1 Stokes flow:

$$f_i = \tau_{ij,j} - p_{,i} \tag{1}$$

translates to in non-tensor notation AND dimensional numbers:

$$\eta \nabla^2 v - \nabla P = g\rho_0 - g\rho_0 \alpha (T - T_0) \tag{2}$$

Since T_0 is 0 (effectively) the equation 2 becomes:

$$\eta \nabla^2 v - \nabla P = g\rho_0 - g\rho_0 \alpha (T) \tag{3}$$

Using scaling relations defined in louis' pdf the scaled equation becomes:

$$\frac{\eta}{h^2} \nabla'^2 (\frac{\kappa}{h} v') - \nabla' P_0 - \frac{\eta\kappa}{h^3} \nabla' P' = g\rho_0 - g\rho_0 \alpha (\Delta T T') \tag{4}$$

Where the non-dimensional variables are indicated by the prime.
Using the relation: $\nabla P_0 = -g\rho_0$ and the scaling relation $P = P_0 + \frac{\eta\kappa}{d^2} P'$

$$\frac{\eta}{h^2} \nabla'^2 \frac{\kappa}{h} v' - \frac{\eta\kappa}{h^3} \nabla' P' = -g\rho_0 \alpha (\Delta T T') \tag{5}$$

Now in terms of the Ra number this becomes:

$$\nabla'^2 v' - \nabla' P' = -Ra T' \tag{6}$$

which in turn is:

$$\nabla'^2 v' - \nabla' P' = -Ra \frac{T}{\Delta T} \tag{7}$$

And to get everything back to dimensional again:

$$h^2 \nabla^2 (\frac{vh}{\kappa}) - h \nabla (\frac{h^2 (P - P_0)}{\eta\kappa}) = -Ra \frac{T}{\Delta T} \tag{8}$$

Multiply by η and κ and take the constants out the front:

$$h^3 \eta \nabla^2 v - h^3 \nabla (P - P_0) = -\kappa\eta Ra \frac{T}{\Delta T} \tag{9}$$

rearrange:

$$h^3 \eta \nabla^2 v - h^3 \nabla P + h^3 \nabla P_0 = -\kappa\eta Ra \frac{T}{\Delta T} \tag{10}$$

again:

$$h^3 \eta \nabla^2 v - h^3 \nabla P = h^3 g\rho_0 - \kappa\eta Ra \frac{T}{\Delta T} \tag{11}$$

and divide by h^3

$$\eta \nabla^2 v - \nabla P = g\rho_0 - \kappa\eta Ra \frac{T}{h^3 \Delta T} \tag{12}$$

So that the dimensional force on the RHS of equation 3 is equal to:

$$g\rho_0 - g\rho_0 \alpha (T) = g\rho_0 - \kappa\eta Ra \frac{T}{h^3 \Delta T} \tag{13}$$

So that:

$$g\rho_0\alpha = \frac{\kappa\eta Ra}{h^3\Delta T} \tag{14}$$

Now using dimensional numbers for everything (from blankenbach): $\alpha = 2.5\times 10^{-5}$, $\kappa = 10^{-6}$, $h = 10^6$, $\rho_0 = 4\times 10^3$, $g = 10$, $\Delta T = 10^3$, $Ra = 10^4$ and $\eta = 2.5\times 10^{19}$
$10\times 4\times 10^3\times 2.5\times 10^{-5} = \frac{10^{-6}\times 2.5\times 10^{19}\times 10^4}{10^18\times 10^3}$

From this relation ρ_0 should be 1.

Now we have found what the density should be, we can run the dimensional convection model to compare it to the dimensionless one in the Blankenbach paper. However, even though the Rayleigh numbers are the same, the actual values of the measurables (eg vrms, Nusselt number) are going to differ by orders of magnitude due to the scaling. How do we find out what the dimensional value of the measurables from that paper should be? We use the scaling relations.

Trying vrms as an example, the scaling of velocity is going to equal the length scaling over the time scaling, that is:
$\delta v = \frac{\delta L}{\delta t}$

The length scaling is 10^6, the time scaling we can find from κ which is m^2t^{-1} in SI units.
$\delta\kappa = \frac{\delta L^2}{\delta t}$

which in turn is $10^{-6} = \frac{10^{12}}{\delta t}$

Therefore δt is 10^{18} so that δv is 10^{-12} so that the dimensional vrms will be $10^{-12}\times$ the non-dimensional vrms from the Blankenbach paper. The same methodology above can applied to all the measurables in the paper.