Particle properties

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Abstract

The standard model of physics classifies particles into elementary leptons and hadrons composed of quarks. In this article the existence of an alternate ordering principle will be demonstrated giving particle energies to be quantized as a function of the fine-structure constant, α. The quantization can be derived using an appropriate wave function that acts as a probability amplitude on the electric field. The value of α itself can be approximated numerically by the gamma functions of the integrals involved.

The model may be used to calculate other particle properties as well. The magnetic moment may be calculated directly from the electromagnetic fields. In the range of femtometer the wave function overlap provides a mechanism for strong interaction. The model gives quantitative terms for strong, Coulomb and gravitational interaction of particles indicating a common base of these three forces.

Necessary input parameters for all calculations can be reduced to elementary charge and electric constant only.

1.1 Introduction

Particle zoo is the informal though fairly common nickname to describe what was formerly known as "elementary particles". The standard model of physics [1] divides these particles into leptons, considered to be fundamental "elementary particles" and the hadrons, composed of two (mesons) or three (baryons) quarks. Well hidden in the data of particle energies lies another ordering principle which can be derived by interpreting particles as electromagnetic objects subject to some general principles of quantum mechanics. The concept of expressing mass in electromagnetic terms is almost as old as Maxwell´s equation, going back as far as 1881 with the work of J.J.Thomson [2]. O.Heaviside [3] and others produced a mass-energy relation for charged particles of $\mathcal{E} = 3/4\ mc_0^2$ later developed further by Poincare and others, dropping factor 3/4 [4][1].
W.Wien was a prominent advocate of reducing mass and consequently gravitation to electromagnetism [5]
In the model presented here, the particles are interpreted as some kind of standing electromagnetic wave localized due to the effects of the strong force and may be visualized as a rotating electromagnetic field with the E-vector pointing towards the origin. Neutral particles are supposed to exhibit nodes[2] separating corresponding equal volume elements of opposite polarity. To obtain quantifiable results, the electromagnetic field will be modified with an appropriate exponential function, Ψ(r, ϑ, φ, e, ε) [3], serving as probability amplitude of the field. The two integrals needed to calculate energy in point charge and photon representation exhibit the following two relations:
1) Their product - resulting from energy conservation - is characterized by containing the product of the two gamma functions $\Gamma(1/3)|\Gamma(-1/3)| \approx \alpha^{-1}/(4\pi)$,
2) their ratio features a quantization of energy states with powers of $1/3^n$ over some base α_0, a relation that can be found in the particle data with $\alpha_0 = \alpha$ as:

$$W_n/W_e \approx 3/2\left(y_l^m\right)^{-1/3}\Pi_{k=0}^{n}\alpha\text{^}(-1/3^k) \qquad n = \{0;1;2;..\} \tag{1}$$

with W_e = energy of electron, W_n = energy of particle n and y_l^m representing the angular part of Ψ(r, ϑ, φ). For spherical symmetry $y_0^0 = 1$ holds, corresponding particles are e, μ, η, p/n, Λ, Σ and Δ [4]. Factor 3/2 is supposed to represent an anomaly of the electron, related to angular momentum, see chpt. 2.5, 2.8.
Apart from calculating particle energies the model may be used to describe other particle properties. The magnetic moment of particles may be calculated directly from the electromagnetic fields modified by Ψ. At distances comparable to particle size, typically femtometer for hadrons, direct interaction of particle wave

1 Here $\mathcal{E}$ denotes energy - in all other parts of this article energy is identified by the letter W while E is for electric field; m = mass ; c_0 = speed of light in vacuum;
2 nodes of positive and negative charge regions will have to coincide with nodes of the wave function but not necessarily vice versa.
3 r = distance from origin, ϑ, φ = angular coordinates, e = elementary charge, ε = electric constant
4 The relation of the masses e, μ, π with α was noted in 1952 by Y.Nambu [6]. M.MacGregor calculated particle mass and constituent quark mass as multiples of α and related parameters [7]. This is an extended, slightly altered and more speculative version of [8].

functions ("overlap") has to be expected. Interpreting this interaction as strong interaction and considering the basic spatial characteristics of the functions may provide a possible explanation why leptons, in particular the tauon, are not subject to this interaction.
Quantitative terms for potential energy of strong and Coulomb interaction as well as particle energy (mass) can be attributed to the terms of the expansion of the incomplete gamma function appearing in the integrals for calculating particle energy and gravitational attraction may be linked to this function as well, suggesting a common base for all three forces.
The following equations basically use two parameters, one for energy (β or τ) based on W_e as reference and free parameter of the model and a second (σ) which is a function of angular momentum, see chpt. 2.5, 2.8. Typical accuracy of the calculations presented is ~ 0.001 (e.g. due to approximations of Γ-functions)[5] which would be also the order of magnitude of possible QED corrections.
The model is an electrostatic approximation of an electromagnetic object implying some asymmetry in its terms, e.g. the electromagnetic units used.
This is a preliminary working paper intended to provide food for thought [6].

1.2 Unit System

The unit system used in this work is SI with the exception of electromagnetic units that are required to be based on their relation to c_0, in the simplest case using a symmetric split of electric and magnetic constant, ε and μ, such as given e.g. in Planck units. In this work SI units are kept with the modification:

$$c_0^2 = (\varepsilon_0\,\mu_0)^{-1} \tag{2}$$

being replaced by

$$c_0^2 = (\varepsilon_c\,\mu_c)^{-1} \tag{3}$$

with
$\varepsilon_c = (2.998\text{E+8 [m}^2\text{/Jm]})^{-1} = (2.998\text{E+8})^{-1}$ [J/m]
$\mu_c = (2.998\text{E+8 [Jm/s}^2\text{]})^{-1} = (2.998\text{E+8})^{-1}$ [s^2/Jm]
i.e. the numerical values for c_0, $1/\varepsilon_c$, $1/\mu_c$ are identical, the units of ε_c, μ_c are expanded by [Jm] for the convenience of this model.
In the following the abbreviation b_0 is used for the Coulomb term $b_0 = e^2/(4\pi\,\varepsilon_0) = e_c^2/(4\pi\varepsilon_c) = 2{,}307\text{E-28}$ [Jm] which is identical in both unit systems, thus all calculations concerning particle energy are not affected except for the definition of τ_{dim}, equ. (40)ff.
From b_0 follows for the square of the elementary charge: $e_c^2 = 9{,}67\text{E-36}$ [J^2].

1.3 Wave function

The model is essentially based on a single assumption:
Particles can be described by using an appropriate exponential wave function, Ψ(r), that acts as a probability amplitude on an electromagnetic field.
An appropriate form of Ψ can be deduced from three boundary conditions:
1.) To be able to apply Ψ to a point charge Ψ(r = 0) = 0 is required, this may be considered by a term such as:

$$\Psi(r) \sim \exp\left(\frac{-\beta/2}{r^y}\right) \tag{4}$$

2.) To ensure integrability an integration limit is needed. This may be achieved by Ψ(r) being the solution of a 2nd order differential equation of approximate general form

$$-\Delta\Psi(r) + \frac{\beta/2}{r^{x+1}}\nabla\Psi(r) - \frac{\beta/2}{\sigma r^{x+2}}\Psi(r) = 0 \tag{5}$$

giving for particle n:

$$\Psi_n(r)=\exp\left(-\left(\frac{\beta_n/2}{r^x}+\left[\left(\frac{\beta_n/2}{r^x}\right)^2-4\frac{\beta_n/2}{\sigma r^x}\right]^{0.5}\right)/2\right) \tag{6}$$

5 Except for energies of higher particle states, see 2.9
6 http://doi.org/10.5281/zenodo.801423 gives a modified, shorter version; this working paper is intended for a broader, more speculative approach.

3.) Ψ should be applicable regardless of the expression chosen to describe the electromagnetic object. In particular requiring a point charge and a photon representation of a localized electromagnetic field (particle) to have the same energy, the exponent of r is required to be x=3 (see (23)), giving finally:

$$\Psi_n(r) = \exp\left(-\left(\frac{\beta_n/2}{r^3} + \left[\left(\frac{\beta_n/2}{r^3}\right)^2 - 4\frac{\beta_n/2}{\sigma r^3}\right]^{0.5}\right)/2\right) \tag{7}$$

Up to the limit of the real solution of (7), $r = r_l$, with

$$r_l = (\sigma\,\beta/8)^{1/3} \tag{8}$$

in all integrals over Ψ(r) given below equ. (9) may be used as approximation for (7)

$$\Psi_n(r < r_l) \approx \exp\left(\frac{-\beta_n/2}{r^3}\right) \tag{9}$$

Phase will be neglected on this approximation level.
The integrals over the approximation of Ψ(r) according to equ. (9) are closely related:

$$\int_0^{r_l} \Psi(r)^2 r^{-(m+1)} dr \quad = \Gamma(m/3,\ \beta/r_l^3)\ \beta^{-m/3}/3 \tag{10}$$

with m = {...;-1;0;1;2;...}. The term $\Gamma(m/3, \beta/r_l^3)$ denotes the upper incomplete gamma function, given by the Euler integral of the second kind:

$$\Gamma(m/3,\ \beta/r_l^3) = \int_{\beta/r_l^3}^{\infty} t^{m/3\ -1} e^{-t} dt \tag{11}$$

It follows from the boundary condition (8) that the integration limit, β/r_l^3, has to be a constant for all particles:

$$\beta_n/r_{l,n}^3 = 2\sigma\tau_n b_0^2/\ r_{l,n}^3 = 8/\sigma \tag{12}$$

For m ≥ 1 the term $\Gamma(m/3, \beta/r_l^3)$ may be approximated by Γ(m/3) [7], for m ≤ 0 the integrals (10), (11) depend critically on the integration limit and have to be integrated numerically.
Coefficient β_n is a particle specific factor, proportional to particle energy W as $\beta_n \sim W_n^{-3}$ (16), for particle n it may be given as partial product of a value for a reference particle, β_{ref} carrying the dimensional term β_{dim} times particle specific dimensionless coefficients, α_n, of succeeding particles representing the ratio of β_n and β_{n+1}:

$$\beta_n = \beta_{ref}\,\Pi_{k=1}^{n}\alpha_k = \beta_{dim}\,\Pi_{k=0}^{n}\alpha_k \tag{13}$$

Coefficient σ is related to the angular part of Ψ(r, ϑ, φ) and thus to angular momentum.
In the following apart from the particle coefficients α_n, parameter β may be analyzed further. To avoid introducing additional parameters one might test an approach giving β as function of b_0 and σ. A suitable expression will be $\beta_n = 2\,\tau_n\,\sigma\,b_0^2/(2\pi)^3$ [8], the particle specific parameter β will be repalced by τ, turning (13) into:

$$\tau_n = \tau_e\,\Pi_{k=0}^{n}\alpha_{\tau,k} = \tau_e\,\Pi_{\tau,n} \tag{14}$$

2 Energy levels of elementary particles

2.1 Calculation of energy - point charge

Particle energy is expected to be equally divided into electric and magnetic part, $W_n = 2W_{n,el} = 2W_{n,mag}$ [9]. To calculate energy the integral over the electrical field E of a point charge is used as a first approximation,

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7 The complete Γ-function Γ(m) of the most frequently used terms Γ(+1/3) |Γ(-1/3)| will be abbreviated to Γ_+ and Γ_-. The sign of the latter arises from the relation between Γ-functions, the relevant integrals, (10), (11), give the positive value.

8 factor $(2\pi)^3$ see 2.8

9 Alternatively $W_n/2$ may be interpreted as $W_n/2 = W_{pot} = W_{kin}$ of a harmonic vibration. The term $W_n/2$ will be used in some relations below.

giving [10]:

$$W_{pc,n} = 2\int_0^\infty \varepsilon_0 E(r)^2 d^3r \;=\; 2\int_0^\infty \frac{e^2}{4\pi\varepsilon_0 r^2}dr \;=\; 2b_0\int_0^\infty r^{-2}dr \qquad (15)$$

The integral for m = 1 is needed to calculate $W_{pc,n}$. Inserting (10) and (11) in equ. (15) will turn out:

$$W_{pc,n} = 2\varepsilon_0\int_0^\infty E(r)^2\Psi_n(r)^2 d^3r \;=\; 2b_0\int_0^{r_{l,n}} \Psi_n(r)^2 r^{-2}dr \;=\; 2\, b_0\, \Gamma_+\, \beta_n^{-1/3}/3 \qquad (16)$$

Equation (16) is the source of β_n, $\tau_n \sim W_n^{-3}$. From (14) and (16) follows:

$$\tau_n/\tau_e = \Pi_{k=0}^{n}\alpha_{\tau,k} \;=\; \Pi_{k=0}^{n}\alpha_{W,k}^{-3} \qquad (17)$$

with $\alpha_{W,k}$ being the coefficients for the general case of a partial product $\Pi_{W,n}$ for particle energies [11]. Through equ. (8) the relations $\tau_n \sim r_{l,n}^3$ and $W_n \sim r_{l,n}^{-1}$ hold.

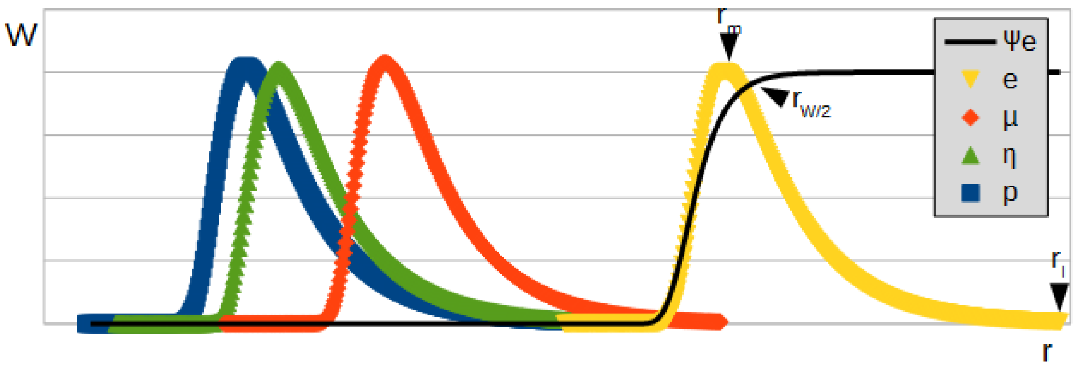

Figure 1: Example for particle energy $W_{n\ calc}(r)$ (normalized) vs lg(r) according to equ. (16); $r_{m,n}$: see (18); $r_{W/2}$ => radius where the integrals of (16) attain half their final value; r_l see (8); black line: $\Psi_e(r)$

$$r_{m,n} = \Gamma_-\, \beta_n^{1/3}/3 \;\approx r_{max,n} \qquad (18)$$ [12]

2.2 Calculation of energy - photon

For m = -1 equations (10), (11) give a relation between radii and Euler-integral:

$$r_{x,n} = \int_0^{r_{x,n}} \Psi_n(r)^2 dr \;=\; \beta_n^{1/3}/3 \int_{\beta/r_{x,n}^3}^{\infty} t^{-4/3}e^{-t}dt \qquad (19)$$

which may be used to express r_x as a function of β, Γ, appropriate for this model. Applying this for the value of the Compton wavelength, λ_C, in the term for the energy of a photon, hc_0/λ_C gives

$$\lambda_{C,n} = \int_0^{\lambda_{C,n}} \Psi_n(r)^2 dr \;=\; \beta_n^{1/3}/3 \int_{\beta/\lambda_{C,n}^3}^{\infty} t^{-4/3}e^{-t}dt \;= 352.97\ \beta_n^{1/3}/3 = 0.9934\ 36\pi^2\ \beta_n^{1/3}/3\ \Gamma_- \qquad (20)$$

According to (16) particle energy is proportional to $\beta_n^{-1/3}$ and $\lambda_{C,n} \sim \beta_n^{1/3}$ has to hold, requiring the lower integration limit of the Euler integral and the factor $\approx 36\pi^2$ to be a constant for all particles. Energy of a photon can be expressed by:

$$W_{Phot,n} = hc_0/\lambda_{C,n} \;=\; \frac{hc_0}{\int^{\lambda_{C,n}} \Psi_n(r)^2 dr} \;=\; \frac{3hc_0}{36\pi^2\Gamma_-\beta_n^{1/3}} \qquad (21)$$

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10 In general it can not be expected that Coulomb's law based on two interacting charges can be used unaltered in this problem. In prior versions a prefactor 4π in (15)f etc. has been used in place of 2, in this version the factor 2π is included in Ψ(r) as $(2\pi)^{-3}$ to recover the correct energy while reducing all values for radii by 1/(2π).

11 Not used in the following. However, coefficients in Ψ, α_τ, should not be confused with those in energy terms, α_W .

12 $r_{m,n} \approx 0.942\ r_{max,n}$

2.3 Relation of integrals for $W_{pc,n}$ and $W_{Phot,n}$ with fine-structure constant α

The energy of a particle has to be the same in both photon and point charge description. From (16) and (21) follows:

$$W_{pc,n} = W_{Phot,n} = 2b_0\, \Gamma_+\, \beta_n^{-1/3}/3 = \frac{3hc_0}{2\pi\, 18\pi\, \Gamma_-\beta_n^{1/3}} \tag{22}$$

which may be rearranged to emphasize the relationship of the gamma functions (Γ_+ = 2.679; Γ_- = 4.062) with α, giving (note: h => ħ):

$$\frac{4\pi\Gamma_+\Gamma_-}{0.998} = \frac{9hc_0}{18\pi b_0} = \frac{\hbar c_0}{b_0} = \alpha^{-1} \tag{23}$$ [13] [14]

Factor k_a = 0.998 will be used in equations below to indicate the deviation from the exact value.

Equation (23) uses two approximations:

1) Γ_+ is used in place of the incomplete Γ-function $\Gamma(1/3, \beta/r_l^3) = 0.996\,\Gamma_+$

2) For the integration limit $\beta_n/r_{x,n}^3 << 0$ the result of the Euler integral in (19) is approximated by

$$\int_{\beta_n/r_{x,n}^3}^{\infty} t^{-4/3} e^{-t}\, dt \approx 3\, (\beta_n/r_{x,n}^3)^{-1/3} \tag{24}$$

yielding $3\,\lambda_{C,n}/(\beta_n^{1/3}\Gamma_-) = 356.0656 = 1.002\; 36\pi^2$ as approximation for $36\pi^2$ [15].

The two factors add up to change the remaining inequality of (23) from 0.998 to 0.996. Calculation errors, approximation residuals as well as possible higher order correction terms of e.g. QED type have to be considered to contribute to the remaining discrepancy.

2.4 Quantization with powers of $1/3^n$ over α

In general a relation between coefficients such as given by equ. (14) is arbitrary. The special form of expression (1) may be derived from the product of the point charge and photon expression of energy, W_n^2,

$$W_n^2 = 2b_0 hc_0 \frac{\int^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr}{\int_{\lambda_{C,n}} \Psi_n(r)^2 dr} \sim \frac{1}{\beta_n^{2/3}} \sim \frac{\alpha_{\tau,0}^{1/3}\alpha_{\tau,1}^{1/3}.....\alpha_{\tau,n}^{1/3}}{\alpha_{\tau,0}\alpha_{\tau,1}....\alpha_{\tau,n}} \tag{25}$$

The last expression of (25) is obtained by expanding the product $\Pi_{\tau,n}^{-2/3}$ included in $\beta_n^{-2/3}$ with $\Pi_{\tau,n}^{1/3}$ From this term it is obvious that a relation $\alpha_{n+1} = \alpha_n^{1/3}$ such as given by equation (1) yields the only non-trivial solution for W_n^2 where all intermediate particle coefficients cancel out and W_n becomes a function of coefficient α_0 only:

$$W_n^2 \sim \frac{\alpha_{\tau,0}^{1/3}\alpha_{\tau,0}^{1/9}....\alpha_{\tau,0}\wedge(1/3^n)\,\alpha_{\tau,0}\wedge(1/3^{n+1})}{\alpha_{\tau,0}\alpha_{\tau,0}^{1/3}\alpha_{\tau,0}^{1/9}....\alpha_{\tau,0}\wedge(1/3^n)} = \alpha_{\tau,0}\wedge(1/3^{n+1})/\alpha_{\tau,0} \qquad n = \{0;1;2;..\} \tag{26}$$

By comparison with experimental data $\alpha_{\tau,0}$ may be identified as $\alpha_{\tau,0} = \alpha_e \approx \alpha^9$ and the α-product can in general be given by:

$$W_n^2 \sim \frac{\alpha^3\alpha^1\alpha^{1/3}....\alpha\wedge(3/3^{n-1})\alpha\wedge(3/3^n)}{\alpha^9\alpha^3\alpha^1....\alpha\wedge(9/3^n)} = \alpha\wedge(3/3^n)/\alpha^9 \qquad n = \{0;1;2;..\} \tag{27}$$

The corresponding term for particle energy will be given by:

13 $\Gamma_+\Gamma_-$ = 3π /sin(π/3) = $3^{0.5}$ 2π = 0.99797 α^{-1}/ 4π.

14 With the unit system of 1.1 follows: $\hbar c_0 4\pi\varepsilon_c/e_c^2 \approx 4\pi\,\Gamma_+\Gamma$ => $\hbar c_0\varepsilon_c = \Gamma_+\Gamma_- e_c^2$ => | ħ'|[J²] ≈ $\Gamma_+\Gamma_-$ e^2[J²]

15 Using λ_C/r_m directly gives the same value. (20) is sensitive to the integration limit, thus altered by the additional factor of 2π compared to previous versions.

$$W_n = \frac{4\pi b_0^2}{\alpha} \frac{\int^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr}{\int_{\lambda_{C,n}} \Psi_n(r)^2 dr} = \left(\frac{(2 b_0)^2 \Gamma_{1/3}^2}{9 \; [\alpha 4\pi \; |\Gamma_{-1/3}| \Gamma_{1/3}] \beta_n^{2/3}} \right)^{0.5} = \qquad n = \{0;1;2;..\} \qquad (28)$$

$$= 2 b_0 \frac{\Gamma_{1/3}}{3 \beta_n^{1/3}} = 2 b_0 \frac{\Gamma_{1/3}}{3 \beta_e^{1/3}} \alpha \wedge (1.5/3^n)/\alpha^{4.5} = W_e \; \frac{3}{2} \; \Pi_{k=0}^{n} \alpha \wedge (-1/3^k)$$

giving equation (1) for spherical symmetry. In the last term of (28) the additional factor 3/2 has to be inserted ad hoc to represent the anomaly in the product (13,14) due to the energy ratio of e, μ, $W_\mu / W_e = 1.5088\ \alpha^{-1}$ [16]. Equation (13, 14) has to be adjusted accordingly:

$$\beta_n = \beta_e (2/3)^3 \; \Pi_{k=0}^{n} \alpha \wedge (3/3^k) = \beta_{dim} \alpha_e (2/3)^3 \; \Pi_{k=0}^{n} \alpha \wedge (3/3^k) = \beta_{dim} \Pi_n \quad ^{17} \qquad n = \{0;1;2;..\} \qquad (29)$$

The coefficients of the partial product for $\Pi_{\tau,n}$ of (14) are given by:

$$\tau_n = \tau_e (2/3)^3 \Pi_{k=0}^{n} \alpha \wedge (3/3^k) = \tau_{dim} \alpha_e (2/3)^3 \; \Pi_{k=0}^{n} \alpha \wedge (3/3^k) = \tau_{dim} \Pi_n \qquad n = \{0;1;2;..\} \qquad (30)$$

A fit of W_e will give $\beta_{dim} = 2.12E\text{-}24\ [m^3]$, $\tau_{dim} = 9.64E{+}25\ [m/J^2]$.
Extending the model to energies below the electron with a coefficient of α^3 in equ. (1), $W_\nu / W_e \approx \alpha^3$, gives a state with energy ~0.2eV which is in a range expected for a neutrino [9].

2.5 Relation of σ and τ with α

2.5.1 σ – spherical symmetric states

According to equation (19) $r_{l,n}$ may be given by :

$$r_{l,n} = \int_0^{r_{l,n}} \Psi_n(r)^2 dr \; = \; \beta_n^{1/3}/3 \int_{8/\sigma}^{\infty} t^{-4/3} e^{-t} dt \;\; \approx 1.5133\ \alpha^{-1} \Gamma_- \beta_n^{1/3}/3 \quad ^{18\ 19} \qquad (31)$$

The coefficient σ is related to factor ≈ $1.5133\ \alpha^{-1}$ by equ. (8) and (31) to be:

$$\sigma = 8\ r_{l,n}^3 / \beta_n = 8(4\pi\ \Gamma_-^3/3)^3 = (1.5133\ \alpha^{-1}\ \Gamma_-\ 2/3)^3 = (k_s\ \alpha^{-1}\ \Gamma_-)^3 = 1.772E{+}8[-] \quad ^{20} \qquad (32)$$

Since the term ≈ $3/2\ \alpha^{-1}$ from (1) is approximately equal to the factor in r_l, ≈ $1.5\ \alpha^{-1}$, these terms will cancel in the expression for $r_{l,\mu}$ (note: $W_n \sim 1/r_{l,n}$) :

$$r_{l,e} \approx 1.51\ \alpha^{-1} \Gamma_- \beta_e^{1/3}/3 \qquad (33)$$

$$r_{l,\mu} \approx 1.51^{-1}\ \alpha^{+1} [1.5\ \alpha^{-1} \Gamma_- \beta_e^{1/3}/3\ ] = \Gamma_- \beta_e^{1/3}/3 = r_{m,e} = 1.51\ \alpha^{-1} \Gamma_- \beta_\mu^{1/3}/3 \qquad (34)$$

2.5.2 Coefficient ~1.5

The value of $1.51\ \alpha^{-1}$ in r_l, σ originates from the relationship with J setting the integration limits in equ. (50) and is obviously close to the ratio $W_\mu/W_e = 206.8 = 1.5088\ \alpha^{-1}$. The source of this anomaly is supposed to be the electron rather than the muon, which is a middle term of product (29)f and the equations have been arranged accordingly in (29)ff by factor $(2/3)^3$ representing a general factor of all particles to be canceled by a factor $(3/2)^3$ in α_e. Several options for ~1.51 involved in this model have been be considered: 3/2, Γ_- / Γ_+ = 1.516, π/2 = 1.571, 1.5088 etc. The value 1.5133 has been chosen due to
1. a possible geometrical interpretation (using(23))

$$1.516\ \alpha^{-1}\ \Gamma_- /3 = \Gamma_- /\Gamma_+ \;\; 4\pi\ \Gamma_-\ \Gamma_+ /0.998 \;\; \Gamma_- /3 = \frac{1}{0.998} \; \frac{4\pi \Gamma_-^3}{3} \qquad (35)$$

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16 While the origin of factor 3/2 is unclear, the difference to 1.5088 can be given in 2.5.2.
17 Factor $(2/3)^3$ reproduces factor 3/2 in (1) to be canceled by an extra $(3/2)^3$ in $\alpha_e \approx (3/2)^3 \alpha^9$ (see (38)f). Π_n for brevity.
18 The term ≈ $1.51\ \alpha^{-1}$ is within the accuracy of the calculations identically to $W_\mu/W_e = 206.8 = 1.509\ \alpha^{-1}$. Calculating factor ≈ 1.5 numerically via the Euler integral of (40) with a value of σ ~ 1.509 gives 1.501, numerical fits of particle energy give values in a range of ~ 1.515 .
19 $\quad r'_{l,n} = 1.5 \;\; \Gamma_-/3 \;\; \alpha^{-1} \;\; \beta_n^{1/3} \; = \lambda_{C,n}/3^{0.5}$
20 Factor $k_s = 1.5133 * 2/3 = 1.0088$ used as abbreviation in the following.

suggests to use

$$1.5133 = 1.516 * 0.998 = 4\pi\Gamma_-^2\alpha^{-1} \quad (36)$$

giving a dimensionless representation of particle volume and connecting the one and three dimensional features of this model.

2. Factor 1.5088 of the ratio W_μ/W_e being subject to a 3rd power relationship of the same kind as the α coefficients:

$$\left(\frac{1.5133}{1.5088}\right)^3 = \left(\frac{1.5133}{1.5}\right) \quad (37)$$

indicating that the particle specific term of β and the components of σ are not correctly separated yet even in the case of spherical symmetric states.

2.5.3 Particle parameter β, τ

In the following according to (37) parameter 1.5133 of σ will be incorporated in the particle specific term τ, giving:

$$\beta_n = \tau_{dim} \frac{2}{(2\pi)^3} \left(\frac{2}{3}\right)^3 \frac{\sigma}{1.5133^3}\; b_0^2 \Pi_{k=0}^{n} \boldsymbol{\left[\alpha^3\left(\frac{1.5133}{1.5}\right)\right]^\wedge\left(\frac{3}{3^k}\right)} \qquad n = \{1,2,...\} \quad (38)$$

for the electron:

$$\beta_e = \tau_{dim} \frac{2}{(2\pi)^3} \left(\frac{2}{3}\right)^3 \frac{\sigma}{1.5133^3}\; b_0^2 \boldsymbol{\left[\frac{3}{2}\,\alpha^3\left(\frac{1.5133}{1.5}\right)\right]^3} \quad (39)$$

the particle specific factor τ/τ_{dim} is given in bold.

Coefficient τ_{dim} will be given by:

$$\tau_{dim} = \frac{1}{e_c \varepsilon_c} = 9.64\text{E+25 [m/J}^2] \quad (40)$$

giving

$$\tau_{dim} b_0^2 = \frac{1}{(4\pi)^2} \left(\frac{e_c}{\varepsilon_c}\right)^3 = 1.520\text{ E-30 [m}^3] \quad (41)$$

ψ will turn into

$$\Psi_n(r) = \exp\left(-\left\{\left(\left(\frac{2}{3}\right)^3 \frac{\sigma\tau_n b_0^2}{(2\pi)^3 r^3}\right) + \left[\left(\left(\frac{2}{3}\right)^3 \frac{\sigma\tau_n b_0^2}{(2\pi)^3 r^3}\right)^2 - \left(\frac{2}{3}\right)^3 \frac{4\tau_n b_0^2}{(2\pi)^3 r^3}\right]^{0.5}\right\}/2\right) \quad (42)$$

and

$$\Psi_n(r < r_l) = \exp\left(-\left(\frac{2}{3}\right)^3 \frac{\sigma \Pi_{\tau,n}}{(2\pi)^3 (4\pi)^2 r^3} \left(\frac{e_c}{\varepsilon_c}\right)^3\right) \quad (43)$$

W_e may be given as:

$$W_e = \frac{e_c^2}{2\pi\varepsilon_c} \int_0^{r_{l,n}} \Psi_e(r)^2 r^{-2}\, dr = \frac{\pi^{2/3}\Gamma_+}{k_s \Gamma_-} \frac{e_c}{\alpha^2} \approx \frac{2^{0.5} e_c}{k_s \alpha^2} \quad (44)$$

2.6 Extension to non-spherical symmetry

Up to here only spherical symmetry and Ψ(r) is considered, introduced through equ. (15)f. For non-spherically symmetric states [21] an appropriate angular term, y_l^m, will be introduced [22]:

$$y_l^m = \int\int \Psi(\varphi,\vartheta)^2 \sin(\vartheta)\, d\varphi\, d\vartheta \;/\; 4\pi \quad (45)$$

giving equation (1) for spherical symmetry.

The ratio of the volume integrals attributed to spherical harmonic Y_1^0 and Y_0^0 gives a factor of 1/3. Assuming

21 see also 4.2

22 Note: the wave function over the E-field will not be normalized to 1.

Y_1^0 to be a sufficient approximation for the next angular term and $W_n \sim 1/r_n \sim 1/V_n^{1/3}$ (V = volume) to be applicable for non-spherically symmetric states as well, will give $W_1^0/W_0^0 = 3^{1/3} = 1.44 = (y_1^0)^{-1}$.
Relation (34) has to turn into:

$$\tau_n = y_l^m\ \tau_e\,(2/3)^3\quad \Pi_{k=0}^{n}\alpha\wedge(3/3^{k}) \qquad\qquad n = \{0;1;2;..\} \tag{46}$$

A change in angular momentum is expected for this transition which is actually observed with ΔJ = ± 1 except for the pair μ/π with Δ J = 1/2.
Results for particles assigned to y_0^0, y_1^0 are presented in table 1.

	n	$W_{n,Lit}$ [MeV]	$\Pi_{k=0}^{n}$ α^(-1/3^k) equ (1)	Π_n equ (45)	W_{calc}/W_{Lit}	J	r_l [fm]	uds
ν	**-1**	**2E-7 ***	**α^{+3}**		**-**	**1/2**	**1.5E+10**	O
e^{+-}	**0**	**0.51**	Reference	**$(3/2)^3\ \alpha^9$**	**1.0001**	**1/2**	**8877**	O
μ^{+-}	**1**	**105.66**	**α^{-1}**	**$\alpha^9\alpha^3$**	**1.0000**	**1/2**	**42.9**	O
π^{+-}	1	139.57	1.44 α^{-1}	$\alpha^9\alpha^3/3$	1.0918	0	29.8	uds
K		495				0		uds
η^0	**2**	**547.86**	**$\alpha^{-1}\alpha^{-1/3}$**	**$\alpha^9\alpha^3\alpha^1$**	**0.9933**	**0**	**8.3**	LC
ρ^0	2	775.26	1.44 $(\alpha^{-1}\alpha^{-1/3})$	$\alpha^9\alpha^3\alpha^1/3$	1.0124	1	5.8	LC
ω^0	2	782.65	1.44 $(\alpha^{-1}\alpha^{-1/3})$	$\alpha^9\alpha^3\alpha^1/3$	1.0028	1	5.8	LC
K*		894				1		uds
p^{+-}	**3**	**938.27**	**$\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}$**	**1.0016**	**1/2**	**4.8**	uds
n	**3**	**939.57**	**$\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}$**	**1.0003**	**1/2**	**4.8**	uds
η'		958				0		LC
Φ^0		1019				1		uds
Λ^0	**4**	**1115.68**	**$\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}$**	**1.0106**	**1/2**	**4.0**	uds
Σ^0	**5**	**1192.62**	**$\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}$**	**1.0046**	**1/2**	**3.8**	uds
Δ	∞	**1232.00**	**$\alpha^{-3/2}$**	**$\alpha^{27/2}$**	**1.0025**	**3/2**	**3.7**	uds
Ξ		1318				1/2		uds
Σ^{*0}	3	1383.70	1.44 $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}/3$	0.9796	3/2	3.3	uds
Ω^-	4	1672.45	1.44 $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}/3$	0.9724	3/2	2.8	uds
N(1720)	5	1720.00	1.44 $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}/3$	1.0046	3/2	2.7	
tau^{+-}	∞	1776.82	1.44 $(\alpha^{-3/2})$	$\alpha^{27/2}/3$	1.0026	1/2	2.5	O
Higgs		1.25 E+5				0		
Max		~2.6-7.0E+5						

Table 1: Particles up to tauon energy [23]; values for $\mathbf{y_0^0}$, y_1^0; col. 3: energy values from literature [10] except *: calculated from model; W_{calc} calculated using (38)f; “uds” in col. 3 indicates particles covered by the quark model, linear combination labeled LC; leptons indicated as O; [24]

2.7 Discussion of particle states

2.7.1 Ground state

Equation (1) may be used to be extended to energy states below the electron. Yet according to (46) the associated parameter $\alpha_{\tau\nu}$ should be α^{-9} i.e. not fitting into the partial product of α_τ parameters. Moreover the relationship of W_e with the Planck-scale, see 5.2.1, also strongly supports the electron as being a ground state.

2.7.2 Lower limit

For extending this model to energies below the electron a coefficient of α^3 is used in equ. (1): $W_\nu / W_e = \alpha^3$. This gives a state with energy 0.2eV which is in an approximate range expected for a neutrino [9].
Yet the final lower limit should be reached soon. While r_l of the hypothetical neutrino is r_l = 3.6E-6 [m], the next lower state would be the last one to fit into the universe, with r_l ~ 1E+13[m] ~ 0,001 light year.

23 up to Σ'^0 all resonance states given in [10] as **** included; Exponent of -3/2, 27/2 for Δ and tau is equal to the limit of the partial products in (1) and (46); r_l calculated with equ. (8);
24 W_{calc}/W_{Lit} values differ from those calculated in reference to the electron in [8]

2.7.3 Upper limit

2.4 Upper limit of energy

In the simple picture sketched in the introduction the rotating E-vector might be interpreted to cover the whole angular range in the case of spherical symmetric states while a p-like state of an Y_1^0-analogue might be interpreted as forming a double cone. Going to higher Y_n^0-analogue states will close the angle of the cone leaving the original vector in the angular limit case, which might be interpreted as an instantaneous snapshot of time-averaged lower states, The maximum of the W(r) curve of spherical symmetric states of fig.1 will shift towards r_l until it reproduces the shape of Ψ(r) itself, i.e. $r_m \Rightarrow r_l$.
This is equivalent to σ approaching approximately unity. Since a bound state requires ~σ > 1 an upper limit for the angular contribution to the particle energy may be given by 1.51 α^{-1} and possible other components included in r_l, σ according to (31)f such as $\Gamma_{-1/3}/3$ or 2. The maximum angular contribution to W_{max} may be estimated as being approximately:

$$1.5133\ \alpha^{-1} < \Delta W_{max,\ angular} < \sigma^{1/3} \qquad (47)$$

From (47) follows an estimate for the total upper limit of energy as:

$$W_e\ 1.5133^2\ \alpha^{-2.5} = 4.12E\text{-}8\ [J] < W_{max} < W_e\ 1.5133\ \alpha^{-1.5}\ \sigma^{1/3} = 1.72E\text{-}7[J] \qquad (48)$$

This corresponds to a factor 2.0 - 5.5 relative to the mass of the Higgs boson [10].
Since non-rest mass energy is not restricted to the particular solutions of the model this limit does not apply to these.

2.7.4 Particle states not in y_0^0 and y_1^0

On the present level the y_0^0 and y_1^0 states of this model cover the 13 particle families of table 1 {e, μ, π, η, ρ/ω, p/n, Λ, Σ, Δ, Σ*, Ω, N(1720), τ} (excluding ν). This may be compared with the number of particle families given by the multiplets of u, d, s quarks in roughly the same energy range which is 13 as well, {π, K, ρ, K*, p/n, Φ, Λ, Σ, Δ, Ξ, Σ*, Ω, Ξ*}(excluding linear combinations).
Apart from particles attributed to y_0^0 and y_1^0 symmetry, assignment of more particle states will be not obvious. The following gives some possible approaches.

2.7.4.1 Partial products

Additional partial product series will have to start with higher exponents n in α^(-1/3^n) giving smaller differences in energy while density of experimentally detected states is high. There might be a tendency of particles to exhibit a lower MLT making experimental detection of particles difficult [25]. To determine the factor y_l^m requires the complete solution of the differential equation yet to be done. All these factors will impede the identification of additional partial product series.
One more partial product might be inferred from the fact that d-like-orbital equivalents with a factor of $5^{1/3}$ as energy ratio relative to η (see 4.2) give the start of an additional partial product series at $5^{1/3}$ W(η) = 937MeV = 0.98 W(η'), i.e. a value that coincides with energy values of the first particles available as starting point, η' or Φ^0, and having Δ(2420) with a spin of 11/2, indicating a high number of nodes, as candidate for being an end point. The difference in energy fits a series, some candidates for intermediate particles exist. However, in general it is not expected that partial products can explain all values of particle energies.

2.7.4.2 Linear combinations and particle compounds

The first particle family that does not fit to the partial product series scheme are the kaons at ~ 495MeV. They might be considered to be an equivalent to linear combination states of classical quantum mechanics. The π-states of the y_1^0 series are expected to be similar to p-orbitals of the H-atom, giving a charge distribution of +|+, -|- and +|-. A linear combination of two π-states would yield the basic symmetry properties of the 4 kaons as:

		+			-			-			+	
K^+	+	+	K^-	-	-	K_S^o	+	+	K_L^o	+	-	(+/- = charge)
		+			-			-			-	

providing two neutral kaons of different structure and parity, implying a decay with different parity and MLT values. For the charged Kaons, K^+, K^-, a configuration for wave function sign equal to the configuration for charge of K_S^o and K_L^o might be possible, giving two variants of P+ and P- parity of otherwise identical

25 Which might explain missing particles of higher n in the y_0^0 and y_1^0 series as well.

particles and corresponding decay modes not violating parity conservation [26].

-                    +
$K^{+/-}$ + + $K^{+/-}$ + - (+/- = wave function sign)
-                    -

The general formalism of such linear combinations might be different from classical quantum mechanics. At least the normalization condition would have to be altered or entirely dropped, which might result in a simple addition of particle energies. This is not the case for two pions adding up to one kaon. However, it has been noted for a long time that simple multiple-mass relations can be found among particle masses [7], [18]. Easy identifiable examples of near integer multiples can be found in particular among mesons, e.g. K, K* or η', η_c and η_b; among baryons e.g. the doubly charged particles stand out.
The latter particles draw attention to another possibility to explain particle resonances. A particle like Δ^{++} (from the reaction of p and π^+) is not expected within this model. Replacing elementary charge e in the equations by 2e would give energies not compatible with other single charged or neutral Δ particles and a whole series of doubly charged particles should exist. A particle of charge 2+ in this energy range would be rather considered to be a compound of n and two π^+, giving an equivalent of the ^3He nucleus (excited state).

2.8 Angular momentum - 2π

A simple relation with angular momentum J for spherical symmetric states, applying a semi-classical approach using

$$J = r_2 x\, p(r_1) = r_2 W_n(r_1)/c_0 \tag{49}$$

and assuming $|r_2| = |r_1|$ and $W_{kin,n} = 1/2\, W_n$, gives the integral:

$$|J| = \int_0^{r_{I,n}} J_n(r)\,dr = 2\frac{b_0}{c_0}\int_0^{r_{I,n}} \Psi_{2n}(r)^2 r^{-1}\,dr \tag{50}$$

From (10)f follows for m = 0:

$$\int_0^{r_{I,n}} \Psi_{2\pi,n}(r)^2 r^{-1}\,dr = 1/3\int_{8/\sigma}^{\infty} t^{-1}e^{-t}dt \approx 5.45 \approx \alpha^{-1}/8\pi \tag{51}$$

yielding the constant $\alpha^{-1}/8\pi$. Inserting (51) in (50) would provide:

$$|J| = 2\frac{b_0}{c_0}\,\frac{\alpha^{-1}}{8\pi} = 1/2\,[\hbar] \qquad 1/(2\pi) \tag{52}$$

To get the expected value of 1/2 [ħ] either assumption $|r_2| = |r_1|$ or the assumption of equ. (15), that the Coulomb law originating from the interaction of 2 particles can be used as first approximation has to be dropped, introducing a factor 2π in either (15)f or (49). The whole complex of angular part of the wave function, wave function phase, angular momentum, magnetic moment needs to be worked out thoroughly before this questions may be settled.

Analogous to the postulate for neutral particles to be composed of volume elements of opposite charge, particles with J = 0, J ≥ 1 are supposed to be composed of a combination of half integer contributions of angular momentum J = ± 1/2, adding up accordingly, implying appropriate multiples for the relation of $|r_2|$ and $|r_1|$ in (49) [27].

2.9 Accuracy of energy calculation

Agreement with experimental values is typically in a range of ± 0.01. There are three major causes preventing a significant improvement of accuracy.
1) Especially in the case of particle families [28], effects on top of the relations given in this work have to play a role to explain different energy levels of differently charged particles. This limits accuracy and the possibility to precisely identify candidates for calculated energies (e.g. both ρ^0 and ω^0 are given for 1.44 $\alpha^{-1}\alpha^{-}$

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26 For the neutral particles charge and wave function signs may not be necessarily independent
27 Going from μ with J=1/2 to η with J=(+1/2 -1/2) = 0, in the case of the proton a contribution of J = 3 |1/2] is needed, i.e. 3 contributions of J = |1/2] each, adding up to total spin of J = 1/2. For this formally $|r_2| = 3\,|r_1|$ in (49) has to hold.
28 Particle families, defined here as possessing the same exponent n in (46) but being different in charge, show a typical spread in energies of 3-4MeV and no dependence on total particle energy.

$^{1/3}$ in tab. 1).
If possible, particles chosen for y_0^0 in table 1 are of charge ± 1. In cases such as Σ with three energy levels, the intermediate energy level is chosen. For the y_1^0 series particles of the same charge as their y_0^0 equivalent are preferred in table 1.
2) The accuracy of the calculations is already in the order of magnitude of expectable QED corrections. Since these originate from the interaction of particles with the vacuum they may not be included in the equations of this model yet may have some influence on experimental values.

As for comparing accuracy of the energy calculation with results from some quark models of the standard model, calculations of simplicity comparable to the model presented here, using the obsolete constituent quarks and spin-spin interaction yield approximately the same accuracy [11]. However, more recent QCD calculations for particle mass use the mass of current quarks as input parameter. For u, d, s quarks, relevant in the energy range dealt with here, this mass is only vaguely defined, e.g. in the case of the u, W_u = 1.8 - 2.8 MeV [12]. Ab initio lattice QCD calculations may give particle mass with an uncertainty of a few percent [13].

3 Other properties

3.1 Magnetic moment [29]

Within this model particles are treated as electromagnetic objects principally enabling a direct calculation of the magnetic moment M from the electromagnetic fields.
The magnetic moment M_e of the electron is given as product of the anomalous g-factor, g_a = 1,00116, Dirac-g-factor, g_D = 2, and the Bohr magneton, $\mu_B = e\,\hbar/(2m_e)$, times the quantum number for angular momentum J = 1/2:

$$M_e = g_a g_D \mu_B/2 = g_a \frac{2ec_0^2}{2W_e}\frac{\hbar}{2} = g_a 9.274\text{E-}24\,[\text{Am}^2] \tag{53}$$

The factor g_a arises from the interaction of the electron with virtual photons as calculated in quantum electrodynamics and should not be part of a calculation of the magnetic moment from the field of the electron itself. Within this model the factor 2 of g_D originates from the fact that particle energy is supposed to be equally divided into contributions of the electric and magnetic field, $W_{el} = W_{mag} = W_n/2$ and only the magnetic field, i.e. W_{mag} contributes to the magnetic moment.
Inserting the term for particle energy of (16) in (53) gives:

$$\frac{M_e}{g_a} = \frac{e\hbar c_0^2}{2W_e} = \frac{e\hbar c_0^2}{2}\frac{3\beta_e^{1/3}}{2b_0\Gamma_+} = e c_0\beta_e^{1/3}\left(\frac{\Gamma_-}{3}\frac{3}{\Gamma_-}\right)\frac{3[\hbar c_0/b_0]}{4\Gamma_+} = ec_0\beta_e^{1/3}\frac{\Gamma_-}{3}\left[\frac{9[\alpha^{-1}]}{4\Gamma_+\Gamma_-}\right] \tag{54}$$

The term on the right is expanded by $\Gamma_-/3$ and turned into a form that will be needed for comparison with a calculation starting directly from the fields as explained in the following.
The relation of the values of E and B in an electromagnetic wave is given by $B = E/c_0$. This gives for the value of M_n :

$$M_n \approx \frac{1}{\mu}\int_0^{r_l} B(r)\Psi_n(r)^2 d^3r = \varepsilon c_0\int_0^{r_l} E(r)\Psi_n(r)^2 d^3r = ec_0\beta_n^{1/3}\frac{\Gamma_-}{3}\,\frac{1.51}{\alpha} \quad {}^{30} \tag{55}$$

Equation (55) neglects contributions to B(r) from other parts of the standing wave and requires an appropriate integration of those. The term in brackets of (54) contains integral terms over $\Psi(r)^2$ that might provide suitable contributions since (expansion with 2π):

$$\frac{9(2\pi)\alpha^{-1}}{8\pi\Gamma_{1/3}|\Gamma_{-1/3}|} = \frac{3\beta_e^{1/3}}{\Gamma_{1/3}}\,\frac{3}{\beta_e^{1/3}|\Gamma_{-1/3}|}\,\frac{2\pi\alpha^{-1}}{8\pi} = \frac{2\pi\int^{r_l}\Psi(r)^2 r^{-1}dr}{\int^{r_l}\Psi(r)^2 r^{-2}dr\left[\int^{r_l}\Psi(r)^2 dr\right]\alpha/1.51} \quad {}^{30} \tag{56}$$

holds.

---

29 Note: to allow for comparison with tabulated values of M in units of [Am^2] the calculations in this chapter use e [C] not e_c [J], conversion factor: [m^{2}C/s] /[m^2 J/s] = e/e_c = 1/19.4 [C/J].
30 The integral in brackets of (56) would produce 1.51/α which is given by its inverse in the equation to compensate for that. The actual integral factor would cancel the same term in (55) .

The comparison of (54) and (55) gives evidence that it is possible to transform the Bohr magneton expression with M ~ 1/m ~1/W ~$1/E^2$ into an electromagnetic expression where M can be calculated directly from the integral over the B-, E-field, M ~ ∫E d^3r .

Some more assumption about symmetry is required to interpret the integrals of (56) and the model may provide an approach to calculate magnetic moments of other particles as well, however, at present the absence of a detailed structure of particles seems to require too much speculation.

	M_Lit [Am²]	|M|_Calc [Am2]	|M|_Calc /|M|_Lit
e^{+-}	-9.28E-24	3.66E-22	-39.433
μ^{+-}	-4.49E-26	1.77E-24	-39.433
p^{+-}	1.41E-26	1.99E-25	14.142
n	-9.66E-27	1.99E-25	-20.637
Λ^0	-3.10E-27	1.68E-25	-54.170

Table 2: Absolute values calculated for magnetic moment with (55) compared to literature [10]

3.2 Particle decay / mean lifetime

To check if the model yields any information about mean lifetimes (MLT) the particles attributed to y_0^0 and y_1^0 are arranged according to their α-exponent index n and indicated for different types of particle families in fig. 2. There seems to be a tendency for charged particles to be significantly more stable than neutral ones and for y_1^0- lifetimes to be lower than y_0^0- lifetimes. [31]

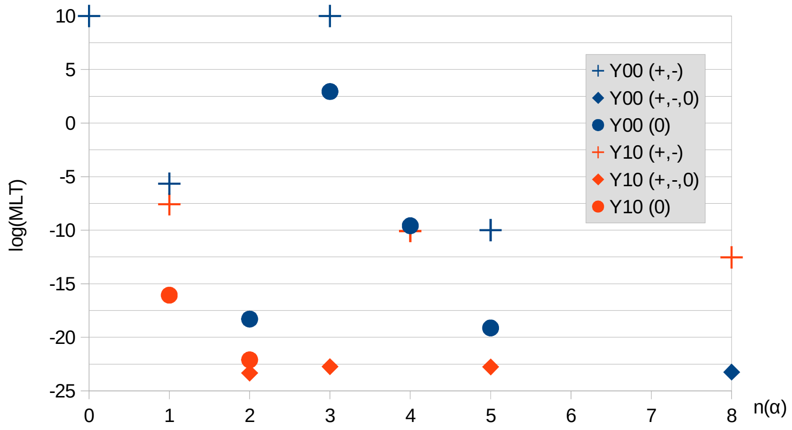

Figure 2: Mean lifetime for y_0^0 (blue) and y_1^0 (red) particles; charged only (+,-), neutral only (0), charged and neutral particle families with near identical MLT (+,-,0).

	MLT [s]	log(MLT)	n(alpha)
e	∞		0
μ	2.20E-06	-5,7	1
η	5.00E-19	-18,3	2
p	∞		3
n	8.80E+02	2,9	3
Λ^0	2.60E-10	-9,6	4
Σ^0	7.40E-20	-19,1	5
Σ^{+-}	8.00E-11	-10,1	5
Δ	5.60E-24	-23,3	∞
π^{+-}	2.60E-08	-7,6	1
π^0	8.50E-17	-16,1	1
ρ+-0	4.50E-24	-23,3	2
ω0	7.80E-23	-22,1	2
$\Sigma 0^{+-}$	1.80E-23	-22,7	3
Ω^-	8.20E-11	-10,1	4
N(1720)	1.70E-23	-22,8	5
tau^{+-}	2.90E-13	-12,5	∞

Table 3: Values for mean lifetime [10] used in figure 2

31 In [7] a dependence of MLT on α is given, however, there seems not to be a direct relation to the α-coefficients of this work.

Differential equation
4.1 Radial part
The approximation $\Psi(r<r_l)$ of equation (9) provides a solution to a differential equation of type

$$-\frac{r}{6\sigma\tau b_0}\frac{d^2\Psi(r)}{dr^2} + \frac{b_0}{2r^3}\frac{d\Psi(r)}{dr} - \frac{b_0}{r^4}\Psi(r) = 0 \quad (57)$$ [32]

However the correct discriminant form of $\Psi(r)$ of equ. (7) would be provided by a slightly different equation (revised by 6 in 2nd, 2 in 1st and σ in 0th order term) :

$$-\frac{r}{\sigma\tau b_0}\frac{d^2\Psi(r)}{dr^2} + \frac{b_0}{r^3}\frac{d\Psi(r)}{dr} - \frac{b_0}{\sigma\ r^4}\Psi(r) = 0 \quad (58)$$

To proceed from the heuristic mathematical approach of equation (57) to one based more on physics the second order term is expected to represent a quantum mechanical term for kinetic energy including the impulse operator. Mass may be replaced by the term $W_e/(2\,c_0^2)$ [33] giving

$$W_{kin}=\left(\frac{2\hbar^2 c_0^2}{2\ W_e}\right)\frac{d^2\Psi(r)}{dr^2} \quad (59)$$

To recover the r-dependence of (57) the following procedures are used as approximation

1.) $W_e \Rightarrow \Gamma_-\Gamma_+\, 2\, b_0/(9\, r)$ which is an approximation for $r \approx r_m$ [34];
2.) Using the first derivative of $\Psi(r)$, $[3\,\sigma\,\tau\, b_0^2\, r^{-4}]$ (and $[3\,\sigma\,\tau\, b_0^2\, r^{-3}]$) to modify the 0th (and 1st order term), i.e. effectively turning them into the next higher derivative, allows for canceling the 2nd order term. Since this term is almost identical to the expression for the supposed term of the strong force, the last term in equ. (65) below, this term is preferred, i.e. $[\sigma\,\tau\, b_0^2\, r^{-4}/2]$ and $[\sigma\,\tau\, b_0^2\, r^{-3}/2]$ will be chosen for the terms of the differential equation [35].
3.) Setting σ in accordance with (58);
4.) Since $\sigma\tau$, technically $\sigma\tau_e$, has to match the resulting expression, τ_e may be redefined as τ_e*. This gives:

$$-\left(\frac{9\hbar^2 c_0^2 r}{\Gamma_-\Gamma_+ 2 b_0}\right)\frac{d^2\Psi(r)}{dr^2} + \frac{b_0(\sigma\tau_e^* b_0^2)}{2r^3}\frac{d\Psi(r)}{dr} - \frac{b_0(\sigma\tau_e^* b_0^2)}{2\sigma\ r^4}\Psi(r) = 0 \quad (60)$$

as differential equation. Equation (42) will turn into (expanded by 4π):

$$\Psi(r)=\exp-\left(\left(\left(\frac{[\Gamma_-\Gamma_+ 4\pi]\sigma\tau_e^* b_0^4}{4\pi 9\hbar^2 c_0^2\ r^4}\right)+\left[\left(\frac{[\Gamma_-\Gamma_+ 4\pi]\sigma\tau_e^* b_0^4}{4\pi 9\hbar^2 c_0^2\ r^4}\right)^2-\frac{4[\Gamma_-\Gamma_+ 4\pi]\tau_e^* b_0^4}{4\pi 9\hbar^2 c_0^2\ r^5}\right]^{0.5}\right)\frac{r}{2}\right) \quad (61)$$

which may be rewritten, using (23), as

$$\Psi(r) = \exp-\left(\left(\left(\frac{k_a\alpha\sigma\tau_e^* b_0^2}{4\pi 9 r^3}\right) + \left[\left(\frac{k_a\alpha\sigma\tau_e^* b_0^2}{4\pi 9 r^3}\right)^2 - \frac{4 k_a\alpha\tau_e^* b_0^2}{4\pi 9 r^3}\right]^{0.5}\right)\frac{1}{2}\right) \quad (62)$$

According to (62) τ_e* has to be defined as:

$$\tau_e^* = \tau_e\frac{18}{(2\pi)^2 k_a\alpha} \approx \tau_e\,\alpha^{-1}/2 \quad (63)$$

---

32 [N15.1] $d\psi(r)/dr = 3\,\sigma\,\tau\, b_0^2\, r^{-4}\,\Psi(r)$
[N15.2] $d^2\psi(k)/dk^2 = 9\,(\sigma\,\tau\, b_0^2)^2\, r^{-8}\,\Psi(r) - 12\,\sigma\,\tau\, b_0^2\, r^{-5}\,\Psi(r) + 6\,\sigma\,\tau\, b_0^2\, r^{-5}\,\Psi(r)$ (polar coordinates)
[N15.1] -[N15.2] inserted in (57) gives:
[N15.3] $r\,(6\,\sigma\,\tau\, b_0)^{-1}\,\{-9\,(\sigma\,\tau\, b_0^2)^2\, r^{-8} + 6\,\sigma\,\tau\, b_0^2\, r^{-5}\} + 3/2\,\sigma\,\tau\, b_0^3\, r^{-7} - b_0\, r^{-4} = 0$
[N15.4] $-3/2\,\sigma\,\tau\, b_0^3\, r^{-7} + b_0\, r^{-4} + 3/2\,\sigma\,\tau\, b_0^3\, r^{-7} - b_0\, r^{-4} = 0$
33 Using $W_{pot,n} = W_{kin,n} = W_n/2$ and (16)
34 β_n in (16) replaced via term of (18)
35 Note: cancelling of factor 2 in β

positioning (60) somewhere between (57) and (58).

4.2 Complete solution / angular part

For the type of differential equation (57)ff a separation of variables will in general not be possible, the spherical harmonics such as $(Y_1^0)^2$ will not be a solution for the differential equation of type (57). However, the factor 3 of Y_1^0 fits not too bad and the symmetry properties of π-particles match those of p-orbitals. In general any wave function corresponding to a rough equivalent of an atomic p-orbital will have to feature a coefficient from the integration over φ, ϑ close to 3 and be accessible to the reasoning in 2.6.

In general other approaches might be better suited to the problem, such as calculating in k-space or using quaternions.

5 Particle-particle interaction

5.1 Relationship between particle energy and strong, Coulomb potential energy

The series expansion of $\Gamma(1/3,\beta_n/r^3)$ in the equation for calculating particle energy (16) gives [14]:

$$\Gamma(1/3,\ \beta_n/(r^3)) \approx \Gamma_{1/3} - 3\left(\frac{\beta_n}{r^3}\right)^{1/3} + \frac{3}{4}\left(\frac{\beta_n}{r^3}\right)^{4/3} = \Gamma_{1/3} - 3\frac{\beta_n^{1/3}}{r} + \frac{3}{4}\frac{\beta_n^{4/3}}{r^4} \tag{64}$$

and for the potential energy part of $W_n(r)$, $W_{n,pot}(r) = W_n(r)/2$:

$$W_{n,pot}(r) \approx W_n/2 - b_0\frac{3\beta_n^{1/3}}{3\beta_n^{1/3}r} + b_0\frac{3}{4}\frac{\beta_n^{4/3}}{3\beta_n^{1/3}r^4} = W_n/2 - \frac{b_0}{r} + b_0\frac{\beta_n}{4r^4} \quad {}^{36} \tag{65}$$

The 2nd term in (65) drops the particle specific factor β_n and gives the electrostatic energy of two elementary charges at distance r. The 3rd term is chosen for the terms of the differential equation given in 4.1. The 0th order term in the differential equation is supposed to represent a potential energy which, though being composed of coefficients originating from electrodynamics, does not represent an electrodynamic or gravitational term but a term which has a high dependence on r and is obviously responsible for the localized character of an electromagnetic object. In 5.3 some arguments are given that demonstrate a relationship of the properties of the wave functions used in this model with the "strong force" of the standard model. It may be assumed that the 3rd term of (65) represents this strong force [37].

5.2 Gravitation

Expressing energy/mass in essentially electromagnetic terms suggests to test if mass interaction i.e. gravitational attraction can be derived from the corresponding terms. Assuming the expansion of the incomplete Gamma function for the integral over r^{-2}, $\Gamma(1/3,\beta_n/r^3)$ (64)f, might be an adequate starting point for gravitational attraction as well, implies that the Coulomb term b_0 will be part of the expression for F_G, i.e. the ratio between gravitational and Coulomb force, e.g. for the electron, $F_{G,e}/F_{C,e} = 2.41\text{E-}43$, should be be a term that can be given as completely separate, self-contained expression.

5.2.1 Planck scale

The same conclusion would arise from assuming that gravitational interaction is a higher order, nonlinear effect of electromagnetic interaction and as such should be of less or equal strength compared to the latter. This suggests to use the expression

$$b_0 = G\, m_{Pl}^2 = G\, W_{Pl}^2 / c_0^4 \tag{66}$$

as definition of Planck terms, giving for the Planck energy W_{Pl}:

$$W_{Pl} = c_0^2\,(b_0/G)^{0.5} = c_0^2\,(\alpha h c_0/G)^{0.5} \tag{67}$$

Using (67) gravitational attraction F_G in the classical limit can be expressed as:

$$F_G = \frac{b_0 W_n W_m}{W_{Pl}^2}\frac{1}{r^2} \tag{68}$$

---

36 signs not adapted to conventional usage of F_C.

37 Apart from the Yukawa-Potential other approaches have been tried, e.g. Heisenberg in a letter to Pauli discusses an exchange energy for p - n interaction $\sim r^{-5}$ https://archiv.heisenberg-gesellschaft.de/heisenberg_0017-064r.pdf

Expression (68) is a restatement of Newton's law with no additional insight unless an expression for W_{Pl} independent of G would be at hand. Expanding relationship (29)f to higher powers of α_e i.e. $\alpha_e^3 = (1.5133^3\ \alpha^9)^3$ provides just that. The relationship is quantitative if using

$$0.9994\,\frac{W_{Pl}}{W_e} = 1.5133^{-2}\,\alpha^{-10}\,2 \quad = 2.039\ \text{E+21} \tag{69}$$

i.e. using $(\alpha_e^3)^{-1/3}$ times the angular limit factor according to (47) in the form 1.5133 α^{-1} *2.
Using (36) to express factor 1.5133 gives (F_G, F_C = gravitational, Coulomb forces):

$$\left(\frac{W_e}{W_{Pl}}\right) = \left(\frac{F_{G,e}}{F_{C,e}}\right)_{calc} = \left[\frac{(4\pi)^2\,\Gamma_-^4\,\alpha^{12}}{2}\right]^2 = 1.0007^2\,\left(\frac{F_{G,e}}{F_{C,e}}\right)_{\exp} = \frac{G\,W_e^2}{c_0^4\,b_0} \tag{70}$$

Using (36),(44) for calculating W_e would give G as:

$$G_{calc} = \frac{c_0^4}{4\pi\,\varepsilon_c}\,\left(\frac{(4\pi)^3\,\Gamma_-^7\ \alpha^{15}}{3\,\pi^{2/3}\,\Gamma_+}\right)^2 = 1.0013\,G_{\exp} \quad ^{38} \tag{71}$$

5.2.2 Virtual superposition states

The results of 5.2.1 have several implications.

From (28) it is not obvious if and at which energy a ground state exists though the electron or neutrino would be obvious choices. The ground state should be distinguished by the existence of a dimensionless parameter α_{ground}^3 that does not represent another particle state but has some more fundamental significance. Relationship (69) seems to provide just that.

Within this model particles might interact via direct contact in place of boson-mediated interaction. The particles are not expected to exhibit a rigid radius. Within the limits of charge and energy conservation a superposition of many states might be conceivable, extending the particle in space with radius ~ $r_{,m,n}$, $r_{l,n}$ appropriate for energy of each virtual superposition state (VSS) [39], enabling interaction at a distance. The wave function of a VSS might contribute an additional factor to lower total Ψ values on site of a second particle thereby reducing particle energy and resulting in an attractive force. In general VSS are not supposed to consist of analogues of e.g. spherical symmetric states covering the complete angular range of 4π but to be an instantaneous, short term extension of the (rotating) E-vector thus requiring the angular limit factor in (69).

The particular definition of the Planck scale used in 5.2.1 is based on the assumption that gravitation is a nonlinear effect in respect to electromagnetics. This model provides a nonlinear term in respect to the Coulomb term in the 3rd term of the expansion of the Γ-function according to (64), $\beta^{4/3}/r^4$, a term that has been attributed to the strong force and provides essentially the potential energy term of the differential equation. This term is supposed to be responsible to keep an electromagnetic object localized i.e. to be responsible for the existence of mass and the curvature of space-time associated with it.

The term $(\Gamma_-\ \alpha^3)^4$ of (70) indicates a relationship of F_G with the SF term of (64), including $(\alpha^9)^{4/3}$ of the electron parameter α_e and the dimensionless parameter Γ_- which is related to the spatial coordinate r. For any particular VSS the term $\beta^{4/3}/r^4$ would yield a well defined value for any $r < r_l$ [40], the r-dependence of F_G would not be based on this term but on the relationship $W_{VSS} \sim 1/r_{VSS}$ of each VSS. In addition there has to be an intensity term being proportional to W_n [41].

In a very simple picture one would have:

1) a VSS providing *energy at* ~ $r_{,m,n}$, $r_{l,n}$, giving according to (65) ~ $b_0\ \beta_{VSS}/r_{m,VSS}^4 \sim b_0/r_{m,VSS}$,
2) the rate at which a VSS would be created should be proportional to particle energy W_n,

---

38 Equ. (71) may be approximated as: $G_{calc} = \dfrac{c_0^4}{4\pi\,\varepsilon_c}\ \dfrac{2}{3}\ \alpha^{24} = 1.00087\,G_{\exp}$

39 The superposition states considered here would be not virtual in a Heisenberg sense, the energy is provided by the source particle.

40 In the simplest case: $\beta_{VSS}^{\ 4/3}/r^4 = \beta_{VSS}^{\ 4/3}/r_{l,VSS}^{\ 4} = 1$

41 To give a simplified example, e and μ might both create neutrino VSS which would result in a neutrino energy /mass term at $r_{l,v}$, i.e. approximately α^3 times weaker at $r_{l,v}$ than at $r_{l,e}$ or $r_{l,\mu}$, providing a corresponding effect in curvature of spacetime. Factor ~ α^3 represents both the ratio of energy and radius of the particles involved. This leaves the question,

3.) the strong force term in (60), (64) etc. is the cause for creating energy and associated curvature of space-time. Assuming that its ground state parameter, ~ α_e, is approximately appropriate for averaging the various factors sketched above and thus for describing the curvature of space-time caused by VSS as well, one might arrive at an equation of type (70).

Replacing W_e by other particle energies W_n in (70) would result in a factor ~ α^x in the denominator and since the exponent of α is the only variable parameter in (70) one ends up at the Planck scale when $W_n \sim \alpha^{-x}$ has used up all of the starting α^{12}.

Such a model seems to suit GRT:

- the curvature of space-time at a distance from a particle / mass is due to the presence of energy in form of VSS,
- energy is intrinsically connected with r (and implicitly t) (=> energy-space-time)
- no non-linearity problems due to extra energy of bosons.

5.2.3 Cosmological implications

5.3 Short range interaction - strong force

In this model, on the length scale of particle radius, the wave functions of two particles should start to overlap and exert some kind of direct interaction. As demonstrated in table 1, col.8, for hadrons the model yields particle radius in the range of femtometer, the characteristic scale for strong interaction and it seems likely to identify strong interaction with the interaction of wave functions. Interaction via overlapping of wave functions constitutes the basis of chemical bonding and has been examined extensively [15]. In general wave functions are signed (not to be confused with electrical charge), for particles above the ground state regions of different sign exist, separated by nodes. There are two major requirements for effective interaction:

1) Comparable size and energy of wave functions ,

2) sufficient net overlap: In the overlap region of two interacting wave functions sign should be the same (bonding) or opposite (antibonding) in all overlapping regions. If regions with same and opposite sign balance to give zero net overlap, no interaction results.

From condition 1) and the data of table 1 it is obvious that the wave functions of neutrino and electron/positron will not show effective interaction with hadrons due to mismatch of size and energy [42]. In the case of the tauon the second rule is crucial. According to this model the tauon is at the end of the partial product series for y_1^0 and should consequently exhibit a high, potentially infinite number of nodes, separating densely spaced volume elements of alternating wave function sign. Though having particle size and energy in the same order of magnitude as other hadrons, such as the proton, the frequent change of sign of the tauon wave function will prohibit net overlap and effective interaction.

This model would however, suggest a smooth transition in the effects of strong interaction. The same reasoning as for the tauon would have to apply e.g. for Δ-particles, for which scattering data are not available. The supporting assumption of the Δ being subject to the strong force based on its short lifetime is not a general distinctive feature of both particle groups and in this model the presence of the strong force, i.e. the wave function character of particle states, is considered a constituent element of all particles anyway. μ/π-scattering might give some information about the presence of the strong force in muon interaction.

Overlap of wave functions should provide a possible description of nuclear bonding as well.

6 Other aspects of the model

6.1 Free particle

Omitting the 0^{th} order term in the differential equations might produce the equation of a free particle. Using the following version of equ. (57) for the electron gives:

$$\frac{r}{6\sigma\tau_e b_0}\frac{d^2\Psi(r)}{dr^2} - \frac{b_0}{2r^3}\frac{d\Psi(r)}{dr} = 0 \qquad (72)$$

---

what would be the source of the difference in the gravitational effect of e and μ at $r_{l,v}$? One obvious choice might be to interpret this intensity in terms of frequency, i.e. the higher rate the rotating E-vector points in a particular direction and having the possibility to create a VSS in that direction.

42 As for energy density ~ W_m/W_n^4 : e/p ~ E-13, μ/p ~ 6E-4; μ/π ~ 1/3, different symmetry may play an additional role.

$$\frac{d^2\Psi(r)}{dr^2} \approx \frac{3\sigma\tau_e b_0^2}{r^4}\frac{d\Psi(r)}{dr} + ... \tag{73}$$

indicating there could exist a function in the general form of (66) for a photon, maybe describing the decrease of the electromagnetic fields perpendicular to wave propagation.

$$\Psi(r) \approx \exp\left(\frac{-\sigma\tau_e b_0^2}{r^3}\right) + ... \tag{74}$$

6.2 Elementary charge

6.2.1 Electrical charge

As $\Psi(r)$ approaches 1 for $r \to r_l$ the Gauss integral $\varepsilon_0 \int E(r)\Psi(r)^2\, dA$ approaches the limit of the elementary charge e. Since for $r \to 0$ the term $E(r)\Psi(r)^2$ goes to zero, there is no 'point charge' at the origin.

At a distance of r_m, (see equ. (18)), marking the approximate maximum of W(r), $\Psi(r)^2$ attains a value of 0.667 yielding a calculated charge of 2/3 e and a value of W_n of $W_n = W_n/4$ [43].

6.2.2 Magnetic charge

The model outlined above should principally be suited to calculate the energy of particles with magnetic charge e_m, i.e. magnetic monopoles. Using the equations above to calculate energies of Dirac magnetic monopoles [16] is straightforward. Replacing e by the magnetic charge e_m

$$e_m = e/(2\alpha) \tag{75}$$

turns b_0 into b_m. The integral (50) yields only minor variations even when changing input parameters by several orders of magnitude. This indicates that a product $2b_0 = xb_m$ of (51)[44] has to be essentially a constant to provide half integer spin. The proportionality $\lambda_{C,n} \sim \beta_n^{1/3}$ has to be applicable for magnetic monopoles as well, yielding the same factor $36\pi^2$ in (20). As a result equ. (23) should hold for both electric and magnetic monopoles. Using the same coefficients τ_n according to equ. (30) as for electric monopoles in equ. (16) would leave $(2\alpha)^{4/3} = 1/280$ as ratio between electric and magnetic particle energies. Assuming $\tau_{0,magn} \sim 1/e_m$ (see (40)) would reduce this ratio to $2\alpha = 2/137$. Both versions place magnetic monopole particles approximately in the same energy range as their electric counterparts.

7 Discussion

7.1 Basic model

The basic idea behind this work is that elementary particles can be considered as a rotating **E**-vector pointing towards the origin and **B** and $\mathbf{V}_{rot}$ [45] being orthogonal to each other, at least on a local scale forming a standing electromagnetic wave. Neutral particles are supposed to exhibit appropriate nodes and corresponding equal volume elements of opposite polarity. Switching direction of the fields will result in the corresponding antiparticles.

Whatever the detailed mechanism of this might be, there are two basic problems to overcome:

1. Since energy of the particle as calculated from electrostatics increases infinitely for $r \to 0$ a function that serves as a damping term is needed to prevent this.
2. $\mathbf{V}_{rot}$ which is considered to be some kind of wave propagation velocity i.e. speed of light c in its broadest sense, has to approach 0 for $r \to 0$.

The function to be modified in this way is of the form

$$W_n(r) \sim b_0\, r^{-1} = \frac{e^2}{4\pi\varepsilon r} \sim e^2 c_0\, r^{-1} \tag{76}$$

Thus the function used to modify this, $\Psi(r)$, has to act on terms that contain r, e, c (or related electromagnetic parameters). Decreasing the value of c_0 obviously is sufficient to meet both requirements.

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43 For the pair e, μ the value of r_m is also distinguished by the relation $r_{l,\mu} = r_{m,e}$, see 2.7.1.

44 or any other constant replacing 4π

45 tangential velocity, not $\boldsymbol{\omega}$

7.2 Relation to standard model of particle physics

The standard model classifies particles into leptons and hadrons, composed of two (mesons) or three (baryons) quarks. The classification into the three groups may be reproduced by this model.

Mesons constitute a distinct group of particles due to their integer angular momentum which is considered to be a combination of half-integer contributions in both models. In the standard model leptons are defined as being particles not subject to strong interaction, being essentially point like. Neutrinos, electron and muon are the particles of lowest mass which in itself might provide an explanation for this quality. The tauon however is outstanding in possessing a mass almost twice that of the proton and major decay channels involving hadrons. The considerations in chpt. 5.3 about overlap and wave function symmetry might provide a consistent explanation for all leptons not to be subject to strong interaction with hadrons which in turn should prohibit detection of internal structure of these particles.

In the model presented the y_0^0 and y_1^0 groups each include all three particle types. The possibility to calculate particle energies with a single model using a uniform set of parameters does not support to identify a special set of particles as more "elementary" than others. However, the standard model of particle physics distinguishes quite rigidly between leptons and hadrons postulating that a set of physical objects characterized by an almost identical set of experimental observables - such as mass, charge, spin, magnetic moment, well defined mean life time and the effects of electromagnetism, weak interaction and gravitation - is based on completely different physical principles. This is quite an extraordinary claim, is it covered by extraordinary evidence ?

The postulate of leptons not being subject to strong interaction is not verifiable beyond experimental accuracy. Neutrino mass is a precedent for the fallacy to confuse a very small value with zero.

The three generation model, attributing a neutrino to each charged lepton, looks like a more solid argument. However, the total number of neutrinos is not beyond doubt (MiniBoone [17], cosmic neutrinos [9]) and neutrino oscillation obscures the earlier assumption of clearly distinct particles. Last not least, a distinctive interaction of neutrinos with the charged leptons might simply be due to the very weak strong interaction of the particles involved not requiring any assumption beyond that.

The standard model describes very successfully hadron properties and the reliability of the model presented here will depend crucially on reproducing the symmetry properties as represented by the various quarks. On a rudimentary level this is the case as demonstrated above.

Except for the reasoning given for "lepton" particles the description of particles as electromagnetic wave structured by nodes implies some kind of measurable substructure though it goes without saying that this substructure does not provide any possibility for a division into smaller entities.

7.3 Relation to classical quantum mechanics

7.3.1 General

Very general, the relation of this model to classical quantum mechanics may be given by interpreting $\Psi(r)$ as probability amplitude. $\Psi(r)$ may be given as the solution of a simple single 2nd order differential equation such as the Schrödinger or Dirac equation, applied directly to the electromagnetic field instead of a particle. The derivation of this model started from working out the function Ψ since it is easier to develop terms for this function by fitting particle properties than to guess a term for the differential equation and in particular the term representing potential energy.

The differential equation may be given as approximately:

$$\left(\frac{\hbar^2 c_0^2}{2 W_{kin}}\right) \Delta \Psi(r) - \sigma W_{pot} r \nabla \Psi(r) + W_{pot} \Psi(r) = 0 \tag{77}$$

There is no eigenvalue for energy, energy as well as other properties have to be calculated by the integral over $\Psi(r)^2$, implying that concepts such as orthonormalization may not be applicable on the level of the differential equation yet alternative orthonormalization conditions may exist [46] or quite general, the equations

46 The comparable values of $r_{l,e}$ and $e/(4\pi\,\varepsilon_c)$ hint at a kind of normalization condition for E(r), e.g.

$$\int_0^{r_{l,e}} E(r)\Psi(r)^2 dr = \frac{e_c}{4\pi\varepsilon_c}\int_0^{r_{l,e}} \Psi(r)^2 r^{-2} dr = \frac{e_c}{4\pi\varepsilon_c}\frac{\Gamma_+}{3\beta_e^{1/3}} = \frac{e_c}{4\pi\varepsilon_c}\frac{\Gamma_+\Gamma_-}{9 r_{m,e}} \approx \frac{(2\pi)^2}{\alpha}$$

giving $\frac{e_c}{4\pi\varepsilon_c} \approx \frac{4}{3}\; 2\pi r_{l,2\pi,e}$

might be considered to be “normalized” to yield the elementary charge for r > r_l.
As a consequence the quantization condition given in 2.4 is not exclusive. The solution of (27)f relates to a set of rest mass of particles of sufficient stability to be observable experimentally but does not prohibit the existence of particles with any other mass.
As for the number of parameters needed to calculate energy states, the model resembles the simplicity of ab initio quantum mechanical models, relying essentially on b_0 and J = 1/2 to yield the expression (1) [47]. Parameter τ_e or more generally β_e is needed to transform the relative energy scale of (1) into an absolute one and may be itself reduced to the elementary form (40), allowing all calculations to be based on e_c and ε_c as sole input parameters.

7.3.2 Quantization condition

Other approaches have been tried to obtain a more definite derivation for the quantization.
A particular simple interpretation may be given using (32) and considering that the ratio $r_{l,n} / r_{l,n+1}^3$ is constant:

$$r_{l,n} / r_{l,n+1}^3 = (\sigma\ \beta_e\ \Pi_{\tau,n}/8)^{1/3}) / (\sigma\ \beta_e\ \Pi_{\tau,n+1}/8) = \text{const} \tag{78}$$

To be valid for all n this implies $\Pi_{\tau,n} \in \Pi_{\tau,n+1}$ and $\Pi_{\tau,n}^{1/3} \in \Pi_{\tau,n+1}$ requiring $\alpha_{\tau,n+1} = \alpha_{\tau,n}^{1/3}$. Since $W_{n+1}^3/W_n \sim \lambda_{C,n}/\lambda_{C,n+1}^3 \sim r_{l,n}/r_{l,n+1}^3$ this result is a restatement of the relations given above though suggesting that some geometrical interpretation in r- or k-space might be conceivable.
Taking the ratio of the two integrals for the particle energy (note ε_0 replaced by ε_c)

$$W_{pc,n} = 2\varepsilon_c \int_0^\infty E(r)^2 \Psi_n(r)^2 d^3r \quad = 2\ b_0\ \Gamma_+ \beta_n^{-1/3}/3 \tag{79}$$

$$W_{pc,n} = 2e_c \int_0^\infty E(r)\, \Psi_n(r)^2 dr \quad = 2\ b_0\ \Gamma_+ \beta_n^{-1/3}/3 \tag{80}$$

gives:

$$\frac{e_c}{\varepsilon_c} = \int_0^\infty E(r)^2 \Psi_n(r)^2 d^3r \ / \int_0^\infty E(r)\, \Psi_n(r)^2 dr \tag{81}$$

and suggests that solutions for E(r) other than the point charge may be used.
It is the angular momentum that most clearly requires some sort of quantization. The term e_c/ε_c suggests that it might be replaced by $e_c c_0 \sim I$ in a term equivalent to (81), maybe providing a more accessible approach to quantization via phase of the wave function, an aspect which has been totally neglected on this level of approximation.

8 Summary

The main results obtained by applying the function Ψ(r) to E(r) will be summarized here using a unit system where energy is given in units of $[e_c]$ = 3.11E-18 [J] and distance by $[e_c/\varepsilon_c]$ = 9.32E-10 [m]. All examples given for the electron.

● wave function:

$$\Psi_e(r<r_l) = \exp\left(-\left(\frac{2}{3}\right)^6 \frac{4\pi}{(2\pi r)^3}\ (\Gamma_- \alpha)^9\right) \tag{82}$$

● fine structure constant:

$$4\pi\ \Gamma_+ \Gamma_- \approx \frac{\hbar c_0}{b_0} = \alpha^{-1} \tag{83}$$

● particle energy (rest energy):

$$W_e = \frac{1}{2\pi} \int_0^{r_{l,n}} \Psi_e(r)^2 r^{-2} dr = \left(\frac{4\pi(\Gamma_- \alpha)^3}{3}\ \frac{2}{\pi^{2/3}\Gamma_+}\right)^{-1} = \left[\frac{2\ 4^{1/3}}{9\pi^{2/3}}(\Gamma_- \alpha)^3\right]^{-1} \left(\frac{1}{2^{1/3} 2\pi} \frac{\Gamma_-}{3}\right)\ [e_c]\ ^{48} \tag{84}$$

47 J = 1/2 and the values ~1.5 and σ are closely related. Factor ~1.5 from the energy ratio µ/e might be considered an additional parameter, yet applies only to this particle pair.
48 The term in square brackets of (84) is the exponential (82) up to -1/3, factors 2 from β, 2π and $\Gamma_+/3$ from (16).

● long range particle interaction (2 electrons):

$$W_{e,pot}(r) = \frac{1}{4\pi \ r}\left[1 - \frac{(4\pi)^4 \Gamma_{-}^{8} \alpha^{24}}{4}\right] \ [e_c] \qquad (85)$$

● constant of gravitation:

$$G_{calc} \approx \frac{1}{4\pi}\ \frac{2}{3}\ \alpha^{24} \qquad (86)$$

Conclusion

Using the exponential function $\Psi(e_c,\varepsilon_c)$ as probability amplitude for the electric field E(r) gives the following results:

- a numerical approximation for the value of the fine-structure constant α,
- a quantization of energy levels given by a partial product of terms α^(-1/3^n),
- a possibility to calculate magnetic moments directly from the electromagnetic fields,
- qualitative explanations for particle properties such as the lepton character of the tauon or the decay of kaons,
- a possibility to quantitatively express gravitational force entirely in electromagnetic terms,
- an indication of a common source for strong force, electromagnetism and gravitation, based on a common set of -electromagnetic- coefficients and the expansion of the incomplete gamma function.

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