OBITUARY NOTICE

PROFESSOR OLAUS HENRICI.

Olaus Magnus Friedrich Erdmann Henrici was born in the year 1840, in Meldorf, on the West Coast of Holstein, where his father held a post in the Danish Civil Service. His father had studied Science and Engineering at the Gewerbeschule in Berlin, which afterwards became the College at Charlottenburg, and he was one of the three who prepared the plans for a canal from Kiel to the mouth of the Elbe. From the age of 9 to 16 Henrici was taught at the Gymnasium in Meldorf. From the age of 16 to 19 he worked as an apprentice in some engineering works at Flensburg. At the age of 19 he went to the Karlsruhe Polytechnicum, where he had the inestimable privilege of coming under the influence of Clebsch, by whose advice he devoted himself entirely to the study of Mathematics. At the age of 22 he went to study under Hesse at Heidelberg, where he obtained the degree of Ph.D. On leaving Heidelberg he went to Berlin, where he studied under Weierstrass and Kronecker. Thence he went to Kiel, where he became a Privat-dozent at the University, but soon found that, being without private means, his prospects were hopeless. So he came to London in 1865, where he struggled for a livelihood, earning a little money by giving private lessons to school boys, until in 1869 he obtained an introduction from Hesse to Sylvester, through whom he became acquainted with Cayley, Clifford, and Hirst. Hirst gave him work in his classes at University College and introduced him to the authorities of Bedford College, where he became Professor of Mathematics. At Easter, 1870, Hirst fell ill and Henrici took over his work, and at the end of the Session was appointed to succeed Hirst, who had resigned.

From 1870 to 1880 he occupied the Chair of Pure Mathematics, which he exchanged in 1880 for that of Applied Mathematics, and this he held until 1884. During his tenure of this last-named Chair he introduced into the course of instruction for English engineers the study of Graphical Statics.

In 1884 he left University College for the Chair of Mechanics and Mathematics at the Central Technical College, where he organised a Laboratory of Mechanics. This was the first of its kind and has been the model of many others. Almost every piece of apparatus is fitted with a device to make extreme accuracy of measurement possible. The Laboratory contains a large number of calculating machines, planimeters, moment integrators, and, most important of all, the Harmonic Analyser. referred to below. In 1911 his work at the Central Technical College came to an end, when his students presented him with a testimonial and founded the Henrici Medal in his honour, to be given annually at the College for proficiency in Mathematics. He retired to Chandler's Ford in Hampshire, where he devoted himself to gardening. He had been in failing health for some time before his death on the 10th August last.

In 1874 he was elected a Fellow of the Royal Society. From 1882 to 1883 he was a member of the Council of the Royal Society, and from 1882 to 1884 President of the Mathematical Society. In 1884 the University of St. Andrews conferred on him the Honorary Degree of LL.D.

In 1877 he married the daughter of the Rev. Dr. Kennedy and sister of Sir Alexander Kennedy, who survives him. There was one child of the marriage, Major E. O. Henrici of the Royal Engineers.

His mathematical writings are as follows:—

(1) " Bemerkung zu ' Hesse, Zerlegung der Bedingung für die Gleichheit der Hauptaxen eines auf einer Oberfläche zweiten Ordnung liegenden Kegelschnittes in die Summe von Quadraten '," *Crelle's Journal*, Vol. 64, 1865.

(2) " Transformation von Differentialausdrücken erster Ordnung zweiten Grades mit Hülfe der verallgemeinerten elliptischen Coordinaten," *Crelle's Journal*, Vol. 65, 1866.

(3) " Skeleton Structures, especially in Application to Building of Steel and Iron Bridges," Atchley & Co., London, 1866.

(4) "On certain Formulæ concerning the Theory of Discriminants, with Applications to Discriminants of Discriminants and to the Theory of Polar Curves," *Proc. London Math. Soc.*, Ser. 1, Vol. 2, 1869.

(5) On Series of Curves, especially on the Singularities of their Envelopes, with Application to Polar Curves," *Proc. London Math. Soc.*, Ser. 1, Vol. 2, 1869.

(6) " On Congruent Figures " (London Series of Science Class Books, Longmans, Green, & Co., 1879).

(7) " Presidential Address to Section A of the British Association," 1883.

(8) "Über Instrumente zur Harmonischen Analyse" (Catalogue of Exhibits at the Exhibition of Mathematical Apparatus at Munich, 1892).

(9) Notices accompanying Exhibits at the Munich Exhibition (Nos. 47, 90, and 161 in the Catalogue).

(10) "On a New Harmonic Analyser" (*Phil. Mag.*, July, 1894).

11) "An Address on the Use of Vectorial Methods in Physics to Section A of the British Association," 1903.

(12) "On Vectors and Rotors" (jointly with Mr. G. C. Turner, 1903).

(13) "On the Theory of Measurement by Metal Tapes and Wires in Catenary" (Ordnance Survey, 1912, jointly with Major Henrici).

(14) Contributions to the *Encyclopædia Britannica*: six articles on (i) Calculating Machines, (ii) Euclidean Geometry, (iii) Projective Geometry, (iv) Descriptive Geometry, (v) Perspective, (vi) Projection.

No. (1). The first paper in the series deals with an investigation by Hesse, who, starting from the well known fact that the discriminant of the quadratic, which determines the lengths of the principal axes of a conic, whose equation is given in a plane, can be expressed as the sum of two squares, endeavoured to obtain a similar result for a conic when it is given as the intersection of a plane and a quadric surface. Hesse expressed the discriminant successively as the sum of 10, 7, 6 and 5 squares, and then said that it was probably impossible to express it as the sum of two squares. Henrici, making use of Euler's expressions for the direction cosines of three orthogonal axes in terms of three independent quantities, succeeded in expressing the discriminant as the sum of two squares in three different ways.

No. (2). The second paper is an extension to n dimensions of a method employed by Hesse in his lectures on Geometry of Space in the discussion of the problem of the Principal Axes of Conics and Quadrics by the aid elliptic coordinates.

No. (3). The third work on the list is a little book in which Henric advocated the use of pin-jointed structures, the principal advantage of which is that each bar is exposed to tension and compression only in the direction of its longitudinal axis, which is the most favourable condition possible.

Nos. (4) and (5). These papers are related to one another. The work depends on the following important theorem:—

If $F(u, v, w)$ be a rational integral function of u, v, w; if Ω be the discriminant of this function with regard to u; and if D be the discriminant of Ω with regard to v; then D is of the form AB^2C^3, and $D = 0$ determines w in such a way that Ω has a double root v; and to these values of v, w correspond for $A = 0$ one double root u of $F = 0$; for $B = 0$ two distinct double roots u of $F = 0$; for $C = 0$ one triple root u of $F = 0$.

The orders of A, B, C are determined, and the results applied to determine the order, the class, and the number of singularities of (1) the envelope of a unicursal series of curves, (2) Steinerian curves, (3) polars of curves, (4) mixed polars of curves, and (5) Steinerians of polar curves.

Though it is not mentioned, the work includes the resolution of the c-discriminant locus of the family of curves $f(x, y, c) = 0$ into the envelope-locus, the node-locus, and the cusp-locus, and accounts for the number of times that each factor occurs.

Nos. (6), (7), and [(14) (ii)]. The article on Euclidean geometry [No. (14) (ii)] is a very clear and in the main a descriptive account of Euclid's *Elements* (excluding Books 7, 8, 9, which are arithmetical, and Book 10 which deals with geometrical irrational quantities). Since it was written a good deal of work on the nature and structure of the Fifth Book (on Proportion) has been published. Henrici's own ideas as to the best methods of treating Elementary Geometry will be found in his little book on "Congruent Figures," No. (6), which in a small compass treats of the matter of Euclid's first four books, excepting the theorems relating to areas. It was written with the object of familiarising students from the very first with those modern methods of which the method of projection and the principle of duality are the most fundamental. These methods are introduced in a very simple way. In his preface Henrici says that "the advantages of the method adopted will however be fully appreciated only in their continuation in the second volume which will treat of areas in connection with what Möbius called 'equal figures' and of similar figures." That second volume was never written. It would appear from the concluding remarks of the address [No. (7)] that Henrici had failed to find any treatment that completely satisfied him. He says, "At present it has not even been settled which series of axioms will be ultimately adopted. Of the various systems which have been proposed since the investigations of Riemann and Helmholtz, I may mention here Clifford's suggestion to replace Euclid's Axiom about parallels by the new one, which maintains that in a plane similar figures exist, or, more completely, that at any part in a plane a figure is possible which is similar to any given figure in that plane. This axiom is somewhat startling as long as we have the usual theory of similar figures in our mind. But the notion of similar figures is truly axiomatic, and it has lately become my conviction that the axiom may be extremely fruitful, and the working out of a syllabus of plane geometry based on it would be very desirable. Possibly many such attempts have still to be made before a new Euclid finds the materials sufficiently prepared for him to raise the hoped for edifice."

The idea of replacing Euclid's postulate of parallels by the postulate that similar figures exist dates back much further than the work of Clifford. It will be found in a lecture by John Wallis in 1663, *Opera Math.*, Vol. 2, pp. 669–678. It reappears in Carnot's *Géométrie de Position* (1803), and in Laplace's *Works*, Vol. 6, p. 472 (1824). Laplace says that the idea of proportionality is a more natural postulate than Euclid's Postulate on Parallels. On the other hand, Bonola (*Non-Euclidean Geometry*, p. 16) says: "The idea of form, independent of the dimensions of the figure, constitutes a hypothesis, which is certainly not more evident than the postulate of Euclid."

The subject seems to be worthy of further research. The idea of similarity is certainly acquired at a very early age. As soon as a child can show that he can recognise a watch from a picture of it he has the idea of similarity. In an observation I once made the age of the child was nine months.

It seems to me a misfortune to science that Henrici did not carry out his idea of writing a book on Similar Figures as he intended.

Nos. (8), (9), (10), and (14) (i). Perhaps the most strikingly original piece of work done by Henrici was the construction of the Harmonic Analyser.

If a function $y = f(x)$ be expressed in a Fourier series

$$y = \tfrac{1}{2}A_0 + \sum_{n=1}^{\infty} A_n \cos(2n\pi x/c) + \sum_{n=1}^{\infty} B_n \sin(2n\pi x/c),$$

then the coefficients are determined by the formulæ

$$cA_n = 2\int_0^c y\cos(2n\pi x/c)\,dx, \quad cB_n = 2\int_0^c y\sin(2n\pi x/c)\,dx.$$

Clifford showed how the integrals could be obtained mechanically thus:—"If the curve to be analysed be stretched out in the direction of the x to n times its base without altering the y, and then wrapped round a cylinder with circumference c so that it goes n times round, then the orthogonal projection of this curve on the meridian plane which passes through the zero point of the curve will enclose an area which is proportional to B_n. Similarly A_n is got by the aid of a plane perpendicular to the first."

Setting out from this idea Henrici transformed the formulæ for the coefficients into the following :—

$$nA_n = -\frac{1}{\pi}\int \sin(2n\pi x/c)\,dy, \qquad nB_n = \frac{1}{\pi}\int \cos(2n\pi x/c)\,dy,$$

where the integration with regard to y has to be taken over the whole of a curve derived from the original curve by making it continuous. The mechanical evaluation of these integrals is easier than that of the integrals first mentioned. The details of the apparatus will be found in the *Philosophical Magazine* for July 1894. It is sufficient to state here that in the original design the curve was drawn on a cylinder, but by an improvement due to Mr. Sharp, one of Henrici's assistants, it was shown that it might be drawn on a flat sheet of paper. Further improvements in the instrument were made by Coradi.

Part of the instrument is as follows :—

There are wheels of radii proportional to

$$1,\ \tfrac{1}{2},\ \tfrac{1}{3},\ \tfrac{1}{4},\ \tfrac{1}{5}.$$

A wire passes round these, and the coefficients for $n = 1, 2, 3, 4, 5$ are obtained by making the tracer go once round the curve.

Coaxial with these wheels and above them are wheels of radii proportional to

$$\tfrac{1}{6},\ \tfrac{1}{7},\ \tfrac{1}{8},\ \tfrac{1}{9},\ \tfrac{1}{10},$$

respectively.

To determine the coefficients for $n = 6, 7, 8, 9, 10$, the wire must be taken off the first set of wheels and be made to pass round the upper set. The coefficients are then obtained in the same way as for $n = 1, 2, 3, 4, 5$.

No. (9). Of the three exhibits referred to under (9) one is a description of an earlier form of the Harmonic Analyser, already noticed. Another was a model of a surface of the ninth order constructed to illustrate Sylvester's complete invariantive determination of the roots of a quintic in his great memoir on the real and imaginary roots of algebraic equations (*Phil. Trans. Roy. Soc.*, 1864). The third was a model of two confocal hyperboloids connected together so that they could be deformed but always remained confocal hyperboloids. This model had a remarkable history. In the year 1873 Henrici asked one of his pupils to construct a model of a hyperboloid of one sheet by taking three rods to represent three generators of one system, and fastening to them rods to represent generators of he other system. He believed that the model so constructed would be rigid. To his surprise he found that it was movable. It was then easy to prove that if the centre and the direction of the axes were fixed the hyperboloid could be deformed into any number of confocal hyperboloids. This model was exhibited to the London Mathematical Society in 1874. Soon after this two such models were constructed and so connected that they moved together but always remained confocal. This is the model exhibited at Munich.

Nos. (11) and (12). In his teaching Henrici made great use of Vectorial Analysis, and with the view of introducing its use into English teaching, he delivered an address before Section A of the British Association in 1903 (No. 11). All that remains of his ideas, in addition to this address, is a little book on "Vectors and Rotors," No. (12), based on his lectures to first year students, which was compiled by his assistant, Mr. G. C. Turner. It treats only of the elementary part of the subject, and was intended to be the first part of a book on Vector Analysis, but that book was never completed.

No. (13). This paper, written jointly with Major Henrici, contains a valuable mathematical investigation, due to Henrici, of the Theory of Measurement by Metal Tapes and Wires in Catenary.

Major Henrici, when calculating the length of a geodetic base line at Lossiemouth from the measurements made there in 1909, was unable to find in any former investigation the correction to be applied for the change in shape of the catenary when on any considerable slope. He submitted the point to his father, and the result was the investigation under consideration. This raised the question of the effect produced on the catenary and on the elastic extension of the tape or wire by changes in the tension. The formulæ given make it possible to measure the straight distance between two points on a slope up to 1 in 3, with a resulting error from the causes above mentioned of less than one in a million, and even this small error can be almost entirely eliminated by taking the mean of two measurements with a standard tension applied first at the upper and secondly at the lower end.

No. (14). Of the contributions to the *Encyclopædia Britannica* I have already referred to the first and second. The third and the sixth on Projective Geometry and Projection have a very high reputation as a clear and lucid exposition of the subject. The fourth and fifth are short articles.

Henrici will be remembered chiefly as a great teacher. During his years of struggle (1865–1869) when he was obliged to spend much time in giving lessons in elementary mathematics to school boys, he had learned to probe the working of the minds of his pupils. This was of great value to him in his subsequent career as a teacher and a writer, for he acquired the faculty of expressing himself with great clearness in both capacities. He published nothing until he felt completely satisfied as to its form. But for this characteristic more of his methods and ideas would have been preserved. I understand that he has left a large amount of manuscript, and it is much to be hoped that some one will be found to go through it with extreme care.

As one of the large number of his pupils I am able to bear testimony to the singular lucidity of his teaching and to his readiness to explain difficulties at all times. With qualities such as these it is easy to understand the mingled respect and affection with which his pupils regarded him. They feel that a great master of his art has passed to his rest.

M. J. M. HILL.