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On the Numerical Value of $\int_0^h e^{x^2}dx$. *By* H. G. DAWSON.

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Starting from the two known equations

$$e^{-x^2}+e^{-(x-a)^2}+e^{-(x+a)^2}+e^{-(x-2a)^2}+e^{-(x+2a)^2}+\&c.$$
$$=\frac{\sqrt{\pi}}{a}\left(1+2e^{-(\pi^2/a^2)}\cos\frac{2\pi x}{a}+2e^{-(4\pi^2/a^2)}\cos\frac{4\pi x}{a}+\&c.\right),$$

$$e^{-x^2}-e^{-(x-a)^2}-e^{-(x+a)^2}+e^{-(x-2a)^2}+e^{-(x+2a)^2}-\&c.$$
$$=\frac{2\sqrt{\pi}}{a}\left(e^{-(\pi^2/4a^2)}\cos\frac{\pi x}{a}+e^{-(9\pi^2/a^2)}\cos\frac{3\pi x}{a}+\&c.\right),$$

we can, by putting $a=1$, and making a few obvious transformations, deduce the two equations

$$e^{-x^2}\phi(x)=\sqrt{\pi}\left(1+\frac{2\cos 2\pi x}{e^{\pi^2}}+\frac{2\cos 4\pi x}{e^{4\pi^2}}+\frac{2\cos 6\pi x}{e^{9\pi^2}}+\&c.\right),$$

$$e^{-x^2}\psi(x)=\sqrt{\pi}\,e^{\frac{1}{4}}\left(1-\frac{2\cos 2\pi x}{e^{\pi^2}}+\frac{2\cos 4\pi x}{e^{4\pi^2}}-\frac{2\cos 6\pi x}{e^{9\pi^2}}+\&c.\right),$$

where $\phi(x) \equiv 1 + \frac{e^{2x}+e^{-2x}}{e} + \frac{e^{4x}+e^{-4x}}{e^4} + \ldots + \frac{e^{2mx}+e^{-2mx}}{e^{m^2}} + \ldots,$

and $\psi(x) = e^x + e^{-x} + \frac{e^{3x}+e^{-3x}}{e^2} + \ldots + \frac{e^{(2m+1)x}+e^{-(2m+1)x}}{e^{m(m+1)}} + \ldots .$

Approximately we have

$$\sqrt{\pi}\, e^{x^2} \left(1 + \frac{2\cos 2\pi x}{e^{\pi^2}}\right) = \phi_1(x),$$

$$e^{\frac{1}{4}} \sqrt{\pi}\, e^{x^2} \left(1 - \frac{2\cos 2\pi x}{e^{\pi^2}}\right) = \psi_1(x);$$

whence $2e^{\frac{1}{4}}\sqrt{\pi}\int_0^h e^{x^2}dx = \int_0^h \{e^{\frac{1}{4}}\phi_1(x) + \psi_1(x)\}\, dx \quad (h > 2);$

therefore $2\sqrt{\pi}\int_0^h e^{x^2}dx = \int_0^h \{\phi_1(x) + e^{-\frac{1}{4}}\psi_1(x)\}\, dx,$

$$2\sqrt{\pi}\int_0^h e^{x^2}dx = \int_0^h \phi_1(x)\, dx + e^{-\frac{1}{4}}\int_0^h \psi_1(x)\, dx.$$

Now $\int_0^h \phi_1(x)\, dx = h + \frac{e^{2h}-e^{-2h}}{2e} + \frac{e^{4h}-e^{-4h}}{4e^4} + \frac{e^{6h}-e^{-6h}}{6e^9} + \ldots \equiv \phi_2(h),$

$$\int_0^h \psi_1(x)\, dx = \frac{e^h - e^{-h}}{1} + \frac{e^{3h}-e^{-3h}}{3e^2} + \frac{e^{5h}-e^{-5h}}{5e^6} + \ldots \equiv \psi_2(h).$$

Now, by aid of the tables given by Professor Newman in *Cambridge Philosophical Transactions*, Vol. XIII., supplemented by those of Dr. Glaisher, which are printed immediately after Professor Newman's, it was found, in general, possible to write down the values of $\phi_2(h)$, $\chi_2(h)$.

When it became necessary to calculate positive powers of e whose index was greater than 2, Dr. Glaisher's Table II. was no longer available, and then use was made of Guderman's *Theorie der Potenzial Functionen*, Tafel II., in conjunction with Vega's *Thesaurus Logarithmorum* (Leipzig, 1794).

The original terms of the series $\phi_2(h)$, $\chi_2(h)$ were taken to eight places of decimals.

By this means the values of $2\sqrt{\pi}\int_0^h e^{x^2}dx$ were found to eight places for values (of h) which increased from 0 to 2 by intervals of magnitude ·01.

The values of $\int_0^h e^{x^2}dx$ were calculated by aid of an ordinary seven-place set of logarithm tables.

In this calculation we have neglected $\int_0^h e^{x^2}\frac{\cos 4\pi x}{e^{4\pi^2}}dx$ and smaller quantities; the value of

$$\int_0^h e^{x^2}\frac{\cos 4\pi x}{e^{4\pi^2}}dx < \frac{1}{4\pi}e^{-4(\pi^2-1)} < \frac{1}{12}e^{-34} < 2\times 10^{-16}.$$

I append tables of the values of $\int_0^h e^{x^2}dx$.

h	$\int_0^h e^{x^2}dx$	h	$\int_0^h e^{x^2}dx$	h	$\int_0^h e^{x^2}dx$
·01	·010000	·37	·387601	·73	·883332
·02	·020003	·38	·399111	·74	·900496
·03	·030008	·39	·410709	·75	·917916
·04	·040021	·40	·422397	·76	·935600
·05	·050042	·41	·434180	·77	·953553
·06	·060073	·42	·446060	·78	·971785
·07	·070114	·43	·458040	·79	·990305
·08	·080171	·44	·470122	·80	1·009120
·09	·090244	·45	·482313	·81	1·028239
·10	·100334	·46	·494614	·82	1·047669
·11	·110445	·47	·507027	·83	1·067419
·12	·120578	·48	·519558	·84	1·087500
·13	·130736	·49	·532209	·85	1·107922
·14	·140920	·50	·544987	·86	1·128696
·15	·151132	·51	·557892	·87	1·149828
·16	·161376	·52	·570930	·88	1·171331
·17	·171652	·53	·584104	·89	1·193216
·18	·181963	·54	·597418	·90	1·215497
·19	·192311	·55	·610876	·91	1·238180
·20	·202699	·56	·624484	·92	1·261280
·21	·213128	·57	·638244	·93	1·284809
·22	·223602	·58	·652163	·94	1·308782
·23	·234121	·59	·666243	·95	1·333207
·24	·244689	·60	·680492	·96	1·358100
·25	·255307	·61	·694912	·97	1·383476
·26	·265979	·62	·709509	·98	1·409349
·27	·276707	·63	·724289	·99	1·435735
·28	·287493	·64	·739255	1·00	1·462652
·29	·298339	·65	·754415	1·01	1·490109
·30	·309248	·66	·769772	1·02	1 518127
·31	·320223	·67	·785335	1·03	1·546721
·32	·331266	·68	·801106	1·04	1·575912
·33	·342381	·69	·817094	1·05	1·605716
·34	·353568	·70	·833303	1·06	1·636153
·35	·364833	·71	·849742	1·07	1·667240
·36	·376176	·72	·866415	1·08	1·699002

h	$\int_0^h e^{x^2}dx$	h	$\int_0^h e^{x^2}dx$	h	$\int_0^h e^{x^2}dx$
1·09	1·731457	1·40	3·240894	1·71	6·886604
1·10	1·764627	1·41	3·312892	1·72	7·076003
1·11	1·798534	1·42	3·386950	1·73	7·272034
1·12	1·833201	1·43	3·463141	1·74	7·474966
1·13	1·868655	1·44	3·541542	1·75	7·685085
1·14	1·904921	1·45	3·622235	1·76	7·902688
1·15	1·942021	1·46	3·705302	1·77	8·128087
1·16	1·979985	1·47	3·790831	1·78	8·361608
1·17	2·018840	1·48	3·878912	1·79	8·603593
1·18	2·058614	1·49	3·969639	1·80	8·854398
1·19	2·099341	1·50	4·063110	1·81	9·114396
1·20	2·141046	1·51	4·159428	1·82	9·383977
1·21	2·183766	1·52	4·258700	1·83	9·663552
1·22	2·227532	1·53	4·361035	1·84	9·953550
1·23	2·272378	1·54	4·466551	1·85	10·254406
1·24	2·318340	1·55	4·575367	1·86	10·566612
1·25	2·365457	1·56	4·687609	1·87	10·890680
1·26	2·413770	1·57	4·803408	1·88	11·227050
1·27	2·463311	1·58	4·922901	1·89	11·576332
1·28	2·514128	1·59	5·046230	1·90	11·939070
1·29	2·566264	1·60	5·173545	1·91	12·315857
1·30	2·619762	1·61	5·304999	1·92	12·703175
1·31	2·674669	1·62	5·440755	1·93	13·114102
1·32	2·731034	1·63	5·580982	1·94	13·536895
1·33	2·788907	1·64	5·725857	1·95	13·976416
1·34	2·848341	1·65	5·875565	1·96	14·433417
1·35	2·909391	1·66	6·030289	1·97	14·908699
1·36	2·972109	1·67	6·190240	1·98	15·403060
1·37	3·036555	1·68	6·355621	1·99	15·917402
1·38	3·102793	1·69	6·526656	2·00	16·45263
1·39	3·170884	1·70	6·703571		

The foregoing table is true to six figures; two tests can be used to try its accuracy; the first is as follows:—if u_h denote $\int_0^h e^{x^2}dx$, then

$$\Delta^2 u_h = 2h\delta\,(1+h\delta)\,\Delta' u_h.$$

The second is $\Delta u_h = e^{h^2}\left\{1+h\delta+\tfrac{1}{3}(1+2h^2)\,\delta^2\right\}\delta,$

where $\delta = \cdot01$, $\Delta u_h = u_{(h+\delta)} - u_h$, $\Delta' u_h = u_h - u_{(h-\delta)}$.