{ ft.}$$

The interpretation of the formula is now that s is physically a result of the combination of the gravity acceleration g with the time t, which enters once in producing the final velocity gt, and mean velocity $\tfrac{1}{2}gt$ and again in combination with this mean velocity to give the distance $\tfrac{1}{2}gt^2$. The essential feature in the application of this plan is the insertion of each quantity as a *quantity*, that is, as *so many times another quantity of the same kind*, and not as a mere "so many."

If in computation the boy should happen to forget to square t, he would get

$$s = \tfrac{1}{2} \times 32 \frac{\text{ft.}}{\text{sec.}^2} \times 3 \text{ sec.} = 48 \frac{\text{ft.}}{\text{sec.}},$$

an obviously impossible *kind* of answer. But if he departs from the above method only in calling $t = \tfrac{1}{20}$ min., he gets

$$s = \tfrac{1}{2} \times 32 \frac{\text{ft.}}{\text{sec.}^2} \times \frac{1}{20^2} \text{ min.}^2 = \frac{1}{25} \frac{\text{ft. min.}^2}{\text{sec.}^2}.$$

To reduce this to simpler terms he has only to substitute 60^2 sec^2 for min^2, exactly as he would perform any other algebraic substitution of equals, and then cancel the sec^2 and finish the computation. Or, if he lets

$$g = 22 \frac{\text{min.}}{\text{hr. sec.}},$$

he gets

$$s = \tfrac{1}{2} \times 22 \frac{\text{min.}}{\text{hr. sec.}} \times 3^2 \text{ sec.}^2$$

$$= \tfrac{1}{2} \times 22 \frac{\text{min.}}{3600 \text{ sec.}^2} \times 3^2 \text{ sec.}^2 = \tfrac{11}{400} \text{ min.},$$

which is as correct an answer as the other. To reduce units the game is simply to substitute equals for equals and cancel. If this does not give the right kind of an answer, it is a sure indication of an error.

Of course, to play the game fairly, we must abolish formulas with lost units, such as $s = 16t^2$. Examples of these are found most frequently in electricity. The old plan would write such a formula as that for the force on a wire in a magnetic field, as $F = IlH$ with a string of restrictions on units, or $F = \tfrac{1}{10}IlH$ with another string. By forgetting the restrictions and using the simpler formula with the most familiar units, the students often achieve remarkable results. On the new plan this would be written $F = KIlH$ where

$$K = \tfrac{1}{10} \frac{\text{dyne}}{\text{amp. cm. gauss.}}$$

and all restrictions are removed. It is of of course true that this form of the equation involves more writing than the others; indeed, it may be noted here that the process of treating all equations as physical statements is not necessarily worth while for trained men doing routine computations, but it is extremely useful for all sorts of cases where the computations are not familiar enough to be classified as routine work. For all such cases it is well worth while to write out the proportionality constant, especially if some one is likely to want l, say, in inches or F in kilograms.

In the detailed application of this principle, there is one point where confusion might arise, though it can readily be avoided. It is the anomalous behavior of the unit, radian, which appears as a perfectly respectable unit when an angular velocity is converted from $\frac{\text{rev}}{\text{min}}$ to $\frac{\text{sec}}{\text{rad}}$ but does not appear when the same angular velocity is found from $\frac{v}{r}$. This anomaly is the only one of its kind, and is not nature's fault, but our own. If we define angle as degree of opening, to be measured in units of the same kind, the substitution method outlined above is the most natural method of converting say $\frac{\text{rev}}{\text{min}}$ to $\frac{\text{rad}}{\text{sec}}$. If, on the other hand, we define angle as a mere ratio of arc to radius it is necessarily a pure number (like a sine or a tangent). If we swap horses in midstream, we shall either miss this unit later or else see it floating up where we do not expect it. This means we must insert or rub out the unit radian whenever it is convenient to do so. Fortunately angle is the only quantity treated in such a way.

For the sake of such mathematical purists as may not approve of the above on philosophical grounds, a few words should be inserted here on the meaning of the term "multiplication." In elementary arithmetic it means merely repeated addition, but with the introduction of irrational numbers the term is extended by mathematicians to an operation that is not strictly repeated addition. The plan here advocated extends the notion of multiplication still further, to cover a physical combination of concrete quantities. In general the definition of multiplication in each individual case amounts to translating into algebra the ordinary verbal definition of the compound quantity involved (area, velocity, work, etc.). This extension is made practicable by the fact that the operation thus defined obeys the same logical postulates as the corresponding algebraic operation on pure numbers. In other words, the machinery of mathematics can be applied not merely to numbers, but to any group of concepts and operations satisfying the same postulates. This fact is accepted intuitively by most students; and incidentally the emphasis it puts on the definitions prevents most of the well-known confusion between acceleration and velocity, power and work, and so on.

To sum up, it seems to me after several years' experience with this system, that it has the following important advantages: (1) It treats equations as neat shorthand statements about real physical things and emphasizes the esthetic side of mathematics in general; (2) It provides an enlarged principle of dimensions by which equations may be checked during computation; and (3) It removes completely all restrictions on the units to be used and enables the student to concentrate his attention on the facts of nature without the disturbing influence of arbitrary rules.

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SPECIAL ARTICLES

ON THE SWELLING AND "SOLUTION" OF PROTEIN IN POLYBASIC ACIDS AND THEIR SALTS

THERE are available only scattered observations on the absorption of water by proteins in the presence of various polybasic acids and their salts. In order to obtain further experimental data in this field, we undertook a rather detailed study of this problem during the past year. As examples of proteins, dried gelatin discs and powdered fibrin were used. For the polybasic acids we chose phosphoric, citric and carbonic. In connection with the swelling of gelatin, we studied also its "solution." The general results of our experiments may be summed up as follows.

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The amounts of water absorbed by gelatin from equimolar solutions of monosodium, disodium and trisodium phosphate depend not only upon which of these salts are present, but upon their concentration. Gelatin absorbs but little more water in a solution of monosodium phosphate than it does in pure water.