Order in the particle zoo

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Abstract

The standard model of physics classifies particles into elementary leptons and hadrons composed of quarks. In this article the existence of an alternate ordering principle will be demonstrated giving particle energies to be quantized as a function of the fine-structure constant, α. The quantization can be derived using an appropriate wave function that acts as a probability amplitude on the electric field. The value of α itself can be approximated numerically by the gamma functions of the integrals for calculating particle energy.
The model may be used to calculate other particle properties as well, in particular particle interaction, giving quantitative terms for strong, Coulomb and gravitational force.
One input parameter derived from the electron mass is required for the calculations.

1 Introduction

Particle zoo is the informal though fairly common nickname to describe what was formerly known as "elementary particles". The standard model of physics [1] divides these particles into leptons, considered to be fundamental "elementary particles" and the hadrons, composed of two (mesons) or three (baryons) quarks. Well hidden in the data of particle energies lies another ordering principle, based on a description of particles as electromagnetic objects.
Particles are interpreted as some kind of standing electromagnetic wave originating from a rotating electromagnetic field with the E-vector pointing towards the origin. Neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of opposite polarity. To obtain quantifiable results, the electromagnetic field will be modified with an appropriate exponential function, $\Psi(r, \vartheta, \varphi)$, serving as probability amplitude of the field. The two integrals needed to calculate energy in point charge and photon representation exhibit the following two relations:
1) Their product - resulting from energy conservation - is characterized by containing the product of the two gamma functions $\Gamma_{+1/3}\,|\Gamma_{-1/3}| \approx \alpha^{-1}/(4\pi)$,
2) their ratio features a quantization of energy states with powers of $1/3^n$ over some base α_0, a relation that can be found in the particle data with $\alpha_0 = \alpha$ as:

$$W_n/W_e \approx 3/2\left(y_l^m\right)^{-1/3}\Pi_{k=0}^{n}\alpha\hat{}(-1/3^k) \qquad n = \{0;1;2;..\} \tag{1}$$

with W_e = energy of electron [1], W_n = energy of particle n and y_l^m representing the angular part of $\Psi(r, \vartheta, \varphi)$. For spherical symmetry $y_0^0 = 1$ holds, corresponding particles are e, μ, η, p/n, Λ, Σ and Δ [2].
Apart from calculating energies the model may be used to describe other particle properties. At distances comparable to particle size, typically femtometer for hadrons, direct interaction of particle wave functions has to be expected. Interpreting this as strong interaction and considering the basic spatial characteristics of the functions may provide a possible explanation why leptons, in particular the tauon, are not subject to this interaction. Expanding the incomplete gamma function appearing in the integrals for calculating particle energy gives quantitative terms for the strong and Coulomb interaction. The model may offer approaches for calculating gravitational interaction as well, in particular the 3rd power relationship of α-coefficients can be expanded to give a value for Planck energy.
The following equations basically use two parameters, one for energy (β) based on W_e as reference and free parameter of the model and a second (σ) which is a function of angular momentum. To focus on the more fundamental relationships the discussion of minor aspects of the model parameters is exiled to an appendix, related topics to be marked as [A]. Typical accuracy of the calculations presented is ~ 0.001 (e.g. due to approximations of Γ-functions) which would be also the order of magnitude of possible QED corrections.

2 Results

2.1 Basic calculations

The model is essentially based on a single assumption:

1 Factor 3/2 is supposed to represent an anomaly of the electron, see [A2,3]
2 The relation of the e, μ, π masses with α was noted in 1952 by Y.Nambu [2]. M.MacGregor calculated particle and constituent quark mass as *multiples* of α and related parameters [3]. This article is a shortened + modified version of [4].

Particles can be described by using an appropriate exponential wave function, Ψ(r), that acts as a probability amplitude on an electromagnetic field.

An appropriate form of Ψ can be deduced from three boundary conditions:

1.) To be able to apply Ψ to a point charge Ψ(r = 0) = 0 is required, this may be considered by a term such as:

$$\Psi(r) \sim \exp\left(\frac{-\beta/2}{r^y}\right) \qquad (2)$$

2.) To ensure integrability an integration limit is needed. This may be achieved by Ψ(r) being the solution of a 2nd order differential equation of approximate general form

$$-\Delta\Psi(r) + \frac{\beta/2}{r^{x+1}}\nabla\Psi(r) - \frac{\beta/2}{\sigma r^{x+2}}\Psi(r) = 0 \qquad (3)$$

giving for particle n:

$$\Psi_n(r)=\exp\left(-\left(\frac{\beta_n/2}{r^x}+\left[\left(\frac{\beta_n/2}{r^x}\right)^2-4\frac{\beta_n/2}{\sigma r^x}\right]^{0.5}\right)/2\right) \qquad (4)$$

3.) Ψ should be applicable regardless of the expression chosen to describe the electromagnetic object. In particular requiring a point charge and a photon representation of a localized electromagnetic field (particle) to have the same energy, the exponent of r is required to be x=3 (see (15)), giving finally:

$$\Psi_n(r)=\exp\left(-\left(\frac{\beta_n/2}{r^3}+\left[\left(\frac{\beta_n/2}{r^3}\right)^2-4\frac{\beta_n/2}{\sigma r^3}\right]^{0.5}\right)/2\right) \qquad (5)$$

Up to the limit of the real solution of (5), $r = r_l$, with

$$r_l = (\sigma\, \beta/8)^{1/3} \qquad (6)$$

in all integrals over Ψ(r) given below equ. (7) may be used as approximation for (5)

$$\Psi_n(r<r_l) \approx \exp\left(\frac{-\beta_n/2}{r^3}\right) \qquad (7)$$

Phase will be neglected on this approximation level, properties of particles will be calculated by the integral over $\Psi(r)^2$ [3] times some function of r and can be given by:

$$\int_0^{r_l}\Psi(r)^2 r^{-(m+1)}dr \approx \int_0^{r_l}\exp(-\beta/r_l^3)r^{-(m+1)}dr = \Gamma(m/3,\beta/r_l^3)\,\frac{\beta^{-m/3}}{3} = \int_{\beta/r_l^3}^{\infty} t^{\frac{m}{3}-1}e^{-t}dt\,\frac{\beta^{-m/3}}{3} \qquad (8)$$

with m = {..-1;0;1;..}. The term $\Gamma(m/3, \beta/r_l^3)$) denotes the upper incomplete gamma function, given by the Euler integral of the second kind with β/r_l^3 as lower integration limit. For m ≥ 1 the complete gamma function $\Gamma_{m/3}$ is a sufficient approximation, for m ≤ 0 the integrals have to be integrated numerically. Coefficient β_n is a particle specific factor, proportional to particle energy W as $\beta_n \sim W_n^{-3}$ (12) , for particle n it may be given as partial product of a value for a reference particle, β_{ref} carrying the dimensional term β_{ref} times particle specific dimensionless coefficients, α_n, of succeeding particles representing the ratio of β_n and β_{n+1}:

$$\beta_n = \beta_{ref}\Pi_{k=1}^{n}\alpha_k = \beta_{dim}\Pi_{k=0}^{n}\alpha_k \qquad (9)$$

Coefficient σ is related to the angular part of the wave function Ψ and thus to angular momentum. The relationship between r_l and σ is given by (6). Replacing r_l by using the Euler integral, equation (8), for m= -1

$r_l \approx 1.5133\, |\Gamma_{-1/3}|\, \beta_n^{1/3}/3\, \alpha^{-1}$ [4] [A1,2] (10)

σ may be given as:

$\sigma = 8\, r_{l,n}^3/\beta_n = 8(1.5133\, |\Gamma_{-1/3}|/3\, \alpha^{-1})^3 = 1.772E+8$ [-] [A1,2] (11)

Particle energy is expected to be equally divided into electric and magnetic part, $W_n = 2W_{n,el} = 2W_{n,mag}$. To

3 hence factor 2 in (2)ff

4 $r_l = 1.5133\, \alpha^{-1}\, r_m$, with $r_m = |\Gamma_{-1/3}|\, \beta_n^{1/3}/3$ being the coordinate of the approximate maximum of W(r), see fig.1

calculate energy, the integral over the electrical field E(r) of a point charge is used as a first approximation. Using (8) for m = 1 gives:

$$W_{pc,n} = 2\varepsilon_0 \int_0^{\infty} E(r)^2 \Psi_n(r)^2 d^3r = 2b_0 \int_0^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr = 2\, b_0\, \Gamma_{1/3}\, \beta_n^{-1/3}/3 \quad (12)$$ [5]

Using equation (8) for m = -1 to calculate the Compton wavelength, λ_C, in the expression for the energy of a photon, hc_0/λ_C, gives the following expression for λ_C :

$$\lambda_{C,n} \approx \int_0^{\lambda_{C,n}} \Psi_n(r)^2 dr = \int_{\beta/\lambda_{C,n}^3}^{\infty} t^{-4/3} e^{-t} dt \;\; \beta_n^{1/3}/3 \approx 36\,\pi^2\, |\Gamma_{-1/3}|\, \beta_n^{1/3}/3 \quad (13)$$

to be used in:

$$W_{Phot,n} = hc_0/\lambda_{C,n} = \frac{hc_0}{\int^{\lambda_{C,n}} \Psi_n(r)^2 dr} = \frac{3\,hc_0}{36\,\pi^2 |\Gamma_{-1/3}|\, \beta_n^{1/3}} \quad (14)$$

The energy of a particle has to be the same in both photon and point charge description. Equating (12) with (14) and rearranging to emphasize the relationship of α with the gamma functions ($\Gamma_{1/3}$ = 2.679; $|\Gamma_{-1/3}|$ = 4.062) gives (note: h => ħ):

$$\frac{4\pi\Gamma_{1/3}|\Gamma_{-1/3}|}{0.998} = \frac{9\,hc_0}{18\,\pi\, b_0} = \frac{\hbar\, c_0}{b_0} = \alpha^{-1} \quad (15)$$

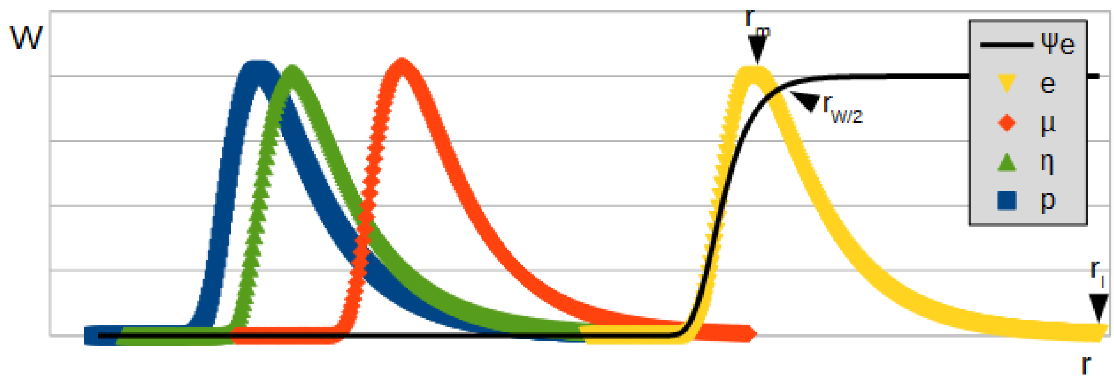

Figure 1: Example for particle energy $W_{n\ calc}$ (r) (normalized) vs lg(r) according to equ. (12); $r_{m,n}$: see note 4; $r_{W/2}$ => radius of the integral (12) at $W_n(r) \sim W_n/2$; r_l see (6); black line: $\Psi_e(r)$

2.2 Quantization with powers of $1/3^n$ over α

Inserting (9) in the product of the point charge and photon expression of energy, W_n^2, gives:

$$W_n^2 = 2\,b_0 hc_0 \frac{\int^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr}{\int^{\lambda_{C,n}} \Psi_n(r)^2 dr} \sim \frac{1}{\beta_n^{2/3}} \sim \frac{\alpha_0^{1/3}\alpha_1^{1/3}.....\alpha_n^{1/3}}{\alpha_0\,\alpha_1\,....\alpha_n} \qquad n = \{0;1;2;..\} \quad (16)$$

The last expression of (16) is obtained by expanding the product $\Pi_n^{-2/3}$ included in $\beta_n^{-2/3}$ with $\Pi_n^{1/3}$ From this term it is obvious that a relation $\alpha_{n+1} = \alpha_n^{1/3}$ such as given by equation (1) yields the only non-trivial solution for W_n^2 where all intermediate particle coefficients cancel out and W_n becomes a function of coefficient α_0 only. By comparison with experimental data α_0 may be identified as $\alpha_0 = \alpha_e \approx \alpha^9$ and the α-product can in general be given by:

$$\frac{\alpha^3\alpha^1....\alpha\wedge(9/3^n)\,\alpha\wedge(3/3^n)}{\alpha^9\,\alpha^3\alpha^1....\alpha\wedge(9/3^n)} = \alpha\wedge(3/3^n)/\alpha^9 \qquad n = \{0;1;2;..\} \quad (17)$$

The corresponding term for particle energies will be given by (using (15)):

5 $b_0 = e^2/(4\pi\varepsilon)$ to be used as abbreviation in the following

$$W_n = \frac{4\pi b_0^2}{\alpha} \frac{\int_{\lambda_{C,n}}^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr}{\int \Psi_n(r)^2 dr} = \left(\frac{(2b_0)^2 \Gamma_{1/3}^2}{9[\alpha 4\pi \ |\Gamma_{-1/3}| \Gamma_{1/3}] \beta_n^{2/3}} \right)^{0.5} = \qquad n = \{0;1;2;..\} \qquad (18)$$

$$= 2b_0 \frac{\Gamma_{1/3}}{3\beta_n^{1/3}} = 2b_0 \frac{\Gamma_{1/3}}{3\beta_{dim}^{1/3}} \alpha \wedge (1.5/3^n)/\alpha^{4.5} \approx W_e \frac{3}{2} \ \Pi_{k=0}^{n} \alpha\wedge(-1/3^k)$$

giving equation (1) for spherical symmetry. In the last term of (18) the additional factor 3/2 has to be inserted ad hoc to represent the anomaly in the product (9) due to the energy ratio of e, μ, $W_\mu / W_e = 1.5088\ \alpha^{-1}$ [A2] [6]. Equation (9) has to be adjusted accordingly:

$$\beta_n = \beta_e (2/3)^3 \ \Pi_{k=0}^{n} \alpha \wedge (3/3^k) = \beta_{dim} \alpha_e (2/3)^3 \ \Pi_{k=0}^{n} \alpha \wedge (3/3^k) = \beta_{dim} \Pi_n \ ^{7} \qquad n = \{0;1;2;..\} \qquad (19)$$

A fit of W_e will give β_{dim} = 2.12E-24 [m^3]. Extending the model to energies below the electron with a coefficient of α^3 in (1) gives a state of energy ~ 0.2eV which is roughly in a range expected for a neutrino [5].

2.3 Non-spherical symmetric states

Up to here only spherical symmetry, y_0^0, and Ψ(r) is considered. The ratio of the volume integrals attributed to spherical harmonic Y_1^0 and Y_0^0 gives a factor of 1/3. Assuming Y_1^0 to be a sufficient approximation for the next angular term and $W_n \sim 1/r_n \sim 1/V_n^{1/3}$ (V = volume) to be applicable for non-spherically symmetric states as well, will give $W_1^0/W_0^0 = 3^{1/3} = 1.44 = (y_1^0)^{-1}$. A change in angular momentum is expected for this transition which is actually observed with ΔJ = ± 1 except for the pair μ/π with Δ J = 1/2.
Results for particles assigned to y_0^0, y_1^0 are presented in table 1.

	n	$W_{n,Lit}$ [MeV]	$\Pi_{k=0}^{n}$ α^(-1/3^k) equ (1)	Π_n equ (19),(37)	W_{calc}/W_{Lit}	J	r_l [fm]
ν	**-1**	**2E-7 ***	**α^{+3}**		**-**	**1/2**	**1.5E+10**
e^{+-}	**0**	**0.51**	Reference	**$(3/2)^3\ \alpha^9$**	**1.0001****	**1/2**	**8877**
μ^{+-}	**1**	**105.66**	**α^{-1}**	**$\alpha^9\alpha^3$**	**1.0000**	**1/2**	**42.9**
π^{+-}	1	139.57	1.44 α^{-1}	$\alpha^9\alpha^3/3$	1.0918	0	29.8
K		495				0	
η^0	**2**	**547.86**	**$\alpha^{-1}\alpha^{-1/3}$**	**$\alpha^9\alpha^3\alpha^1$**	**0.9933**	**0**	**8.3**
ρ^0	2	775.26	1.44 $(\alpha^{-1}\alpha^{-1/3})$	$\alpha^9\alpha^3\alpha^1/3$	1.0124	1	5.8
ω^0	2	782.65	1.44 $(\alpha^{-1}\alpha^{-1/3})$	$\alpha^9\alpha^3\alpha^1/3$	1.0028	1	5.8
K*		894				1	
p^{+-}	**3**	**938.27**	**$\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}$**	**1.0016**	**1/2**	**4.8**
n	**3**	**939.57**	**$\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}$**	**1.0003**	**1/2**	**4.8**
η'		958				0	
Φ^0		1019				1	
Λ^0	**4**	**1115.68**	**$\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}$**	**1.0106**	**1/2**	**4.0**
Σ^0	**5**	**1192.62**	**$\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}$**	**1.0046**	**1/2**	**3.8**
Δ	**∞**	**1232.00**	**$\alpha^{-3/2}$**	**$\alpha^{27/2}$**	**1.0025**	**3/2**	**3.7**
Ξ		1318				1/2	
Σ^{*0}	3	1383.70	1.44 $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}/3$	0.9796	3/2	3.3
Ω^-	4	1672.45	1.44 $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}/3$	0.9724	3/2	2.8
N(1720)	5	1720.00	1.44 $(\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}/3$	1.0046	3/2	2.7
tau^{+-}	∞	1776.82	1.44 $(\alpha^{-3/2})$	$\alpha^{27/2}/3$	1.0026	1/2	2.5

Table 1: Particles up to tauon energy[8]; values for **y_0^0 (bold)**, y_1^0; col. 3: energy values of [6] except*: W_{calc} calculated using the slighty more precise [A(37)] in place of (19); ** Using [A, 40]

6 While the origin of factor 3/2 is unclear, the difference to 1.5088 can be given by [A(36)].
7 Factor $(2/3)^3$ reproduces factor 3/2 in (1,) to be canceled by an extra $(3/2)^3$ in $\alpha_e \approx (3/2)^3 \alpha^9$ (see (38)). Π_n for brevity.
8 up to Σ'⁰ all resonance states given in [6] as **** included; Exponent of -3/2, 27/2 for Δ and tau is equal to the limit of the partial products in (1) and (19); r_l calculated with (6);

2.4 Upper limit of energy

In the simple picture sketched in the introduction the rotating E-vector might be interpreted to cover the whole angular range in the case of spherical symmetric states while a p-like state of an Y_1^0-analogue might be interpreted as forming a double cone. Going to higher Y_n^0-analogue states will close the angle of the cone leaving the original vector in the angular limit case, which might be interpreted as an instantaneous snapshot of time-averaged lower states, The maximum of the W(r) curve of spherical symmetric states of fig.1 will shift towards r_l until it reproduces the shape of Ψ(r) itself, i.e. r_m, => r_l.

This is equivalent to σ approaching approximately unity. Since a bound state requires σ > 1 an upper limit for the angular contribution to the particle energy may be given by 1.51 α^{-1} and possible other components included in r_l, σ according to (10), (11) such as $\Gamma_{-1/3}/3$ or 2. The maximum angular contribution to W_{max} may be estimated as being approximately:

$$1.5133\ \alpha^{-1} < \Delta W_{max,\ angular} < \sigma^{1/3} \tag{20}$$

From (19) follows an estimate for the total upper limit of energy as:

$$W_e\ 1.5133^2\ \alpha^{-2.5} = 4.12E\text{-}8\ [J] < W_{max} < W_e\ 1.5133\ \alpha^{-1.5}\ \sigma^{1/3} = 1.72E\text{-}7[J] \tag{21}$$

This corresponds to a factor 2.0 - 5.5 relative to the mass of the Higgs boson [6].

2.5 Expansion of the incomplete gamma function $\Gamma(1/3,\beta_n/r^3)$

The series expansion of $\Gamma(1/3,\beta_n/r^3)$ in the equation for calculating particle energy (12) gives [7]:

$$\Gamma(1/3,\ \beta_n/(r^3)) \approx \Gamma_{1/3} - 3\left(\frac{\beta_n}{r^3}\right)^{1/3} + \frac{3}{4}\left(\frac{\beta_n}{r^3}\right)^{4/3} = \Gamma_{1/3} - 3\frac{\beta_n^{1/3}}{r} + \frac{3}{4}\frac{\beta_n^{4/3}}{r^4} \tag{22}$$

and for the potential energy / electrostatic part of $W_n(r)$, $W_{n,pot}(r) = W_n(r)/2$:

$$W_{n,pot}(r) \approx W_n/2 - b_0\frac{3\beta_n^{1/3}}{3\beta_n^{1/3}r} + b_0\frac{3}{4}\frac{\beta_n^{4/3}}{3\beta_n^{1/3}r^4} = W_n/2 - \frac{b_0}{r} + b_0\frac{\beta_n}{4r^4} \tag{23}$$

The 2nd term in (23) drops the particle specific factor β_n and gives the electrostatic energy of two elementary charges at distance r. The 3rd term is an approximately appropriate choice for the 0th order term of the differential equation below and is supposed to be responsible for the localized character of an electromagnetic object. In chpt. 3.3 some arguments are given that demonstrate a relationship of the properties of the wave functions used in this model with the "strong force" of the standard model. It may be assumed that the 3rd term of (23) represents this strong force.

2.6 Differential equation

The approximation $\Psi(r<r_l)$ of equation (7) provides a solution to a differential equation of type

$$-\frac{r}{6}\frac{d^2\Psi(r)}{dr^2} + \frac{\beta_n/2}{2r^3}\frac{d\Psi(r)}{dr} - \frac{\beta_n/2}{r^4}\Psi(r) = 0 \tag{24}$$

However the correct discriminant form of Ψ(r) of equ. (5) would be provided by a slightly different equation:

$$-r\frac{d^2\Psi(r)}{dr^2} + \frac{\beta_n/2}{r^3}\frac{d\Psi(r)}{dr} - \frac{\beta_n/2}{\sigma\ r^4}\Psi(r) = 0 \tag{25}$$

To test if this type of differential equation is related to conventional terms used in quantum mechanics, a quantum mechanical term for kinetic energy for the 2nd order term, using W_e => $\Gamma_-\Gamma_+\ 2\ b_0\ /(9\ r)$ which is an approximation for $r \approx r_m$ to replace $m_e = W_e/c_o^2$, and a slightly altered ($1/\sigma$, r^{-3}) 3rd term of (23) for the 0th and 1st order term are used to give a rough approximation for the differential equation as

$$-\left(\frac{9\hbar^2 c_0^2 r}{\Gamma_{-1/3}\Gamma_{+1/3}2b_0}\right)\frac{d^2\Psi(r)}{dr^2} + \frac{b_0\beta_e}{4r^3}\frac{d\Psi(r)}{dr} - \frac{b_0\beta_e}{4\sigma\ r^4}\Psi(r) = 0 \tag{26}$$

In particular for σ -> 1 this yields roughly appropriate order of magnitude results for r ~ r_m. In general better suited mathematical approaches for providing a term such as (5), in particular as far as the angular term is concerned, have to be considered.

2.7 Gravitation

Defining the Planck-energy W_{Pl} as

$$W_{Pl} = c_0^2\,(b_0/G)^{0.5} = c_0^2\,(\alpha h c_0/G) \tag{27}$$

gravitational attraction F_G in the classical limit can be expressed as:

$$F_G = \frac{b_0 W_n W_m}{W_{Pl}^2}\frac{1}{r^2} \tag{28}$$

Expression (28) is a restatement of Newton's law with no additional insight unless an expression for W_{Pl} independent of G would be at hand. Expanding relationship (19) to higher powers of α_e i.e. $\alpha_e^3 = (1.5133^3\,\alpha^9)^3$ provides just that. The relationship is quantitative if using

$$0.9994\frac{W_{Pl}}{W_e} = 1.5133^{-2}\alpha^{-10}2 \quad = 2.039\text{ E+21} \tag{29}$$

i.e. using $(\alpha_e^3)^{-1/3}$ times the angular limit factor according to (20) in the form 1.5133 α^{-1} *2.
Using [A(35)] to express factor 1.5133 gives (F_G, F_C = gravitational, Coulomb forces):

$$\left(\frac{W_e}{W_{Pl}}\right) = \left(\frac{F_{G,e}}{F_{C,e}}\right)_{calc} = \left[\frac{(4\pi)^2\,\Gamma_-^4\,\alpha^{12}}{2}\right]^2 = 1.0007^2\left(\frac{F_{G,e}}{F_{C,e}}\right)_{\exp} = \frac{G\,W_e^2}{c_0^4\,b_0} \quad {}^{9} \tag{30}$$

3 Discussion

3.1 Additional particle states

In general it is not expected that partial products can explain all values of particle energies and linear combination states have to be considered.
The first particle family that does not fit to the partial product scheme are the kaons at ~ 495MeV. Assuming them to be a linear combination of two π-states with a supposed charge distribution of +|+, -|- and +|- would yield the basic symmetry properties of the four kaons as given below, providing two neutral kaons of different structure and parity:

K^+	K^-	K_S^o	K_L^o	
+	-	-	+	
+ +	- -	+ +	+ -	(+/- = charge)
+	-	-	-	

Analogous, for the charged kaons, K^+, K^-, a configuration for wave function sign equal to the configuration for charge of K_S^o and K_L^o might be possible, giving two analogous variants of + and - parity of otherwise identical particles. Such configurations for the kaons might give a simple explanation for the unusual decay modes observed in the experiments.

3.2 Gravitation

In chapter 2.7 an example using the Planck energy is given that the model may be used to construct simple models for gravitational force that provide values in the correct range of order of magnitude.

From (18) it is not obvious if and at which energy a ground state exists though the electron or neutrino would be obvious choices. The ground state should be distinguished by the existence of a dimensionless parameter α_{ground}^3 that does not represent another particle state but has some more fundamental significance. Relationship (29) seems to provide just that.
For giving a detailed mechanism within this model particles might interact via direct contact in place of boson-mediated interaction. The particles are not expected to exhibit a rigid radius. Within the limits of charge and energy conservation a superposition of many states might be conceivable, extending the particle in space with radius $r_{l,n}$ appropriate for energy of each superposition state, enabling interaction at distance r_l. The wave function of a particle might contribute an additional factor to lower total Ψ values on site of a second particle thereby reducing particle energy and resulting in an attractive force. The appearance of the angular limit term of (20) in (29) may reflect the instantaneous character of such an interaction [10].

9 Using [A, 39] for calculating W_e would give G as:

$$G_{calc} = \frac{c_0^4}{\varepsilon_c}\frac{2^{10}\pi^{11/3}}{9}\frac{\Gamma_-^{14}}{\Gamma_+^2}\alpha^{30} = 1.0013\,G_{\exp}$$

10 The term $(\Gamma_{-1/3}\,\alpha^3)^4$ in expression (30) indicates a relationship with the 3rd term of (22).

3.3 Short range interaction - strong force

In this model, on the length scale of particle radius, the wave functions of two particles should start to overlap and exert some kind of direct interaction. As demonstrated in table 1, last column, for hadrons the model yields particle radius in the range of femtometer, the characteristic scale for strong interaction and it seems likely to identify strong interaction with the interaction of wave functions. Interaction via overlapping of wave functions constitutes the basis of chemical bonding and has been examined extensively [8].In general wave functions are signed (not to be confused with electrical charge), for particles above the ground state regions of different sign exist, separated by nodes. There are two major requirements for effective interaction:

1) Comparable size and energy of wave functions,

2) sufficient net overlap: In the overlap region of two interacting wave functions sign should be the same (bonding) or opposite (antibonding) in all overlapping regions. If regions with same and opposite sign balance to give zero net overlap, no interaction results.

From condition 1) and the data of table 1 it is obvious that the wave functions of neutrino and electron will not show effective interaction with hadrons due to mismatch of size and energy. In the case of the tauon the second rule is crucial. According to this model the tauon is exceptional by being at the end of the partial product series for y_1^0 and should consequently exhibit a high, potentially infinite number of nodes, separating densely spaced volume elements of alternating wave function sign. Though having particle size and energy in the same order of magnitude as other hadrons, such as the proton, the frequent change of sign of the tauon wave function will prohibit net overlap and effective interaction.

Conclusion

Using the exponential function $\Psi(r,\vartheta,\varphi)$ as probability amplitude for the electric field $E(r)$ gives the following results:

- a numerical approximation for the value of the fine-structure constant α,
- a quantization of energy levels given by a partial product of terms α^$(-1/3^n)$,
- qualitative explanations for particle properties such as the lepton character of the tauon or the decay of kaons,
- a possibility to quantitatively express the gravitational constant in terms of electromagnetic constants and electron energy,
- an indication of a common base for strong force, electromagnetism and mass/gravitation, given by a common set of coefficients and the expansion of the incomplete gamma function.

References

[1] Griffiths, D.J. Introduction to Elementary Particles. John Wiley & Sons 2nd. Ed.; 2008

[2] Nambu,Y. Progress of theoretical physics 7, 595-596; 1952

[3] MacGregor, M. The power of alpha, Singapore: World Scientific; 2007

[4] Schindelbeck, T. http://doi.org/10.5281/zenodo.832957; 2018

[5]Neutrinos in Cosmology, J. Lesgourgues (RWTH Aachen U.) and L.Verde (U. of Barcelona & ICREA), http://pdg.lbl.gov/2017/reviews/rpp2017-rev-neutrinos-in-cosmology.pdf

[6] Mohr, P.J., Newell, D.B.,Taylor, B.N. CODATA Recommended Values; doi: 10.1103/RevModPhys.88.035009; 2014

[7] Olver, F.W.J. et al. NIST Handbook of Mathematical Functions, Cambridge University Press, 2010; http://dlmf.nist.gov/8.7.E3

[8] Kutzelnigg, W. Einführung in die Theoretische Chemie, Wiley-VCH, Weinheim; 2002

Appendix

[A1] Angular momentum / 2π

A simple relation with angular momentum J for spherical symmetric states, applying a semi-classical approach using

$$J = r_2 x\, p(r_1) = r_2 W_n(r_1)/c_0 \tag{31}$$

and assuming $|r_2| = |r_1|$ and $W_{kin,n} = 1/2\ W_n$, gives the integral:

$$|J| = \int_0^{r_{l,n}} J_n(r)\,dr = 2\frac{b_0}{c_0}\int_0^{r_{l,n}} \Psi_{2n}(r)^2 r^{-1}\,dr \tag{32}$$

From (8) follows for m = 0:

$$\int_0^{r_{l,n}} \Psi_{2\pi,n}(r)^2 r^{-1}\,dr = 1/3\int_{8/\sigma}^{\infty} t^{-1}e^{-t}dt \approx 5.45 \approx \alpha^{-1}/8\pi \tag{33}$$

yielding the constant $\alpha^{-1}/8\pi$. Inserting (33) in (32) would provide:

$$|J| = 2\frac{b_0}{c_0}\frac{\alpha^{-1}}{8\pi} = 1/2\ [\hbar]\ 1/(2\pi) \tag{34}$$

To get the expected value of 1/2 [ħ] either assumption $|r_2| = |r_1|$ or the assumption of equ. (12), that the Coulomb law originating from the interaction of 2 particles can be used as first approximation has to be dropped, introducing a factor 2π in either (32) or (12). The whole complex of angular part of the wave function, wave function phase, angular momentum, magnetic moment needs to be worked out thoroughly before this questions may be settled.

[A2] Coefficient ~1.5

The value of 1.51 α^{-1} in r_l, σ originates from the relationship with J through equ. (33) and is obviously close to the ratio $W_\mu/W_e = 206.8 = 1.5088\ \alpha^{-1}$. The source of this anomaly is supposed to be the electron rather than the muon, which is a middle term of product (19) and the equations will be arranged accordingly in (19) by factor $(2/3)^3$ representing a general factor of all particles to be canceled by a factor $(3/2)^3$ in α_e. Several options for ~1.51 involved in this model have been be considered: 3/2, $\Gamma_- /\Gamma_+ = 1.516$, $\pi/2 = 1.571$, 1.5088 etc. The value 1.5133 has been chosen due to

1. a possible geometrical interpretation (using(15))

$$1.516\ \alpha^{-1}\ \Gamma_- /3 = \Gamma_- /\Gamma_+\ 4\pi\ \Gamma_-\ \Gamma_+/0.998\ \ \Gamma_- /3 = \frac{1}{0.998}\ \frac{4\pi\Gamma_-^3}{3} \quad \Rightarrow \quad 1.5133 = 1.516 * 0.998 \tag{35}$$

connecting the one and three dimensional features of this model.

2. Factor 1.5088 of the ratio W_μ/W_e being subject to a 3rd power relationship of the same kind as the α coefficients:

$$\left(\frac{1.5133}{1.5088}\right)^3 = \left(\frac{1.5133}{1.5}\right) \tag{36}$$

indicating that the particle specific term of β and the components of σ are not correctly separated yet even in the case of spherical symmetric states.

[A3] Particle parameter β

Apart from the particle coefficients α_n, parameter β may be analyzed further. To avoid introducing additional parameters one might test an approach giving β as function of b_0 and σ. A suitable expression will be:
$\beta \sim \sigma\ b_0^2$ and since (36) will be used within the particle specific factor, coefficient 1.5133 of σ will be placed there, giving for the general term: (i.e. excluding the electron) [11]:

$$\beta_n = \beta^{\#}_{dim}\ \frac{2}{(2\pi)^3}\ \frac{\sigma}{1.5133^3}\ b_0^2 \Pi_{k=0}^{n}\left[\boldsymbol{\alpha}^3\left(\frac{\mathbf{1.5133}}{\mathbf{1.5}}\right)\right]^\wedge\left(\frac{\mathbf{3}}{\mathbf{3}^k}\right) \qquad n = \{1,2,...\} \tag{37}$$

for the electron:

$$\beta_e = \beta^{\#}_{dim} * 2(2\pi)^3\ \frac{\sigma}{1.5133^3}\ b_0^2\left[\ \frac{\mathbf{3}}{\mathbf{2}}\ \boldsymbol{\alpha}^3\left(\frac{\mathbf{1.5133}}{\mathbf{1.5}}\right)\right]^{\mathbf{3}} \tag{38}$$

the particle specific factor is given in bold.
Using this expression the remaining term $\beta^{\#}_{dim}$ = 2.856 E+25 [m/J²] yields a simple term if a unit system with symmetric splitting of c_0 into constants ε and μ, is used, i.e. in SI units the modification:

$$c_0^2 = (\varepsilon_c\ \mu_c)^{-1} \tag{39}$$

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11 Factor 2π in (36)f is related to the topic in [A1]

with
ε_c = (2.998E+8 [m²/Jm])$^{-1}$ = (2.998E+8)$^{-1}$ [J/m]
μ_c = (2.998E+8 [Jm/s²])$^{-1}$ = (2.998E+8)$^{-1}$ [s^2/Jm]
i.e. the numerical values for c_0, $1/\varepsilon_c$, $1/\mu_c$ are identical, the units of ε_c, μ_c are expanded by [Jm] for the convenience of this model. From b_0 follows for the square of the elementary charge: e_c^2 = 9,67E-36 [J^2]
This allows to give β_{dim}* as:

$$\beta^{\#}_{dim} = \left(\frac{2}{3}\right)^3 \frac{1}{e_c \varepsilon_c} \quad = 2.856 \text{ E+25 [m/J}^2] \tag{40}$$

and

$$\beta^{\#}_{dim} b_0^2 = \left(\frac{2}{3}\right)^3 \frac{1}{(4\pi)^2} \left(\frac{e_c}{\varepsilon_c}\right)^3 \quad = 1.520 \text{ E-30 [m}^3] \tag{41}$$

turning all equations into ab initio expressions using electromagnetic constants only.