

XXIV. *On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of Life Contingencies. In a Letter to FRANCIS BAILY, Esq. F. R. S. &c. By BENJAMIN GOMPERTZ, Esq. F. R. S.*

Read June 16, 1825.

DEAR SIR,

THE frequent opportunities I have had of receiving pleasure from your writings and conversation, have induced me to prefer offering to the Royal Society through your medium, this Paper on Life Contingencies, which forms part of a continuation of my original paper on the same subject, published among the valuable papers of the Society, as by passing through your hands it may receive the advantage of your judgment.

I am, Dear Sir, yours with esteem,

BENJAMIN GOMPERTZ.

9th June 1825.

CHAPTER I.

ARTICLE 1. **I**N continuation of Art. 2. of my paper on the valuation of life contingencies, published in the Philosophical Transactions of this learned Society, in which I observed the near agreement with a geometrical series for a short period of time, which must pervade the series which expresses the number of living at ages in arithmetical progression, pro-

ceeding by small intervals of time, whatever the law of mortality may be, provided the intervals be not greater than certain limits: I now call the reader's attention to a law observable in the tables of mortality, for equal intervals of long periods; and adopting the notation of my former paper, considering L_x to express the number of living at the age x , and using λ for the characteristic of the common logarithm; that is, denoting by $\lambda(L_x)$ the common logarithm of the number of persons living at the age of x , whatever x may be, I observe that if $\lambda(L_n) - \lambda(L_{n+m})$, $\lambda(L_{n+m}) - \lambda(L_{n+2m})$, $\lambda(L_{n+2m}) - \lambda(L_{n+3m})$, &c. be all the same; that is to say, if the differences of the logarithms of the living at the ages $n, n+m; n+m, n+2m; n+2m, n+3m; \&c.$ be constant, then will the numbers of living corresponding to those ages form a geometrical progression; this being the fundamental principle of logarithms.

Art. 2. This law of geometrical progression pervades, in an approximate degree, large portions of different tables of mortality; during which portions the number of persons living at a series of ages in arithmetical progression, will be nearly in geometrical progression; thus, if we refer to the mortality of DEPARCIEUX, in Mr. BAILY's life annuities, we shall have the logarithm of the living at the ages 15, 25, 35, 45, and 55 respectively, 2,9285; 2,88874; 2,84136; 2,79379; 2,72099, for $\lambda(L_{15})$; $\lambda(L_{25})$; $\lambda(L_{35})$; &c. and we find $\lambda(L_{25}) - \lambda(L_{15}) = ,04738$ $\lambda(L_{35}) - \lambda(L_{25}) = ,04757$, and consequently these being nearly equal (and considering that for small portions of time the geometrical progression takes place very nearly) we observe that in those tables the numbers of

living in each yearly increase of age are from 25 to 45 nearly, in geometrical progression. If we refer to Mr. MILNE'S table of Carlisle, we shall find that according to that table of mortality, the number of living at each successive year, from 92 up to 99, forms very nearly a geometrical progression, whose common ratio is $\frac{3}{4}$; thus setting out with 75 for the number of living at 92, and diminishing continually by $\frac{1}{4}$, we have to the nearest integer 75, 56, 42, 32, 24, 18, 13, 10, for the living at the respective ages 92, 93, 94, 95, 96, 97, 98, 99, which in no part differs from the table by $\frac{1}{37}$ th part of the living at 92.

Art. 3. The near approximation in old age, according to some tables of mortality, leads to an observation, that if the law of mortality were accurately such that after a certain age the number of living corresponding to ages increasing in arithmetical progression, decreased in geometrical progression, it would follow that life annuities, for all ages beyond that period, were of equal value; for if the ratio of the number of persons living from one year to the other be constantly the same, the chance of a person at any proposed age living to a given number of years would be the same, whatever that age might be; and therefore the present worth of all the payments would be independent of the age, if the annuity were for the whole life; but according to the mode of calculating tables from a limited number of persons at the commencement of the term, and only retaining integer numbers, a limit is necessarily placed to the tabular, or indicative possibility of life; and the consequence may be, that the value of life annuities for old age, especially where they are

deferred, should be deemed incorrect, though indeed for immediate annuities, where the probability of death is very great, the limit of the table would not be of so much consequence, for the present value of the first payment would be nearly the value of the annuity.

Such a law of mortality would indeed make it appear that there was no positive limit to a person's age; but it would be easy, even in the case of the hypothesis, to show that a very limited age might be assumed to which it would be extremely improbable that any one should have been known to attain.

For if the mortality were, from the age of 92, such that $\frac{1}{4}$ of the persons living at the commencement of each year were to die during that year, which I have observed is nearly the mortality given in the Carlisle tables between the ages 92 and 99,* it would be above one million to one that out of three millions of persons, whom history might name to have reached the age of 92, not one would have attained to the age of 192, notwithstanding the value of life annuities of all ages above 92 would be of the same value. And though the limit to the possible duration of life is a subject not likely ever to be determined, even should it exist, still it appears interesting to dwell on a consequence which would follow, should the mortality of old age be as above described. For, it would follow that the non-appearance on the page of history of a single circumstance of a person having arrived

* If from the Northampton tables we take the numbers of living at the age of 88 to be 83, and diminish continually by $\frac{1}{4}$ for the living, at each successive age, we should have at the ages 88, 89, 90, 91, 92, the number of living 83; 61.3; 45.9; 34.4; 25.8; almost the same as in the Northampton table.

at a certain limited age, would not be the least proof of a limit of the age of man ; and further, that neither profane history nor modern experience could contradict the possibility of the great age of the patriarchs of the scripture. And that if any argument can be adduced to prove the necessary termination of life, it does not appear likely that the materials for such can in strict logic be gathered from the relation of history, not even should we be enabled to prove (which is extremely likely to be the state of nature) that beyond a certain period the life of man is continually becoming worse.

Art. 4. It is possible that death may be the consequence of two generally co-existing causes ; the one, chance, without previous disposition to death or deterioration ; the other, a deterioration, or an increased inability to withstand destruction. If, for instance, there be a number of diseases to which the young and old were equally liable, and likewise which should be equally destructive whether the patient be young or old, it is evident that the deaths among the young and old by such diseases would be exactly in proportion of the number of young to the old ; provided those numbers were sufficiently great for chance to have its play ; and the intensity of mortality might then be said to be constant ; and were there no other diseases but such as those, life of all ages would be of equal value, and the number of living and dying from a certain number living at a given earlier age, would decrease in geometrical progression, as the age increased by equal intervals of time ; but if mankind be continually gaining seeds of indisposition, or in other words, an increased liability to death (which appears not to be an unlikely supposition with respect to a great part of life, though

the contrary appears to take place at certain periods) it would follow that the number of living out of a given number of persons at a given age, at equal successive increments of age, would decrease in a greater ratio than the geometrical progression, and then the chances against the knowledge of any one having arrived to certain defined terms of old age might increase in a much faster progression, notwithstanding there might still be no limit to the age of man.

Art. 5. If the average exhaustions of a man's power to avoid death were such that at the end of equal infinitely small intervals of time, he lost equal portions of his remaining power to oppose destruction which he had at the commencement of those intervals, then at the age x his power to avoid death, or the intensity of his mortality might be denoted by aq^x , a and q being constant quantities; and if L_x be the number of living at the age x , we shall have $aL_x \times q \cdot \dot{x}$ for the fluxion of the number of deaths $= -(L_x) \cdot$; $\therefore abq^x = -\frac{\dot{L}_x}{L_x}$, $\therefore abq^x = -\text{hyp. log. of } b \times \text{hyp. log. of } L_x$, and putting the common logarithm of $\frac{1}{b} \times$ square of the hyperbolic logarithm of 10 $= c$, we have $c \cdot q^x = \text{common logarithm of } \frac{L_x}{d}$; d being a constant quantity, and therefore L_x or the number of persons living at the age of $x = d \cdot \overline{g}^x$; g being put for the number whose common logarithm is c . The reader should be aware that I mean \overline{g}^x to represent g raised to the power q^x and not g^q raised to the x power; which latter I should have expressed by $\overline{g^q}^x$, and which would evidently be equal to g^{q^x} . I take this opportunity to make this observation, as algebraists are sometimes not sufficiently precise in their notation of exponentials.

This equation between the number of the living, and the age, becomes deserving of attention, not in consequence of its hypothetical deduction, which in fact is congruous with many natural effects, as for instance, the exhaustions of the receiver of an air pump by strokes repeated at equal intervals of time, but it is deserving of attention, because it appears corroborated during a long portion of life by experience; as I derive the same equation from various published tables of mortality during a long period of man's life, which experience therefore proves that the hypothesis approximates to the law of mortality during the same portion of life; and in fact the hypothesis itself was derived from an analysis of the experience here alluded to.

Art. 6. But previously to the interpolating the law of mortality from tables of experience, I will premise that if, according to our notation, the number of living at the age x be denoted by L_x , and λ be the characteristic of a logarithm, or such that $\lambda(L_x)$ may denote the logarithm of that number, that if $\lambda(L_a) - \lambda(L_{a+r}) = m$, $\lambda(L_{a+r}) - \lambda(L_{a+2r}) = mp$, $\lambda(L_{a+2r}) - \lambda(L_{a+3r}) = m^2 p$; and generally $\lambda(L_{a+n-r}) - \lambda(L_{a+n}) = m \cdot p^{\frac{n}{r}-1}$; that by continual addition we shall have $\lambda(L_a) - \lambda(L_{a+n}) = m(1 + p + p^2 + p^3 + \dots + p^{\frac{n}{r}-1}) = m \cdot \frac{1-p^{\frac{n}{r}}}{1-p}$; and therefore if $p^{\frac{1}{r}} = q$, and ϵ be put equal to the number whose common logarithm is $\frac{m}{1-q^n}$, we shall have $\lambda(L_{a+n}) = \lambda(L_a) - \lambda(\epsilon) \times (1 - q^n) = \lambda\left(\frac{L_a}{\epsilon}\right) + \lambda(\epsilon) \cdot q^n$; $\therefore L_{a+n} = \frac{L_a}{\epsilon} \times \epsilon^{\lambda^{-1}(q^n)}$; and this equation, if for $a + n$ we write x , will give $L_x = \frac{L_a}{\epsilon} \cdot \epsilon^{\lambda^{-1}(q^{-a} \times q^x)}$; and consequently if $\frac{L_a}{\epsilon}$ be put

$= d$, and $\bar{\varepsilon}^{\bar{q}} = g$, the equation will stand $L_x = d \cdot \bar{g}^{\bar{q}^x}$, and $\lambda(g) = \lambda(\varepsilon) \times \bar{q}^{-a} = \frac{m \bar{q}^{-a}}{1 - \bar{q}^r}$; and I observe that when q is affirmative, and $\lambda(\varepsilon)$ negative, that $\lambda(g)$ is negative. The equation $L_x = d \cdot \bar{g}^{\bar{q}^x}$ may be written in general $\lambda(L_x) = \lambda(d) \pm$ the positive number whose common logarithm is $\{\lambda^2(g) \pm x \lambda(g)\}$, the upper or under sign to be taken according as the logarithm of g is positive or negative, λ^2 standing for the characteristic of a second logarithm; that is, the logarithm of a logarithm, $\lambda(q) = \frac{1}{r} \times \lambda(p)$, $\lambda^2(g) = \lambda^2(\varepsilon) - a \cdot \lambda(q) = \lambda\left(\frac{m}{1-p}\right) - a \cdot \lambda(q) = \lambda(m) - \lambda(1-p) - a \lambda(q)$; also $\lambda(d) = \lambda(L_a) - \frac{m}{1-p}$.

Art. 7. Applying this to the interpolation of the Northampton table, I observe that taking $a = 15$ and $r = 10$ from that table, I find $\lambda(L_a) - \lambda(L_{a+r}) = ,0566 = m$, $\lambda(L_{a+r}) - \lambda(L_{a+2r}) = ,0745$, $\lambda(L_{a+2r}) - \lambda(L_{a+3r}) = ,0915$, and $\lambda(L_{a+3r}) - \lambda(L_{a+4r}) = ,1228$; now if these numbers were in geometrical progression, whose ratio is p , we should have respectively $m = ,0566$; $m p = ,0745$; $m p^2 = ,0915$; $m p^3 = ,1228$. No value of p can be assumed which will make these equations accurately true; but the numbers are such that p may be assumed, so that the equation shall be nearly true; for resuming the first and last equations we have $p^3 = \frac{1228}{566}$; \therefore logarithm of $p = \frac{1}{3}(\text{logarithm of } 1228 - \text{logarithm of } 566) = ,11213$, $\therefore \lambda(q) = ,011213$ and $p = 1,2944$. And to examine how near this is to the thing required, continually to the logarithm of ,0566 namely $\bar{2},75282$, adding ,11213 which is the logarithm of p , we have respectively for the

logarithms of $m p$, of $m p^2$, of $m p^3$ the values $\bar{2},8649$, $\bar{2},9771$, $\bar{1},0892$; the numbers corresponding to which are ,07327; ,09486; ,1228; and consequently m , $m p$, $m p^2$, and $m p^3$ respectively equal to ,0566; ,07327; ,09486, and ,1228 which do not differ much from the proposed series ,0566; ,07327; ,09486, and ,1228; and according to our form for interpolation, taking $m = ,0566$ and $p = 1,2944$; we have $\frac{m}{1-p} = -\frac{,0566}{,2944} = -,1922$; and $\lambda(L_{15})$ agreeably to the Northampton tables, being $= 3,7342$ we have $\lambda(d) = 3,7342 + ,1922 = 3,9264$, $d = 8441$, $\lambda^2(q)$, that is to say, the logarithm of the logarithm of $q = \lambda\left(\frac{m}{1-p}\right) - a \lambda(q) = \bar{1},28375 - ,16819 = \bar{1},1156$, $\lambda(g) = -,130949 = \bar{1},8695$, the negative sign being taken because $\lambda(g) = \lambda(\epsilon) \times q^{-a} = \frac{m}{1-q} \cdot q^{-a}$, and $g = ,7404$. And therefore x being taken between the limits, we are to examine the degree of proximity of the equation $L_x = 8441 \times \sqrt[1,0261^x]{7404}$ or $\lambda(L_x)$, that is, the logarithm of the number of living at the age $x = 3,9264$ — number whose logarithm is $(\bar{1},11556 + x \times ,011213)$, as the logarithm of g is negative. The table constructed according to this formula, which I shall lay before the reader, will enable him to judge of the proximity it has to the Northampton table; but previously thereto shall show that the same formula, with different constants, will serve for the interpolations of other tables.

Art. 8. To this end let it be required to interpolate DEPARCIEUX'S tables, in Mr. BAILY'S life annuities, between the ages 15 and 55.

The logarithms of the living at the age of

15	are 2,92840	differences = ,03966	= $\lambda(L_{15}) - \lambda(L_{25})$
25	2,88874	,04738	= $\lambda(L_{25}) - \lambda(L_{35})$
35	2,84136	,04757	= $\lambda(L_{35}) - \lambda(L_{45})$
45	2,79379	,07280	= $\lambda(L_{45}) - \lambda(L_{55})$
55	2,72099		

Here the three first differences, instead of being nearly in geometrical progression are nearly equal to each other, showing from a remark above, that the living, according to these tables, are nearly in geometrical progression; and the reader might probably infer that this table will not admit of being expressed by a formula similar to that by which the Northampton table has been expressed between the same limits, but putting,

on the supposition of the possibility, though the thing cannot be accurately true,

$$\left\{ \begin{array}{l} \lambda(L_{15}) = \dots = 2,92840 \\ \lambda(L_{25}) = \lambda(L_{15}) - m \dots = 2,88874 \\ \lambda(L_{35}) = \lambda(L_{15}) - m - mp \dots = 2,84136 \\ \lambda(L_{45}) = \lambda(L_{15}) - m - mp - mp^2 \dots = 2,79379 \\ \lambda(L_{55}) = \lambda(L_{15}) - m - mp - mp^2 - mp^3 = 2,72099 \end{array} \right\} \text{ and we shall have}$$

$\lambda(L_{15}) - \lambda(L_{35})$ or its equal $m + mp = ,08704$, and $\lambda(L_{35}) - \lambda(L_{55})$ or its equal $p^2 \times m + pm = ,12037$; $\therefore p^2 = \frac{12037}{8704}$ and the log. of $p = \frac{\log. \text{ of } 12037 - \log. \text{ of } 8704}{2} = ,0703997$ and $p = 1,176$, $m = \frac{,08704}{1 + p} = \frac{,08704}{2,176} = ,04$. And to see how these values of m and p will answer for the approximate determination of the logarithms above set down of the numbers of living at the ages 15, 25, 35, 45, and 55, we have the following easy calculation by continually adding the logarithm of p

Logarithm of $m = \bar{2},6020600$		$\lambda(L_{15}) = 2,92840$
Log. of $p = 0,0703997$	therefore $mp = ,047039$	$-m = - ,04$
Log. of $mp = \bar{2},6724597$	$mp^2 = ,055317$	$\lambda(L_{25}) = 2,88840$
Log. of $mp^2 = 2,7428594$	$mp^3 = ,065051$	$-mp = - ,04704$
Log. of $mp^3 = 2,8128591$		<hr style="width: 100%;"/>
		2,84136
		$-mp^2 = - ,05532$
		<hr style="width: 100%;"/>
		2,78604
		$-mp^3 = - ,06505$
		<hr style="width: 100%;"/>
		2,72099

These logarithms of the approximate number of living at the ages 15, 25, 35, 45 and 55, are extremely near those proposed, and the numbers corresponding to these give the number of living at the ages 15, 25, 35, 45 and 55, respectively, 848; 773,4; 694; 612,3; and 526; differing very little from the table in Mr. BAILY'S life annuities; namely, 848; 774; 694; 622 and 526. And we have $a = 15$, $r = 10$, $m = ,04$; $\lambda(m) = \bar{2},60206$; $1 - p = - ,176$; $\lambda q = \frac{1}{10} \lambda(p) = ,00703997$; $\lambda(g) = \frac{m q^{-a}}{1-p} = - \frac{,04 \times q^{-a}}{,176}$, and is negative; $\lambda \lambda(g) = \lambda(,04) - 15 \times ,00704 - \lambda(,176) = \bar{1},25095$; $\lambda(d) = \lambda(L_a) - \frac{m}{1-p} = 2,9284 + ,22727 = 3,1557$; $\therefore \lambda(L_x) = 3,1557 -$ number whose log. is $(\bar{1},25095 + ,00704 x)$, for the logarithm of living in DEPARCIEUX' table in Mr. BAILY'S annuities, between the limits of age 15 and 55. The table which we shall insert will afford an opportunity of appreciating the proximity of this formula to the table.

Art. 9. To interpolate the Swedish mortality among males between the ages of 10 and 50, from the table in Mr. BAILY'S annuities :

Here $\lambda(L_{10}) = 3,779091$

$\lambda(L_{20}) = 3,746868$ to be assumed $= \lambda(L_{10}) - m$

$\lambda(L_{30}) = 3,703205$. . . $= \lambda(L_{10}) - m - mp$

$\lambda(L_{40}) = 3,648165$. . . $= \lambda(L_{10}) - m - mp - mp^2$

$\lambda(L_{50}) = 3,564192$. . . $= \lambda(L_{10}) - m - mp - mp^2 - mp^3$

Consequently $m + mp = \lambda(L_{10}) - \lambda(L_{30}) = ,075886$, and
 $\lambda(L_{30}) - \lambda(L_{50}) = p^2 \times \frac{m + mp}{1 + p} = ,139013$; therefore
 $p^2 = \frac{,139013}{,075886}$, and $\lambda(p) = ,1314468$; $\therefore p = 1,3535$; $m = \frac{,075886}{1 + p} =$
 $\frac{,075886}{2,3535}$; $\lambda(m) = \bar{2},5084775$; $m = ,032244$; $a = 10$; $r = 10$;

$\lambda(q) = ,01314468$; $\lambda g = \frac{m \cdot q^{-10}}{,3535}$, negative; $\lambda \lambda(g) = \lambda(m)$
 $- 10 \lambda(q) - \lambda(,3535) = \bar{2},82861$; $\lambda(d) = \lambda L_a - \frac{m}{1 - p} =$
 $3,779091 + ,091218 = 3,8703$; consequently this will give
 between the ages 10 and 50 of Swedish males,

$\lambda(L_x)$ or the logarithm of the living at the age of $x =$
 $3,8703 - \text{number, whose logarithm is } (\bar{2},82861 + ,013145 x)$.

A table will also follow to show the proximity of this with
 Mr. BAILY'S table.

Art. 10. For Mr. MILNE'S table of the Carlisle mortality
 we have, as given by that ingenious gentleman,

$$\lambda(L_{10}) = 3,81023$$

$$\lambda(L_{20}) = 3,78462$$

$$\lambda(L_{30}) = 3,75143$$

$$\lambda(L_{40}) = 3,70544$$

$$\lambda(L_{50}) = 3,64316$$

$$\lambda(L_{60}) = 3,56146$$

And the difference of these will form a series nearly in
 geometrical progression, whose common ratio is $\frac{4}{3}$, and in
 consequence of this, the first method may be adopted for the

interpolations. Thus because $\lambda(L_{10}) - (L_{20}) = ,02561$, the first term of the differences, and $\lambda(L_{50}) - \lambda(L_{60}) = ,0817$, the fifth term of the differences: take the common ratio $= \frac{817}{256} \Big| \frac{1}{2}$, and $m = ,0256$; $\therefore \lambda(m) = \bar{2},40824$. These will give $\lambda(p) = ,126$; $p = 1,3365$; $a = 10$, $r = 10$, $\lambda(q) = ,0126$, $\lambda(\epsilon) = \frac{m}{1-p} = -\frac{,0256}{,3365}$; $\therefore \lambda(g)$ negative; $\lambda \lambda g = 2,40824 - \lambda(,3365) - ,126 = \bar{2},75526$; and $\lambda(d) = \lambda(L_{10}) + \frac{,0256}{,3365} = 3,88631$, and accordingly, to interpolate the Carlisle table of mortality for the ages between 10 and 60, we have for any age x ,

$$\lambda(L_x) = 3,88631 - \text{number whose logarithm is } (\bar{2},88126 + ,0126 x).$$

Here we have formed a theorem for a larger portion of time than we had previously done. If by the second method the theorem should be required from the data of a larger portion of life, we must take r accordingly larger; thus if a be taken 10, $r = 12$, then the interpolation would be formed from an extent of life from 10 to 58 years; and referring to Mr. MILNE'S tables, our second method would give $\lambda(L_x) = 3,89063$ — the number whose logarithm is $(\bar{2},784336 + ,0120948 x)$; this differs a little from the other, which ought to be expected.

If the portion between 60 and 100 years of Mr. MILNE'S Carlisle table be required to be interpolated by our second method, we shall find $p = 1,86466$; $\lambda(m) = \bar{1},30812$; $m = ,20329$, &c. and we shall have $\lambda(L_x) = 3,79657$ — the number whose logarithm is $(\bar{3},74767 + ,02706 x)$.

This last theorem will give the numbers corresponding to the living at 60, 80, and 100, the same as in the table; but for the ages 70 and 90, they will differ by about one year:

the result for the age of 70 agreeing nearly with the living corresponding to the age 71; and the result for the age 90, agreeing nearly with the living at the age 89 of the Carlisle tables.

Art. 11. Lemma. If according to a certain table of mortality, out of a , persons of the age of 10, there will arrive b , c , d , &c. to the age 20, 30, 40, &c.; and if according to the tables of mortality, gathered from the experience of a particular society, the decrements of life between the intervals 10 and 20, 20 and 30, 30 and 40, &c. is to the decrements in the aforesaid table between the same ages, proportioned to the number of living at the commencement of those intervals respectively, as 1 to n , 1 to n' , 1 to n'' , &c. it is required to construct a table of mortality of that society, or such as will give the above data.

Solution. According to the first table, the decrements of life from 10 to 20, 20 to 30, 30 to 40, &c. respectively, will be found by multiplying the number of living at the commencement of each period by $\frac{a-b}{a}$, $\frac{b-c}{b}$, $\frac{c-d}{c}$, &c., and therefore, in the Society proposed, the corresponding decrements will be found by multiplying the number of living at those ages by $\frac{a-b}{a} n$; $\frac{b-c}{b} n'$; $\frac{c-d}{c} n''$ &c.; and the number of persons who will arrive at the ages 20, 30, 40, &c. will be the numbers respectively living at the ages 10, 20, 30, &c. multiplied respectively by $\frac{1-n \cdot a + nb}{a}$, $\frac{1-n' \cdot b + n'c}{b}$, $\frac{1-n'' \cdot c + n''d}{c}$, &c.; hence out of the number a , living at the age 10, there will arrive at the age 10, 20, 30, 40, 50, &c. the numbers $\frac{1-n \cdot a + nb}{a}$; $\frac{1-n \cdot a + nb}{a} \times \frac{1-n' \cdot b + n'c}{b}$; $\frac{1-n \cdot a + nb}{a} \times \frac{1-n' \cdot b + n'c}{b} \times \frac{1-n'' \cdot c + n''d}{c}$; &c. and the numbers for the intermediate ages must be found by interpolation.

In the ingenious Mr. MORGAN'S sixth edition of PRICE'S Annuities, p. 183, vol. i. it is stated, that in the Equitable Assurance Society, the deaths have differed from the Northampton tables; and that from 10 to 20, 20 to 30, 30 to 40, 40 to 50, 50 to 60, and 60 to 80, it appears that the deaths in the Northampton tables were in proportion to the deaths which would be given by the experience of that society respectively, in the ratios of 2 to 1; 2 to 1; 5 to 3; 7 to 5, and 5 to 4. According to this, the decrements in 10 years of those now living at the ages 10, 20, 30, and 40, will be the number living at those ages multiplied respectively by ,0478; ,0730; ,1024; ,1284; and the deaths in twenty years of those now living at the age of 60, would be the number of those living multiplied by ,3163. And also, taking, according to the Northampton table, the living at the age of 10 years equal to 5675, I form a table for the number of persons living at

the ages . . .	10	20	30	40	50	60	70	80
being	5675	5403,5	5010	4496	3919	3116	*	1197
and the log. of the number of persons living	3,75612	3,73268	3,69984	3,65283	3,59318	3,49360	*	

Consequently, if $a = 20$, $r = 10$, we have $\lambda(L_{20}) = 3,73268$; $\lambda(L_{40}) = \lambda(L_{20}) - m - mp = 3,65283$; $\lambda(L_{60}) = L_{20} - m - mp - \frac{m p^2}{1+p} - m p^3 = 3,49360$; $m \cdot 1 + p = ,07985$; and $m p^3 \times 1 + p = 3,65283 - 3,49360 = ,15923$; hence $\lambda(p) = \frac{1}{2} \lambda\left(\frac{,15923}{,07985}\right) = ,149875$; and $p = 1,412131$; $\lambda(m) = \lambda(,07985) - \lambda(2,41243) = \bar{2},519874$; and $m = ,033013$; $\therefore \lambda(\epsilon) = \frac{-m}{,412131}$ negative; $\therefore \lambda(g)$ is negative; $\lambda \lambda(g) = \lambda m - \lambda,412131 - ,0149875 \times 20 = \bar{2},6051$; $\lambda(d) = \lambda(L_{20}) - \lambda(\epsilon) = 3,73268 - ,080302 = 3,813$ sufficiently near; and our formula for the

mortality between the ages of 20 and 60, which appears to me to be the experience of the Equitable Society, is $\lambda(L_x) = 3,813$ —the number whose log. is $(\bar{2}.6051 + ,0149875x)$.

This formula will give

At the ages .	10	20	30	40	50	60	70	80
No. of living .	5703,2	5403,5	5007	4496	3862	3116	*	1500
Differs from the } proposed by }	28,2	0	+ 3	0	- 57	0	*	303

In the table of Art. 12, the column marked 1, represents the age; column marked 2, represents the number of persons living at the corresponding age; column marked 3, the error to be added to the number of living deduced from the formula, to give the number of living of the table for which the formula is constructed; column marked 4, gives the error in age, or the quantity to be added to the age in column 1, that would give the number of living in the original table, the same as in column 2. It may be proper to observe, that where the error in column 3 and 4 is stated to be 0, it is not meant to indicate that a perfect coincidence takes place, but that the difference is too small to be worth noticing.

CHAPTER II.

ARTICLE 1. The near proximity to the geometrical progression of the series expressing the number of persons living at equal small successive intervals of time during short periods, out of a given number of persons living at the commencement of those intervals, affords a very convenient mode of calculating values connected with life contingencies, for short limited periods; by offering a manner of forming general tables, applicable (by means of small auxiliary tables of the particular mortalities) to calculations for any particular mortality; and by easy repetition, to calculate the values for any length of period for any table of mortality we please.

If, for instance, it were required to find the value of an annuity of an unit for p years, on three lives of the age b, c, d , the rate of interest being such that the present value of an unit to be received at the expiration of one year, be equal to r , then the value of the first payment would be $\frac{L_{b+1}}{L_b} \times \frac{L_{c+1}}{L_c} \times \frac{L_{d+1}}{L_d} \times r$; and of the p^{th} payment the present value would be $\frac{L_{b+p}}{L_b} \times \frac{L_{c+p}}{L_c} \times \frac{L_{d+p}}{L_d} \times r^p$; but if $L_{b+p} = L_b \times \left(\frac{L_{b+1}}{L_b}\right)^p$ whether p be 1, 2, 3, &c. which will be the case when $L_b, L_{b+1}, L_{b+2},$ &c. form a geometrical progression, and similarly, if $L_{c+p} = L_c \times \left(\frac{L_{c+1}}{L_c}\right)^p$, and also, $L_{d+p} = L_d \times \left(\frac{L_{d+1}}{L_d}\right)^p$, the pre-

sent value of the p^{th} payment will be $\left(\frac{L_{1:b,c,d}}{L_{b,c,d}} r\right)^p$; hence, if $\frac{L_{1:b,c,d}}{L_{b,c,d}} r$ be put $= a$, the value of the annuity will be $a + a^2 + a^3 + a^4 \dots a^p = \frac{a-a^{p+1}}{1-a} = \frac{1-a^{p+1}}{a-1}$.

Art. 2. Consequently, let a general table be formed of the logarithm of $\frac{1-a^p}{a-1}$ for every value of the log. of a^p ; and also let a particular table be formed for every value of the log. of $\frac{L_{x+p}}{L_x}$ according to the particular table of mortality to be adopted; from the last table take the log. of $\frac{L_{b+p}}{L_b}$, $\frac{L_{c+p}}{L_c}$, $\frac{L_{d+p}}{L_d}$; and also from a table constructed for the purpose, take the log. of r^p , add these four logs. together, and the sum will be the log. of \overline{a}^p , which being sought for in the general table, will give the log. of $\left(\frac{1-a^p}{a-1}\right)$ which will be the log. of the annuity sought for the term p , on supposition of the geometrical progression being sufficiently near. Here I remark, that were it not for more general questions than the above, it would be preferable to have general tables formed for the values of $\frac{1-a^p}{a-1}$, instead of the log. of such values; but from the consideration that for most purposes a table of the logs. of $\frac{1-a^p}{a-1}$ will be found most convenient, I have had them calculated in preference.

Art. 3. The shorter the periods are, the nearer does the series of the number of persons living at the equal intervals of successive ages approximate to the geometrical progression; and consequently this mode, by the assumption of sufficiently short periods, and frequent repetitions, will answer

for any degree of accuracy the given table of mortality will admit of, but then the labour will be increased in proportion.

Art. 4. There are different modes of obviating, in a great measure, this inconvenience, by assuming an accommodated ratio for the given age, instead of the real ratio, from amongst which I shall only for the present select a few. The first is

as follows : find for every value of a , the log. of $\frac{1}{p} \left[\frac{L_{x+y}}{L_x} \right]^y$, that

is, the log. of $\frac{L_{x+1}}{L_x} + \frac{L_{x+2}}{L_x} + \frac{L_{x+3}}{L_x} \dots \dots \frac{L_{x+p}}{L_x}$; seek

this value in the general table, which will give the corresponding value of the log. of a^p ; and construct a table of such values for every value of x , and adopt these values for log. of a^p , instead of the abovenamed values of the log. of $\frac{L_{x+p}}{L_x}$,

for the determination of the values of the limited periods: the preference of this to the first proposed method consists in this; that if the series $\frac{1}{L_b} \times (L_{b+1} + L_{b+2} + L_{b+3} \dots L_{b+p}) =$

$\epsilon + \epsilon^2 + \epsilon^3 + \dots \epsilon^p$, the series $\frac{L_{b+1}}{L_b}, \frac{L_{b+2}}{L_b}, \&c.$ being nearly

in geometrical progression, and $\frac{L_{b+1}}{L_b} - \epsilon = \epsilon_1, \frac{L_{b+2}}{L_b} - \epsilon^2 = \epsilon_2,$
 $\&c. \epsilon_1, \epsilon_2, \&c.$ will be small, and $\epsilon_1 + \epsilon_2 + \epsilon_3 \dots \epsilon_p = 0$, and there-

fore, if the series $\frac{L_{c+1}}{L_c}, \frac{L_{c+2}}{L_c}, \frac{L_{c+3}}{L_c}, \&c.$ and $\frac{L_{d+1}}{L_d}, \frac{L_{d+2}}{L_d}, \&c.$

formed accurately geometrical progressions, and the value of $\frac{L_{c+1} \times L_{d+1}}{L_c \times L_d} . r = m$, the value of the annuity for the term,

would be accurately equal to $m \epsilon + m^2 \epsilon^2 + m^3 \epsilon^3 \dots \dots + m^p \epsilon^p + m \epsilon_1 + m^2 \epsilon_2 + m^3 \epsilon_3 \dots \dots + m^p \epsilon_p$, but because in

general $\frac{L_{c+1}}{L_c}$, $\frac{L_{d+1}}{L_d}$ and r differ very little from unity, m will not differ much from unity; and therefore if p be not great, m , m^2 , m^3 , &c. will not differ much from unity; and consequently, as ϵ_1 , ϵ_2 , ϵ_3 , &c. are small, $m\epsilon_1 + m^2\epsilon_2 + m^3\epsilon_3 \dots m^p\epsilon_p$ will not differ much from $\epsilon_1 + \epsilon_2 + \epsilon_3 + \dots + \epsilon_p$; but this has been shown to be 0; consequently $m\epsilon_1 + m^2\epsilon_2 + m^3\epsilon_3 \dots + m^p\epsilon_p$ differs very little from 0, or in other words is very small; and consequently, the value of the annuity differs very little from $m\epsilon + m^2\epsilon^2 + m^3\epsilon^3 \dots + m^p\epsilon^p$; and the same method of demonstration would apply with any one of the other ages, the remaining ages being supposed to possess the property of the accurate geometrical progression; notwithstanding this, however, as none of them probably will contain that property, but in an approximate degree, a variation in the above approximations may be produced of a small quantity of the second order; that is, if the order of the product of two small quantities; but, as in this approximation, I was only aiming at retaining the quantities of the first order, I do not consider this as affecting the result as far as the approximation is intended to reach: thus far with regard to the first accommodated ratios.

Art. 5. Moreover, on the supposition that L_c , L_{c+1} , L_{c+2} , \dots , L_{c+p} , and also L_d , L_{d+1} , L_{d+2} \dots , L_{d+p} are series in geometrical progression, and that $r \cdot \frac{L_{c+1}}{L_c} \times \frac{L_{d+1}}{L_d} = m = n.q.$ Since the annuity for p years on the three lives is equal to $\frac{L_{b+1}}{L_b} \cdot m + \frac{L_{b+2}}{L_b} \cdot m^2 + \dots + \frac{L_{b+p}}{L_b} \cdot m^p$ it follows

that if $\frac{L_{b+1}}{L_b}.n + \frac{L_{b+2}}{L_b}.n^2 + \frac{L_{b+3}}{L_b}.n^3 \dots \dots \dots \frac{L_{b+p}}{L_b}.n^p =$
 $\xi.n + \xi^2.n^2 + \xi^3.n^3 \dots \dots + \xi^p.n^p$ that if n be very nearly equal
to m , $\frac{L_{b+1}}{L_b}.n.q + \frac{L_{b+2}}{L_b}.n^2.q^2 + \&c. \dots \dots \dots \frac{L_{b+p}}{L_b}.n^p.q^p$ which
will be the value of the annuity on the three lives, will be
nearly $= \xi.n.q + \xi^2.n^2.q^2 + \&c. \dots \dots \dots \xi^p.n^p.q^p$. If q were
equal to unity, or, which is the same thing, $m=n$, the
equality would be accurate; but it may not be so when m
differs from 1; but the nearer n is to m , at least when the
difference does not exceed certain limited small quantities,
the nearer will be the coincidence. It appears therefore,
that if instead of taking the accommodated ratio for ξ^p so that
 $\frac{1}{L_b} \times (L_{b+1} + L_{b+2} + L_{b+3} \dots L_{b+p}) = \xi + \xi^2 + \xi^3 \dots \xi^p$
it will be preferable generally to take it so that $\frac{1}{L_b} \times (n.L_{b+1} +$
 $n^2.L_{b+2} + n.L_{b+3} \&c. \dots n^p.L_{b+p}) = \xi + \xi^2 + \xi^3 \&c. \dots \xi^p$ in
which n is between m and 1, the nearer m the better generally,
though possibly not universally so throughout the whole limit.
And the second method I use for increasing the accuracy, is to
adopt an accommodated ratio, or ξ^p , so that $\frac{1}{L_b} \times (1,05^{-1}L_{b+1} +$
 $1,05^{-2}L_{b+2} + \&c. \dots 1,05^p L_{b+p}) = 1,05^{-1} \xi + 1,05^{-2} \xi^2 + 1,05^{-3} \xi^3$
 $\dots 1,05^{-p} \xi^p$. Another method which might have its peculiar
advantage, is to assume $\xi^p = \frac{L_{b+\frac{1}{2}p}}{L_b}$ under the idea of using
a mean ratio.

The General Tables.*

Art. 6. I have had three general tables calculated for
fixed periods, Numbers 1, 2, and 3. Number 1, for pe-

* The chief of the arithmetical operations in the constructions of most of the
tables were performed under my direction, by Mr. DAVID JONES, of N^o. 10, King-
street, Soho; and, as far as my leisure would allow, I have endeavoured to assure
myself of their accuracy by different inspections.

riods of ten years; that is, for $\lambda \left(\frac{1-a^{10}}{a^{-1}-1} \right)$, corresponding to a given value of $\lambda (a^{10})$. N^o. 2, for seven years, or for $\lambda \left(\frac{1-a^7}{a^{-1}-1} \right)$, corresponding to $\lambda (a^7)$, and the 3d for five years, or for $\lambda \left(\frac{1-a^5}{a^{-1}-1} \right)$, corresponding to $\lambda (a^5)$; calculated (whether $p = 10, 7$ or 5) for every value of $\lambda (a^p)$, answering to $\bar{3},00$; $\bar{3},01$; $\bar{3},02$, &c. . . . 0. The first column containing the aforesaid value of $\lambda (a^p)$, corresponding to which, in an horizontal line, is placed the log. of $\frac{1-a^p}{a^{-1}-p}$, and between each successive value is placed the difference, retaining a decimal figure more; at the head of the other columns for the proportional parts of the differences, are placed a column showing the number of cyphers to be prefixed to the differences entered in the column following, which are headed

$\left\{ \begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 9 & 8 & 7 & 6 & 5 \end{matrix} \right\}$ nearest under 1, 2, 3, 4 and 5, and opposite the number; suppose $\bar{2},16$ of table log. of a^{10} , stands

$\left. \begin{matrix} 0275, 0550, 0826, 1101, \\ 2477, 2202, 1926, 1651, \end{matrix} \right\}$ the upper, with the addition of the two cyphers, give the proportional parts for ,001, 002, 003, ,004, 005: and the under, with the two cyphers, shows the proportional parts for ,009, 008, 007, 006; and the reason of choosing this arrangement, is the advantage which it offers of proof of correctness; thus the sum of the higher and lower numbers of each of the above row with the two cyphers = 002752, which is double ,001376, and equal to the whole difference between the successive terms.

Let it be required to find the logarithm of $\left(\frac{1-a^{10}}{a^{-1}-1} \right)$, corresponding to log. of $a^{10} = \bar{1}.7954$. In the General Table I,

Opposite to $\bar{1}.79$ we have . . .	,88868
For ,005 we have proportional part . . .	256
For ,0004 . . . ditto . . .	20

The sum . ,89144 is the answer.

If log. of a^p is less than $\bar{3},00$, then it will be necessary to calculate $\lambda\left(\frac{1-a^p}{a^p-1}\right)$ by common methods, as the tables do not go lower. And generally it will be then sufficient, omitting a^p , only to calculate the value of $-\lambda(a^{-1}-1)$; but from this, if more accuracy be required, subtract the number whose common logarithm is $(\bar{1},6378 + \lambda(r)^p)$.

If $\lambda\left(\frac{1-a^p}{a^p-1}\right)$ be given, and $\lambda(a)$ be required, proceed thus, $\lambda\left(\frac{1-a^{10}}{a^{10}-1}\right)$ being = ,89144 for example. In Table I, the next value of

$\lambda\left(\frac{1-a^{10}}{a^{10}-1}\right)$ is ,88868 to which $\lambda(a^{10})$ corresponding is $\bar{1},79$

Difference ,00256 belonging to $\bar{1},79$ gives . . . ,005

Difference ,00020 . . . ditto . . . ,0004

\therefore if $\lambda\left(\frac{1-a^{10}}{a^{10}-1}\right) = ,89144$ then we have $\lambda(a^{10}) = \bar{1},7954$

If $\lambda(a^p)$ is less than $\bar{3}$, proceed thus: put the given value of $\lambda\left(\frac{1-a^p}{a^p-1}\right) = \lambda q$, and we have the common logarithm of $a = -p \times \lambda(1 + q^{-1}) +$ a small correction if great accuracy be required; which correction is nearly equal to

$p \times$ the number whose common log. is $\{1,6378 - \lambda q - \overline{p+1} \cdot (1 + q^{-1})\}$

These methods and tables only apply immediately to $\lambda\left(\frac{1-a^p}{a^p-1}\right)$ when a is a proper fraction; but if a be greater than unity, put it equal to b^{-1} , then will b be a proper fraction;

but $\frac{1-a^p}{a^p-1} = \frac{a^p-1}{1-a^p} = \frac{b^{-p}-1}{1-b^{-p}} = b^{-p} \times \frac{1-b^p}{b^{-1}-1} = a^{p+1} \times \left(\frac{1-b^p}{b^{-1}-1}\right)$; conse-

quently $\lambda\left(\frac{1-a^p}{a^p-1}\right) = \overline{p+1} \cdot \lambda(p) + \lambda\left(\frac{1-b^p}{b^{-1}-1}\right)$ I have likewise

had Table IV. calculated, which is a general table, for the com-

mon log. of $\left(\frac{1}{a^{-1}-1}\right)$, corresponding to a given value of λa ,

commencing with $\lambda(a) = 1.7; 1,701; 1,702, \&c.$ with the differences between them. I have not, in this table, had the proportional parts inserted, though it would be attended with advantage, as the table is not meant to be of general use; but only given to be applied for rough purposes, or where accuracy is not particularly required for calculating at once the value of a life annuity for the whole term of life, or the whole remaining terms of life, after a given term, by considering the present value of each successive payment to form the successive terms of a geometrical progression whose first term and common ratio are each equal to a . And as $\lambda\left(\frac{1}{a-1}\right)$ will represent the log. of the sum of the said geometrical progression, it will likewise express approximatively the logarithm of the value required. For many purposes, a table of $\frac{1}{a-1}$, answering to given values of a , would be preferable, but not for general purposes.

Art. 7. I have already, in Art. 4 and 5, Chap. II, introduced the term accommodated ratios, or chances, and endeavoured to explain the methods to be adopted to reap the advantage of the ideas there expressed. Table V, for Carlisle, Deparcieux, and Northampton, are the logarithms of tenth terms of the accommodated ratios, or the logarithms of the accommodated chances for living ten years, calculated according to a mode laid down in Art. 5, Chap. II; that is, it expresses for every age, or value of b , the logarithm of ϵ^{10} , when $\frac{1}{L_b} \times (1,05^{-1} L_{b+1} + 1,05^{-2} L_{b+2} + \&c. \dots 1,05^{-p} L_{b+p})$ is equal to $1,05^{-1} \epsilon + 1,05^{-2} \epsilon^2 + \&c. \dots 1,05^{-10} \epsilon^{10}$. and to show, by example, how these are calculated, let it be required to find the logarithm of the accommodated chance for living

ten years, for the age 20, calculated according to the Carlisle table upon the consideration of interest at 5 per cent. Accord-

ing to the Carlisle tables, I find $\lambda_{10}^{\frac{1,05^{-1}}{1}} \overline{20}$; that is, the logarithm of the annuity of one pound on a life of 20, for ten years, at 5 per cent = ,87176, and putting $a = 105^{-1} \cdot \beta$, by hypothesis

we shall have $\lambda_{10}^{\frac{x}{1}} a^x$; that is the logarithm of $(a + a^2 + a^3 \dots a^{10}) = ,87176$; that is, $\lambda \left(\frac{1-a^{10}}{a^{-1}-1} \right) = ,87176$; hence proceeding, as shown above, to find from General Table I. $\lambda(a^{10})$

Having given . . . ,87176 = $\lambda \left(\frac{1-a^{10}}{a^{-1}-1} \right)$

We have next less = ,86842 corresponding to 1.75

,00334 difference

,00302 proportional part ,006

30 . . ditto ,0006

2 . . ditto ,00004

,87176 corresponds to . $\lambda(a^{10}) = \overline{1.75664}$
 $\lambda(1,05^{10}) = \underline{\underline{,21189}}$

$\overline{1.96853}$ for the log. of the accommodated chance to live 10 years at the Carlisle mortality.

In the same way may the accommodated chance be found for any other term, when general tables for the term are constructed, and from any other base of interest. I may observe, that by using different rates of interest, as a base for determining the accommodated chances, different degrees of accuracy may be obtained. See Art. 5. Chap. II.

Art. 8. Table VI. is the logarithm of the accommodated chances \mathcal{E} at every age, b for living one year, where \mathcal{E} is of such value that the sum of the geometrical progression $\frac{\mathcal{E}}{1,05} + \frac{\mathcal{E}^2}{1,05^2} + \&c.$ ad infinitum, or, which is the same thing,

$\frac{1}{\frac{1.05}{6}-1}$ shall be equal to the value of the whole life annuity at five per cent. at such age, namely $\overset{1.05^{-1}}{\underset{1}{\lceil} b}$; consequently $\frac{6}{1.05^{-1}} \times$

$$\left(1 + \overset{1.05^{-1}}{\underset{1}{\lceil} b}\right) = \overset{1.05^{-1}}{\underset{1}{\lceil} b}; \therefore \lambda \text{ } \mathcal{E} = \lambda \left(\overset{1.05^{-1}}{\underset{1}{\lceil} b}\right) + \lambda(1.05) - \lambda \left(\overset{1.05^{-1}}{\underset{0}{\lceil} b}\right).$$

This table is constructed for Carlisle, Deparcieux, and Northampton, and is to be used in conjunction with Table IV., where only a rough value of the contingency is required; and though this table applies as the other tables of accommodated chances, to different rates of interest, still it would be of advantage more particularly *here* for the greater approximation to have similar tables constructed from the

formula $\lambda(\mathcal{E}) = \lambda \left(\overset{r}{\underset{1}{\lceil} b}\right) + \lambda(r^{-1}) - \lambda \left(\overset{r}{\underset{0}{\lceil} b}\right)$ for different values of r .

Art. 9. In calculating the value of life annuities for long periods, by means of adding together the values of portions of those periods, the portions of the distant periods contain factors of the real chance of living to these periods, and likewise of the discounted value of the money of which the payment is not immediate; thus if t be greater than 10,

$$\overset{r}{\underset{t}{\lceil} a, b, c} = \overset{r}{\underset{10}{\lceil} a, b, c} + \overset{r}{\underset{t}{\lceil} a, b, c} = \overset{r}{\underset{10}{\lceil} a, b, c} + \frac{L_{a, b, c}}{L_{a, b, c}} \cdot r^{10} \times$$

$\overset{r}{\underset{(t-10)}{\lceil} a+10, b+10, c+10}$. It will be therefore convenient to have a table of the logarithm of the real chance of living 10, 20, 30 years, &c. and also for other terms; and some of these are given by Tables VII., VIII., IX.

Time will not allow me, for the present, to offer more than a very few examples of the method to be employed in calculating by these tables, which are as follow :

Example 1. Required, according to the Carlisle table, the value of a life annuity, for ten years, on the joint lives 30 and 40, at 3 per cent interest.

In Table VIII. for Carlisle, log. of accommodated	
chance for 10 years, at the age 30 . . .	= $\bar{1}.9552$
Ditto 40	= $\bar{1}.9383$
Ditto λ 1,03	= $\bar{1}.8716$
	Sum . . . $\bar{1}.7651 = \lambda (a^{10})$

In Table I, $\bar{1}.76$ corresponds to8734
In proportional parts ,005 corresponds to	.253
Ditto . . . 0001 corresponds to	. 5
	Consequently $\bar{1}.7651$ corresponds to . . .87604

which is the log. of the required value : the number corresponding to this is 7,5169, for the value of the annuity, according to the Carlisle mortality, at 3 per cent. on the joint lives 30 and 40; and by calculation from Mr. MILNE's tables, I find the value should be 7,5168 ; the difference of the two is evidently insignificant. In this way I calculated the log. of the value of the life annuity, at the Carlisle mortality, at 3 per cent. for 10 years, for the joint lives 0 and 10, 10 and 20, 20 and 30, 30 and 40, 40 and 50, 50 and 60, to be ,76580 ; ,90247 ; ,89139 ; ,87604 ; ,86295 ; ,81067 ; and the annuity, or the numbers corresponding to the said logarithms,

5,8318 ; 7,9874 ; 7,7874 ; 7,5169 ; 7,2937 ; 6,4665 ;

and, according to calculation from Mr. MILNE's tables, I get

5,8595 ; 7,992 ; 7,7906 ; 7,5168 ; 7,2916 ; 6,4679.

The difference between the two sets is insignificant, except

perhaps in the values of $\frac{1}{10} \overbrace{1}^{1.05^{-1}} \underline{0, 10}$; that is, the value of the annuity on the joint life of a child just born, with one of the age of 10, at 3 per cent. Had we divided the period in portions, the value might have been obtained as near as we pleased; or we should likewise have obtained greater accuracy, had we assumed an accommodated chance deduced at a more appropriate interest than 5 per cent. See Art. 5, Chap. II.

Example 2. Let it be required to find the value of a life annuity at 3 per cent. for 10 years, at the Carlisle mortality, for the five lives of the age 20, 30, 40, 45 and 50.

In Table VIII. log. of accom. chance for 10 years at age 20 = $\bar{1}.9685$

Ditto	30 = $\bar{1}.9552$
Ditto	40 = $\bar{1}.9383$
Ditto	45 = $\bar{1}.9367$
Ditto	50 = $\bar{1}.9292$
					$\lambda 1.05^{-10} = \bar{1}.8716$
					$\lambda (a^{10}) = \bar{1}.5995$

This sought in Table I.; thus, $\bar{1}.59$ giving ,79035
 ,009 427
 ,0005 23

gives ,79485 the N^o to which log. is 6,2352

for the value of $\frac{1}{10} \overbrace{1}^{1.03^{-1}} \underline{20, 30, 40, 45, 50}$.

Example 3. Let it be required to find the value of $\frac{1}{1} \overbrace{1}^{1.03^{-1}} \underline{b, b+10}$ Carlisle mortality, when $b = 10$, that is, for the whole joint lives of 10 and 20. By dividing the whole in portions of ten

years, the operation will stand thus for $\frac{1}{10} \overbrace{1}^{1.03^{-1}} \underline{b, b+10}$.

	b=10	b=20	b=30	b=40	b=50	b=60	b=70	b=80	
Log. of accom. ratio } for 10 years = } $\lambda(1,03^{-10}) =$	$\bar{1}.9768$ $\bar{1}.9685$ $\bar{1}.8716$	$\bar{1}.9685$ $\bar{1}.9552$ $\bar{1}.8716$	$\bar{1}.9552$ $\bar{1}.9383$ $\bar{1}.8716$	$\bar{1}.9383$ $\bar{1}.9292$ $\bar{1}.8716$	$\bar{1}.9292$ $\bar{1}.8318$ $\bar{1}.8716$	$\bar{1}.8318$ $\bar{1}.6689$ $\bar{1}.8716$	$\bar{1}.6689$ $\bar{1}.3134$ $\bar{1}.8716$	$\bar{1}.3134$ $\bar{2}.6695$ $\bar{1}.8716$	} from Tab.VIII. Carlisle.
sum . . =	$\bar{1}.8169$	$\bar{1}.7953$	$\bar{1}.7651$	$\bar{1}.7391$	$\bar{1}.6326$	$\bar{1}.3723$	$\bar{2}.8539$	$\bar{3}.8545$	
No ^s corresponding } to sum in Table I. }	.90247	.89139	.87604	.86295	.81067	.69156	.48781	.19146	
Log. of ratios for 10 years = } $\lambda 1,03^{-20}$. . =	$\bar{1}.97438$ $\bar{1}.96681$ $\bar{1}.87163$	$\bar{1}.94120$ $\bar{1}.92082$ $\bar{1}.74325$	$\bar{1}.89520$ $\bar{1}.85854$ $\bar{1}.61488$	$\bar{1}.83292$ $\bar{1}.77684$ $\bar{1}.48651$	$\bar{1}.75123$ $\bar{1}.59577$ $\bar{1}.35814$	$\bar{1}.57016$ $\bar{1}.19448$ $\bar{1}.22977$	$\bar{1}.16886$ $\bar{2}.36767$ $\bar{1}.10139$		
The log. of the present } worth of each portion }		.70421	.48131	.23157	$\bar{1}.90694$	$\bar{1}.39670$	$\bar{2}.48222$	$\bar{4}.82938$	

And the present worth of each, or the numbers corresponding to the last logarithms are arranged below.

For first 10 years	7.9886	As the method by which the logarithms of the present worth of the different portions are found, may not be seen by every reader, I will explain the operation in the third portion; that is, when the logarithm of the portion first found is anticipated for 20 years. Resume Table VII. log. of real chance for age } 10 living 20 years . . . } Ditto 20 years living . . . } $\lambda(1,03^{-20})$ }	.87604
2nd ditto	5.0607		$\bar{1}.94120$
3d d ^o	3.0291		$\bar{1}.92082$
4th d ^o	1.7044		$\bar{1}.74325$
5th d ^o	.8071		
6th d ^o	.2492		
7th d ^o	.0303		
8th d ^o	.0007		
sum	18.8701	.48131	

which differs but insignificantly from Mr. MILNE's table, which gives 18.873. In a similar way, I find the value of the joint lives for ages 20 and 30, at 3 per cent. and Carlisle mortality to be 16.745; which, according to Mr. MILNE's table, should be 16.749; which appears to be an insignificant difference.

Example 4. To find, when particular accuracy is not required, according to the formula for the whole of life,

the approximate value of $\frac{1,03^{-1}}{1} \int_a^{a+10}$ at the Carlisle mortality, when $a = 10, 20, 30,$ &c. call the logarithm of accommodated ratios for an unlimited time at the age a, R_a standing for the accommodated ratio in Table VI. at the age a .

$a =$	10	20	30	40	50	60	70	80	90
R_a	$\bar{1}.99529$	$\bar{1}.99455$	$\bar{1}.99265$	$\bar{1}.98991$	$\bar{1}.98546$	$\bar{1}.97514$	$\bar{1}.95755$	$\bar{1}.92461$	$\bar{1}.86660$
R_{a+10}	$\bar{1}.99455$	$\bar{1}.99265$	$\bar{1}.98991$	$\bar{1}.98546$	$\bar{1}.97514$	$\bar{1}.95755$	$\bar{1}.92461$	$\bar{1}.86660$	$\bar{1}.81282$
$\lambda 1,03^{-1}$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$
	$\bar{1}.97700$	$\bar{1}.97436$	$\bar{1}.96972$	$\bar{1}.96253$	$\bar{1}.94776$	$\bar{1}.91985$	$\bar{1}.86932$	$\bar{1}.77837$	$\bar{1}.66658$
Log. which corresponds to	$\bar{1}.26451$	$\left. \begin{array}{l} 1.20975 \\ .00631 \end{array} \right\}$	$\left. \begin{array}{l} 1.13083 \\ .01062 \end{array} \right\}$	$\left. \begin{array}{l} 1.03886 \\ .00641 \end{array} \right\}$	$\left. \begin{array}{l} .88674 \\ .00667 \end{array} \right\}$	$\left. \begin{array}{l} .68817 \\ .00502 \end{array} \right\}$	$\left. \begin{array}{l} .45337 \\ .00123 \end{array} \right\}$	$\left. \begin{array}{l} .17571 \\ .00093 \end{array} \right\}$	
		1.21606	1.14145	1.04527	$.89341$	$.69319$	$.45460$	$.17664$	
Numbers . .	18.387	16.446	13.850	11.099	7.824	4.9339	2.8485	1.5019	
Instead of .	18.873	16.749	14.449	11.954	8.729	5.565	3.229	1.589	

To find the value corresponding to $\bar{1}.66658$, not in the table, find the number corresponding complement of the log. $\bar{1}.6658$, which number is 2,159; subtract 1, and find the complement of the log. which is $= \bar{1}.9359165$, whose number is ,8628. Mr. MILNE's table gives .979. But as it is not always the same rate of interest which gives the best accommodated ratios, in order to try when, for instance, the interest of money is 3 per cent. what rate of interest should be used in determining the ratios, use the following table:*

Interest.

1.08	$\lambda (1.08^{-1} \times 1.03) = \bar{1}.979$	} nearly;
1.07	$\lambda (1.07^{-1} \times 1.03) = \bar{1}.983$	
1.06	$\lambda (1.06^{-1} \times 1.03) = \bar{1}.987$	
1.05	$\lambda (1.05^{-1} \times 1.03) = \bar{1}.991$	
1.04	$\lambda (1.04^{-1} \times 1.03) = \bar{1}.996$	

* This is not given as a perfect and unerring rule, but as a method in many cases useful, and which would be perfect for the accommodated ratio of one of the lives, if the other lives followed an exact geometrical ratio throughout; and that the real geometrical ratios were in that case used for them, provided that instead of comparing the said sum with the small table, we take for the base of interest the number whose logarithm is $-\lambda (1,03)$, when the interest is 3 per cent.; and it is to be recollected that the methods is only given as a rough approximation.

Add the logarithm of accommodated ratios, as given in the Table VI. of all the lives but one in question, together, and see which of those rates of interest it nearest agrees with, and use that to calculate the life left, and proceed so for

every life ; thus for $\overset{1,03^{-1}}{1} \left| \begin{array}{l} 30, 40 \end{array} \right.$; to find the rate of interest for 30, I observe that $R_{40} = \bar{1}.9899$ agrees nearest with 6 per cent. in the little table, and $R_{30} = \bar{1}.99265$ agrees nearest with 5 per cent., I therefore take 6 per cent. for the age 30, and for the other I take 5 per cent. : proceed thus :

Example 5.

R if calculated at 6 per cent.	$\bar{1}.99316$
R_{40} per table	$\bar{1}.98991$
$\lambda 1,05^{-1}$	$= \bar{1}.98716$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$\bar{1}.97023$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
Proportionate parts	1.14558
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
To which logarithm	1.14885
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
The N ^o corresponding is	14.088
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
Instead of	14.449

Example 6.

	$\overset{1,03^{-1}}{1} \left \begin{array}{l} 40, 50 \end{array} \right.$
R at 6 per cent.	$\bar{1}.99060$
R_{40} at 6 per cent.	$\bar{1}.98632$
R_{50} at 6 per cent.	$\bar{1}.98716$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$\bar{1}.96408$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$\bar{1}.05336$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$.00102$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	1.06438
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	11.598
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	Instead of 11.954

Example 7.

	$\overset{1,03^{-1}}{1} \left \begin{array}{l} 50, 60 \end{array} \right.$
R at 8 per cent.	$= \bar{1}.98759$
R_{50} at 6 per cent.	$= \bar{1}.97599$
R_{60} at 6 per cent.	$= \bar{1}.98716$
$\lambda 1,03^{-1}$	$= \bar{1}.98716$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$\bar{1}.95074$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$.91357$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$.00687$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
	$.92044$
	<hr style="width: 50%; margin-left: auto; margin-right: 0;"/>
which log. corresponds	$.8318$
instead of	$.8729$

I observe that I have not given any table of the logarithm of the accommodated ratios for an unlimited term, except that calculated with 5 per cent. as a radix ; but by the assistance of a table of life annuities, for single life at different rates per cent., this will enable us, independent of certain exceptions, to derive the quantity for the same rates per cent. for any radix at the per cent. contained in the second table ; thus to find R Carlisle mortality, radix 8 per cent. I look to the Carlisle table of single lives at 8 per cent., and I find the value of the annuity on the life of 50 = 8.987, I search the age to which this will correspond at 5 per cent. and I find sufficiently nearly 59,82 for the age corresponding, to which from my table (with the radix at 5 per cent.) for the log. of ratios I find $\bar{1}.97536$; to this I add log. of $\frac{1.08}{1.05}$; that is, ,01223, and we get $\bar{1}.98759$, the same as given on the other side. This method is accurately consistent with the definition of accommodated ratios for unlimited periods ; and if this description of accommodated ratios at a certain rate per cent. be given for one table, for which at the same rate per cent. we have the value of single lives, we may find the same description of accommodated ratios for any other table of mortality for which, at the same rate per cent. we have a table of the value of single lives : thus, suppose the logarithm of this description of accommodated ratios be given for the Carlisle table at five per cent., and the same be required for the Northampton for the age 60, at the same rate ; $\frac{1.05}{1} \overline{1} 60$ Northampton = 8,392, this being sought in the Carlisle

table for $1.05^{-1} \int_1^x$ gives $x = 62,41$ for the corresponding age; seek the logarithm of accommodated ratios for an unlimited term, corresponding to this for Carlisle, for the age 6,241, and we have $\bar{1}.9723$, agreeing with the table given.

Previously to concluding this chapter, I shall add a small table, which will be found very useful in the application of the methods here proposed.

n	Log. of $1,03^{-n}$	Log. of $1,035^{-n}$	Log. of $1,04^{-n}$	Log. of $1,045^{-n}$	Log. of $1,05^{-n}$
1	$\bar{1}.9871628$	$\bar{1}.9850597$	$\bar{1}.9829667$	$\bar{1}.9808837$	$\bar{1}.9788107$
2	$\bar{1}.9743256$	$\bar{1}.9701193$	$\bar{1}.9659333$	$\bar{1}.9617674$	$\bar{1}.9576214$
3	$\bar{1}.9614883$	$\bar{1}.9551790$	$\bar{1}.9489000$	$\bar{1}.9426511$	$\bar{1}.9364321$
4	$\bar{1}.9486511$	$\bar{1}.9402386$	$\bar{1}.9318666$	$\bar{1}.9235348$	$\bar{1}.9152428$
5	$\bar{1}.9358139$	$\bar{1}.9252983$	$\bar{1}.9148333$	$\bar{1}.9044185$	$\bar{1}.8940535$
6	$\bar{1}.9229767$	$\bar{1}.9103579$	$\bar{1}.8978000$	$\bar{1}.8853023$	$\bar{1}.8728642$
7	$\bar{1}.9101394$	$\bar{1}.8954176$	$\bar{1}.8807666$	$\bar{1}.8661860$	$\bar{1}.8516749$
8	$\bar{1}.8973022$	$\bar{1}.8804772$	$\bar{1}.8637333$	$\bar{1}.8470697$	$\bar{1}.8304856$
9	$\bar{1}.8844650$	$\bar{1}.8655369$	$\bar{1}.8466999$	$\bar{1}.8279534$	$\bar{1}.8092963$
10	$\bar{1}.8716278$	$\bar{1}.8505965$	$\bar{1}.8296666$	$\bar{1}.8088371$	$\bar{1}.7881070$

General Table I. $\lambda(a^{10}), \lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$.

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$		1 9	2 8	3 7	4 6	5
$\bar{3}.00$,00163	,00	0200	0399	0599	0798	0998	$\bar{3}.25$,05295	,00	0212	0423	0635	0847	1059
	,00199,6		1796	1597	1397	1198			,00211,7		1905	1694	1482	1270	
$\bar{3}.01$,0036,2		0200	0401	0601	0801	1002	$\bar{3}.26$,05506		0212	0424	0637	0849	1061
	,00200,3		1803	1602	1402	1202			,00212,2		1910	1698	1485	1273	
$\bar{3}.02$,00563		0201	0401	0602	0802	1003	$\bar{3}.27$,05719		0213	0425	0638	0851	1064
	,00200,6		1805	1605	1404	1204			,00212,7		1914	1702	1489	1276	
$\bar{3}.03$,00763		0201	0402	0603	0804	1005	$\bar{3}.28$,05931		0213	0426	0640	0853	1066
	,00201,0		1809	1608	1407	1206			,00213,2		1919	1706	1492	1279	
$\bar{3}.04$,00964		0202	0403	0605	0806	1008	$\bar{3}.29$,06144		0214	0427	0641	0855	1069
	,00201,5		1814	1612	1411	1209			,00213,7		1923	1710	1496	1282	
$\bar{3}.05$,01166		0202	0404	0606	0808	1010	$\bar{3}.30$,06358		0214	0429	0643	0857	1072
	,00202,0		1818	1616	1414	1212			,00214,3		1929	1714	1500	1286	
$\bar{3}.06$,01368		0202	0405	0607	0810	1012	$\bar{3}.31$,06572		0215	0430	0644	0859	1074
	,00202,4		1822	1619	1417	1214			,00214,8		1933	1718	1504	1289	
$\bar{3}.07$,01570		0203	0406	0608	0811	1014	$\bar{3}.32$,06787		0215	0431	0646	0861	1077
	,00202,8		1825	1622	1420	1217			,00215,3		1938	1722	1507	1292	
$\bar{3}.08$,01773		0203	0407	0610	0814	1017	$\bar{3}.33$,07002		0216	0432	0648	0864	1080
	,00203,4		1831	1627	1424	1220			,00216,0		1944	1728	1512	1296	
$\bar{3}.09$,01976		0204	0408	0611	0815	1019	$\bar{3}.34$,07218		0216	0433	0649	0866	1082
	,00203,8		1834	1630	1427	1223			,00216,4		1948	1731	1515	1298	
$\bar{3}.10$,02180		0204	0409	0613	0817	1022	$\bar{3}.35$,07435		0217	0434	0651	0868	1085
	,00204,3		1839	1634	1430	1226			,00217,0		1953	1736	1519	1302	
$\bar{3}.11$,02384		0205	0409	0614	0819	1024	$\bar{3}.36$,07652		0218	0435	0653	0870	1089
	,00204,7		1842	1638	1433	1228			,00217,5		1958	1740	1523	1305	
$\bar{3}.12$,02589		0205	0410	0616	0821	1026	$\bar{3}.37$,07869		0218	0436	0654	0872	1090
	,00205,2		1847	1642	1436	1231			,00218,0		1962	1744	1526	1308	
$\bar{3}.13$,02794		0206	0411	0617	0823	1029	$\bar{3}.38$,08087		0219	0437	0656	0875	1094
	,00205,7		1851	1646	1440	1234			,00218,7		1968	1750	1531	1312	
$\bar{3}.14$,03000		0206	0412	0618	0824	1031	$\bar{3}.39$,08306		0219	0438	0658	0877	1096
	,00206,1		1855	1649	1443	1237			,00219,2		1973	1754	1534	1315	
$\bar{3}.15$,03206		0207	0413	0620	0826	1033	$\bar{3}.40$,08525		0220	0439	0659	0879	1099
	,00206,5		1859	1654	1446	1239			,00219,7		1977	1758	1538	1318	
$\bar{3}.16$,03407		0207	0415	0622	0829	1037	$\bar{3}.41$,08745		0220	0441	0661	0882	1102
	,00207,3		1866	1658	1451	1244			,00220,4		1984	1763	1543	1322	
$\bar{3}.17$,03620		0208	0415	0623	0830	1038	$\bar{3}.42$,08965		0221	0442	0663	0884	1105
	,00207,6		1868	1661	1453	1246			,00221,0		1989	1768	1547	1326	
$\bar{3}.18$,03827		0208	0416	0624	0832	1041	$\bar{3}.43$,09186		0221	0443	0664	0886	1107
	,00208,1		1873	1665	1457	1249			,00221,4		1993	1771	1551	1328	
$\bar{3}.19$,04036		0209	0417	0626	0834	1043	$\bar{3}.44$,09408		0222	0444	0666	0888	1111
	,00208,6		1877	1669	1460	1252			,00222,1		1999	1777	1555	1333	
$\bar{3}.20$,04244		0209	0418	0627	0836	1046	$\bar{3}.45$,09630		0223	0445	0668	0890	1113
	,00209,1		1882	1673	1464	1255			,00222,6		2003	1781	1558	1336	
$\bar{3}.21$,04453		0210	0419	0629	0838	1048	$\bar{3}.46$,09852		0223	0446	0670	0893	1116
	,00209,6		1886	1677	1467	1258			,00223,2		2009	1786	1562	1339	
$\bar{3}.22$,04663		0210	0420	0630	0840	1051	$\bar{3}.47$,10076		0224	0448	0671	0895	1119
	,00210,1		1891	1681	1471	1261			,00223,8		2014	1790	1567	1343	
$\bar{3}.23$,04873		0211	0421	0632	0842	1053	$\bar{3}.48$,10300		0224	0449	0673	0898	1122
	,00210,6		1895	1685	1474	1264			,00224,4		2020	1795	1571	1346	
$\bar{3}.24$,05084		0211	0422	0633	0844	1056	$\bar{3}.49$,10524		0225	0450	0675	0900	1125
	,00211,1		1900	1689	1478	1267			,00225,0		2025	1800	1575	1350	

General Table I. $\lambda(a^{10}), \lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$.

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$	1 9	2 8	3 7	4 6	5		
$\bar{3}.50$,10749 ,00225,6	,00	0226 2030	0451 1805	0677 1579	0902 1354	1128	$\bar{3}.75$,16581 ,00242,0	,00	0242 2178	0484 1936	0726 1694	0968 1452	1210
$\bar{3}.51$,10975 ,00226,2		0226 2036	0452 1810	0679 1583	0905 1357	1131	$\bar{3}.76$,16823 ,00242,7		0243 2184	0485 1942	0728 1699	0971 1456	1214
$\bar{3}.52$,11201 ,00226,8		0227 2041	0454 1814	0680 1588	0907 1361	1134	$\bar{3}.77$,17065 ,00243,5		0244 2192	0487 1948	0731 1705	0974 1461	1218
$\bar{3}.53$,11428 ,00227,5		0228 2048	0455 1820	0683 1593	0910 1365	1138	$\bar{3}.78$,17309 ,00244,2		0244 2198	0488 1954	0733 1709	0977 1465	1221
$\bar{3}.54$,11655 ,00228,1		0228 2053	0456 1825	0684 1597	0912 1369	1141	$\bar{3}.79$,17553 ,00244,9		0245 2204	0490 1959	0735 1714	0980 1469	1225
$\bar{3}.55$,11883 ,00228,7		0229 2058	0457 1830	0686 1601	0915 1372	1144	$\bar{3}.80$,17798 ,00245,6		0246 2210	0491 1965	0737 1719	0982 1474	1228
$\bar{3}.56$,12112 ,00229,2		0229 2063	0458 1834	0688 1604	0917 1375	1146	$\bar{3}.81$,18044 ,00246,4		0246 2218	0493 1971	0739 1725	0986 1478	1232
$\bar{3}.57$,12341 ,00230,1		0230 2071	0460 1841	0690 1611	0920 1381	1151	$\bar{3}.82$,18290 ,00247,1		0247 2224	0494 1977	0741 1730	0988 1482	1236
$\bar{3}.58$,12571 ,00230,6		0231 2075	0461 1845	0692 1614	0922 1384	1153	$\bar{3}.83$,18537 ,00247,8		0248 2230	0496 1982	0743 1735	0991 1487	1239
$\bar{3}.59$,12802 ,00231,2		0231 2081	0462 1850	0694 1618	0925 1387	1156	$\bar{3}.84$,18785 ,00248,6		0249 2237	0497 1989	0746 1740	0994 1492	1243
$\bar{3}.60$,13033 ,00231,9		0232 2087	0464 1855	0696 1623	0928 1391	1160	$\bar{3}.85$,19034 ,00249,3		0249 2244	0499 1994	0748 1745	0997 1496	1247
$\bar{3}.61$,13265 ,00232,5		0233 2093	0465 1860	0698 1628	0930 1395	1163	$\bar{3}.86$,19284 ,00250,1		0250 2251	0500 2001	0750 1751	1000 1501	1251
$\bar{3}.62$,13497 ,00233,0		0233 2097	0466 1864	0699 1631	0932 1398	1165	$\bar{3}.87$,19533 ,00250,9		0251 2258	0502 2007	0753 1756	1004 1505	1255
$\bar{3}.63$,13730 ,00233,8		0234 2104	0468 1870	0701 1637	0935 1403	1169	$\bar{3}.88$,19784 ,00251,6		0252 2264	0503 2013	0755 1761	1006 1510	1258
$\bar{3}.64$,13964 ,00234,5		0235 2111	0469 1876	0704 1642	0938 1407	1174	$\bar{3}.89$,20035 ,00252,4		0252 2272	0505 2019	0757 1767	1010 1514	1262
$\bar{3}.65$,14199 ,00235,1		0235 2116	0470 1881	0705 1646	0940 1411	1176	$\bar{3}.90$,20288 ,00253,2		0253 2279	0506 2026	0760 1772	1013 1519	1266
$\bar{3}.66$,14434 ,00235,8		0236 2122	0472 1886	0707 1651	0943 1414	1179	$\bar{3}.91$,20541 ,00254,1		0254 2287	0508 2033	0762 1779	1016 1525	1271
$\bar{3}.67$,14670 ,00236,6		0237 2129	0473 1893	0710 1656	0946 1420	1183	$\bar{3}.92$,20795 ,00254,7		0255 2292	0509 2038	0764 1783	1019 1528	1274
$\bar{3}.68$,14906 ,00237,1		0237 2134	0474 1897	0711 1660	0948 1423	1186	$\bar{3}.93$,21050 ,00255,7		0256 2301	0511 2046	0767 1790	1023 1534	1279
$\bar{3}.69$,15143 ,00237,9		0238 2141	0476 1903	0714 1665	0952 1427	1190	$\bar{3}.94$,21306 ,00256,3		0256 2307	0513 2050	0769 1794	1025 1538	1282
$\bar{3}.70$,15381 ,00238,5		0239 2147	0477 1908	0716 1670	0954 1431	1193	$\bar{3}.95$,21562 ,00257,2		0257 2315	0514 2058	0772 1800	1029 1543	1286
$\bar{3}.71$,15620 ,00239,2		0239 2153	0478 1914	0718 1674	0957 1435	1196	$\bar{3}.96$,21819 ,00258,0		0258 2322	0516 2064	0774 1806	1032 1548	1290
$\bar{3}.72$,15859 ,00239,9		0240 2159	0480 1919	0720 1679	0960 1439	1200	$\bar{3}.97$,22077 ,00258,8		0259 2329	0518 2070	0776 1812	1035 1553	1294
$\bar{3}.73$,16099 ,00240,6		0241 2165	0481 1925	0722 1684	0962 1444	1203	$\bar{3}.98$,22336 ,00259,6		0260 2336	0519 2077	0779 1817	1038 1558	1298
$\bar{3}.74$,16339 ,00241,3		0241 2172	0483 1930	0724 1689	0965 1448	1207	$\bar{3}.99$,22559 ,00260,4		0260 2344	0521 2083	0781 1823	1042 1562	1302

General Table I. $\lambda (a^{10}), \lambda \left(\frac{1-a^{10}}{a^{-1}-1} \right)$.

$\lambda(a^{10})$	$\lambda \left(\frac{1-a^{10}}{a^{-1}-1} \right)$	1	2	3	4	5	$\lambda(a^{10})$	$\lambda \left(\frac{1-a^{10}}{a^{-1}-1} \right)$	1	2	3	4	5
		9	8	7	6	5			9	8	7	6	5
$\bar{z}.00$,22856 ,00261,4	,00 0261	0523	0784	1046	1307	$\bar{z}.25$,29652 ,00283,9	,00 0284	0568	0852	1136	1420
$\bar{z}.01$,23117 ,00262,0	0262	0524	0786	1048	1310	$\bar{z}.26$,29936 ,00284,9	2555	2271	1987	1703	1425
$\bar{z}.02$,23379 ,00263,0	2358	2096	1834	1572	1315	$\bar{z}.27$,30221 ,00285,9	0285	0570	0855	1140	1430
$\bar{z}.03$,23642 ,00263,8	0263	0526	0789	1052	1319	$\bar{z}.28$,30507 ,00286,8	2564	2279	1994	1709	1434
$\bar{z}.04$,23906 ,00264,7	2367	2104	1841	1578	1324	$\bar{z}.29$,30794 ,00287,9	0286	0572	0858	1144	1440
$\bar{z}.05$,24171 ,00265,6	0264	0528	0791	1055	1328	$\bar{z}.30$,31081 ,00288,9	2573	2287	2001	1715	1445
$\bar{z}.06$,24036 ,00266,3	2374	2110	1847	1583	1332	$\bar{z}.31$,31370 ,00289,9	0287	0574	0860	1147	1450
$\bar{z}.07$,24703 ,00267,0	0265	0529	0794	1059	1335	$\bar{z}.32$,31660 ,00290,9	2581	2294	2008	1721	1455
$\bar{z}.08$,24970 ,00268,5	2382	2118	1853	1588	1343	$\bar{z}.33$,31951 ,00291,9	0288	0576	0864	1152	1460
$\bar{z}.09$,25238 ,00269,0	0266	0531	0797	1062	1345	$\bar{z}.34$,32243 ,00293,0	2591	2303	2015	1727	1465
$\bar{z}.10$,25507 ,00269,9	2390	2125	1859	1594	1350	$\bar{z}.35$,32536 ,00294,0	0289	0578	0867	1156	1470
$\bar{z}.11$,25777 ,00270,8	0266	0533	0799	1065	1354	$\bar{z}.36$,32830 ,00295,0	2600	2311	2022	1733	1475
$\bar{z}.12$,26048 ,00271,7	2397	2130	1864	1598	1359	$\bar{z}.37$,33125 ,00296,1	0290	0580	0870	1160	1481
$\bar{z}.13$,26320 ,00272,6	0267	0534	0801	1068	1363	$\bar{z}.38$,33421 ,00297,1	2609	2319	2029	1739	1485
$\bar{z}.14$,26592 ,00273,5	2403	2136	1869	1602	1368	$\bar{z}.39$,33718 ,00298,2	0291	0582	0873	1164	1491
$\bar{z}.15$,26866 ,00274,6	0269	0537	0806	1074	1373	$\bar{z}.40$,34016 ,00299,3	2618	2327	2036	1745	1497
$\bar{z}.16$,27140 ,00275,2	2417	2148	1880	1611	1376	$\bar{z}.41$,34316 ,00300,3	0292	0584	0876	1168	1502
$\bar{z}.17$,27415 ,00276,3	0270	0540	0810	1080	1382	$\bar{z}.42$,34616 ,00301,4	2627	2335	2043	1751	1507
$\bar{z}.18$,27692 ,00277,2	2421	2152	1883	1614	1386	$\bar{z}.43$,34917 ,00302,5	0293	0586	0879	1172	1513
$\bar{z}.19$,27969 ,00278,1	0274	0542	0812	1083	1391	$\bar{z}.44$,35220 ,00303,6	2637	2344	2051	1758	1518
$\bar{z}.20$,28247 ,00279,1	2429	2159	1889	1619	1396	$\bar{z}.45$,35523 ,00304,7	0294	0588	0882	1176	1524
$\bar{z}.21$,28526 ,00280,1	0275	0549	0824	1098	1401	$\bar{z}.46$,35828 ,00305,8	2646	2352	2058	1764	1529
$\bar{z}.22$,28806 ,00281,1	2437	2166	1896	1625	1406	$\bar{z}.47$,36134 ,00306,9	0295	0590	0885	1180	1535
$\bar{z}.23$,29087 ,00281,9	0276	0553	0829	1105	1410	$\bar{z}.48$,36441 ,00308,1	2655	2360	2065	1770	1541
$\bar{z}.24$,29369 ,00282,9	2445	2174	1902	1630	1415	$\bar{z}.49$,36749 ,00309,2	0296	0592	0888	1184	1546

General Table I. $\lambda(a^{10}), \lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$.

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^{-1}-1}\right)$	1 9	2 8	3 7	4 6	5		
$\bar{2}.50$,37058 ,00310,3 ,37368	,00	0310 2794	0621 2482	0931 2172	1241 1862	1552	$\bar{2}.75$,45174 ,00340,8	,00	0341 3067	0682 2726	1022 2386	1363 2045	1704
$\bar{2}.51$,00311,5 ,37680		0312 2804	0623 2492	0935 2181	1246 1869	1558	$\bar{2}.76$,45515 ,00342,3		0342 3081	0685 2738	1027 2396	1369 2054	1712
$\bar{2}.52$,00312,6 ,37993		0313 2813	0625 2501	0938 2188	1250 1876	1563	$\bar{2}.77$,45857 ,00343,5		0343 3092	0687 2748	1031 2405	1374 2061	1718
$\bar{2}.53$,00313,8 ,38306		0314 2824	0628 2510	0941 2197	1255 1883	1569	$\bar{2}.78$,46201 ,00345,0		0345 3105	0690 2760	1035 2415	1380 2070	1725
$\bar{2}.54$,00314,9 ,38621		0315 2834	0630 2519	0945 2204	1260 1889	1575	$\bar{2}.79$,46546 ,00346,1		0346 3115	0692 2769	1038 2423	1384 2077	1731
$\bar{2}.55$,00316,1 ,38937		0316 2845	0632 2529	0948 2213	1265 1897	1581	$\bar{2}.80$,46892 ,00347,5		0348 3128	0695 2780	1043 2433	1390 2085	1738
$\bar{2}.56$,00317,3 ,39255		0317 2856	0635 2538	0952 2221	1269 1904	1587	$\bar{2}.81$,47239 ,00348,9		0349 3140	0698 2791	1047 2442	1396 2093	1745
$\bar{2}.57$,00318,5 ,39573		0319 2867	0637 2548	0956 2230	1274 1911	1593	$\bar{2}.82$,47588 ,00350,2		0350 3152	0700 2802	1051 2451	1410 2101	1751
$\bar{2}.58$,00319,6 ,39893		0320 2876	0639 2557	0959 2237	1278 1918	1598	$\bar{2}.83$,47938 ,00351,6		0352 3164	0703 2813	1055 2461	1406 2110	1758
$\bar{2}.59$,00320,8 ,40213		0321 2887	0642 2566	0962 2246	1283 1925	1604	$\bar{2}.84$,48290 ,00353,0		0353 3177	0706 2824	1059 2471	1412 2118	1765
$\bar{2}.60$,00322,0 ,40536		0322 2898	0644 2576	0966 2254	1288 1932	1610	$\bar{2}.85$,48643 ,00354,3		0354 3189	0709 2834	1063 2486	1417 2126	1772
$\bar{2}.61$,00323,3 ,40859		0323 2910	0647 2586	0970 2263	1293 1940	1617	$\bar{2}.86$,48997 ,00355,8		0356 3202	0712 2846	1067 2491	1423 2135	1779
$\bar{2}.62$,00324,5 ,41183		0325 2921	0649 2596	0974 2272	1298 1947	1623	$\bar{2}.87$,49353 ,00357,1		0357 3214	0714 2857	1071 2500	1428 2143	1786
$\bar{2}.63$,00325,6 ,41509		0326 2930	0651 2605	0977 2279	1302 1954	1628	$\bar{2}.88$,49710 ,00358,5		0359 3227	0717 2868	1076 2510	1434 2151	1793
$\bar{2}.64$,00326,8 ,41836		0327 2941	0654 2614	0980 2288	1307 1961	1634	$\bar{2}.89$,50069 ,00360,0		0360 3240	0720 2880	1080 2520	1440 2160	1800
$\bar{2}.65$,00328,3 ,42164		0328 2955	0657 2626	0985 2298	1313 1970	1642	$\bar{2}.90$,50429 ,00361,4		0361 3253	0723 2891	1084 2530	1446 2168	1807
$\bar{2}.66$,00329,4 ,42493		0329 2965	0659 2635	0988 2306	1318 1976	1647	$\bar{2}.91$,50790 ,00362,7		0363 3264	0725 2902	1088 2539	1451 2176	1814
$\bar{2}.67$,00330,6 ,42833		0331 2975	0661 2645	0992 2314	1322 1984	1653	$\bar{2}.92$,51153 ,00364,2		0364 3278	0728 2914	1093 2549	1457 2185	1821
$\bar{2}.68$,00331,9 ,43156		0332 2987	0664 2655	0996 2323	1328 1991	1660	$\bar{2}.93$,51517 ,00365,6		0366 3290	0731 2925	1097 2559	1462 2194	1828
$\bar{2}.69$,00333,2 ,43490		0333 2999	0666 2666	1000 2332	1333 1999	1666	$\bar{2}.94$,51883 ,00367,1		0367 3304	0734 2937	1101 2570	1468 2203	1836
$\bar{2}.70$,00334,4 ,43824		0334 3010	0669 2675	1003 2341	1338 2006	1672	$\bar{2}.95$,52250 ,00368,5		0369 3317	0737 2948	1106 2580	1474 2211	1843
$\bar{2}.71$,00335,7 ,44159		0336 3021	0671 2686	1007 2350	1343 2014	1679	$\bar{2}.96$,52618 ,00369,9		0370 3329	0740 2959	1110 2589	1480 2219	1850
$\bar{2}.72$,00337,1 ,44496		0337 3034	0674 2697	1011 2360	1348 2023	1686	$\bar{2}.97$,52988 ,00371,4		0371 3343	0743 2971	1114 2600	1486 2228	1857
$\bar{2}.73$,00338,2 ,44835		0338 3044	0676 2706	1015 2367	1353 2029	1691	$\bar{2}.98$,53360 ,00372,9		0373 3356	0746 2983	1119 2610	1492 2237	1865
$\bar{2}.74$,00339,6		0340 3056	0679 2717	1019 2377	1358 2038	1698	$\bar{2}.99$,53732 ,00374,4		0374 3370	0749 2995	1123 2620	1498 2246	1872

General Table I. $\lambda(a^{10}), \lambda\left(\frac{1-a^{10}}{a^1-1}\right)$.

$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^1-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^{10})$	$\lambda\left(\frac{1-a^{10}}{a^1-1}\right)$		1 9	2 8	3 7	4 6	5
1.00	,54107 ,00375,8	,00	0376 3382	0752 3006	1127 2631	1503 2255	1879	1.25	,63964 ,00415,3	,00	0415 3748	0831 3322	1246 2907	1661 2492	2077
1.01	,54483 ,00377,4		0377 3397	0755 3019	1132 2642	1510 2264	1887	1.26	,64380 ,00416,7		0417 3750	0833 3334	1250 2917	1667 2500	2084
1.02	,54860 ,00378,7		0379 3408	0757 3030	1136 2651	1515 2273	1894	1.27	,64796 ,00418,4		0418 3766	0837 3347	1255 2929	1674 2510	2092
1.03	,55239 ,00380,3		0380 3423	0761 3042	1141 2662	1521 2282	1902	1.28	,65215 ,00420,0		0420 3780	0840 3360	1260 2940	1680 2520	2100
1.04	,55619 ,00381,9		0382 3437	0764 3055	1146 2673	1528 2291	1910	1.29	,65635 ,00421,7		0422 3795	0843 3374	1265 2952	1687 2530	2109
1.05	,56001 ,00383,3		0383 3450	0767 3066	1150 2683	1533 2300	1917	1.30	,66056 ,00423,3		0423 3810	0847 3386	1270 2963	1693 2540	2117
1.06	,56384 ,00384,9		0385 3464	0770 3079	1155 2694	1540 2309	1925	1.31	,66480 ,00425,0		0425 3825	0850 3400	1275 2975	1700 2550	2125
1.07	,56769 ,00386,4		0386 3478	0773 3091	1159 2705	1546 2318	1932	1.32	,66905 ,00426,7		0427 3840	0853 3414	1280 2987	1707 2560	2134
1.08	,57156 ,00388,0		0388 3492	0776 3104	1164 2716	1552 2328	1940	1.33	,67331 ,00428,4		0428 3856	0857 3427	1285 2999	1714 2570	2143
1.09	,57544 ,00389,4		0389 3505	0779 3115	1168 2726	1558 2336	1947	1.34	,67760 ,00430,1		0430 3871	0860 3441	1290 3011	1720 2581	2151
1.10	,57933 ,00391,0		0391 3519	0782 3128	1173 2737	1564 2346	1955	1.35	,68190 ,00431,7		0432 3885	0863 3454	1295 3022	1727 2590	2159
1.11	,58324 ,00392,5		0393 3533	0785 3140	1178 2748	1570 2355	1963	1.36	,68622 ,00434,0		0434 3906	0868 3472	1302 3038	1736 2604	2170
1.12	,58717 ,00394,2		0394 3548	0788 3154	1183 2759	1577 2365	1971	1.37	,69056 ,00434,7		0435 3912	0869 3478	1304 3043	1739 2608	2174
1.13	,59111 ,00395,9		0396 3563	0792 3167	1188 2771	1584 2375	1980	1.38	,69490 ,00437,0		0437 3933	0874 3496	1311 3059	1748 2622	2185
1.14	,59507 ,00397,1		0397 3574	0794 3177	1191 2780	1588 2383	1986	1.39	,69927 ,00438,7		0439 3948	0877 3510	1316 3071	1755 2632	2194
1.15	,59904 ,00398,8		0399 3589	0798 3190	1196 2792	1595 2393	1994	1.40	,70366 ,00440,4		0440 3964	0881 3523	1321 3083	1762 2642	2202
1.16	,60303 ,00400,5		0401 3605	0801 3204	1202 2804	1602 2403	2003	1.41	,70806 ,00442,1		0442 3979	0884 3537	1326 3095	1768 2653	2211
1.17	,60703 ,00402,1		0402 3619	0804 3217	1206 2815	1608 2413	2011	1.42	,71249 ,00443,8		0444 3994	0888 3550	1331 3107	1775 2663	2219
1.18	,61105 ,00403,7		0404 3633	0827 3230	1211 2826	1615 2422	2019	1.43	,71692 ,00445,5		0445 4005	0890 3560	1335 3115	1780 2670	2225
1.19	,61509 ,00405,3		0405 3648	0811 3242	1216 2837	1621 2432	2027	1.44	,72137 ,00448,0		0448 4032	0896 3584	1344 3136	1792 2688	2240
1.20	,61914 ,00406,9		0407 3662	0814 3255	1221 2848	1628 2441	2035	1.45	,72585 ,00449,1		0449 4042	0898 3593	1347 3144	1796 2695	2246
1.21	,62321 ,00408,5		0409 3677	0817 3268	1226 2860	1634 2451	2043	1.46	,73035 ,00450,8		0451 4057	0902 3606	1352 3156	1803 2705	2254
1.22	,62729 ,00410,1		0410 3691	0820 3281	1230 2871	1640 2461	2051	1.47	,73485 ,00452,7		0453 4074	0905 3622	1358 3169	1811 2716	2264
1.23	,63140 ,00411,8		0412 3706	0824 3294	1235 2883	1647 2471	2059	1.48	,73938 ,00454,4		0454 4090	0909 3635	1363 3181	1818 2726	2272
1.24	,63551 ,00413,1		0413 3718	0826 3305	1239 2892	1652 2489	2066	1.49	,74393 ,00456,1		0456 4105	0912 3649	1368 3193	1824 2737	2281

General Table II. $\lambda(a^7), \lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$.

$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$	1	2	3	4	5	$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^{-1}-1}\right)$	1	2	3	4	5		
		9	8	7	6	5			9	8	7	6	5		
$\bar{3}.00$	$\bar{1},77356$ $,00227,0$,00	0227	0454	0681	0908	1135	$\bar{3}.25$	$\bar{1},83164$ $,00238,5$,00	0239	0477	0716	0954	1193
$\bar{3}.01$	$\bar{1},77583$ $,00227,4$		0227	0455	0682	0910	1137	$\bar{3}.26$	$\bar{1},83403$ $,00239,0$		0239	0478	0717	0956	1195
$\bar{3}.02$	$\bar{1},77810$ $,00227,8$		0228	0456	0683	0911	1139	$\bar{3}.27$	$\bar{1},83641$ $,00239,5$		0240	0479	0719	0958	1198
$\bar{3}.03$	$\bar{1},78038$ $,00228,3$		0228	0457	0685	0913	1142	$\bar{3}.28$	$\bar{1},83881$ $,00240,0$		0240	0480	0720	0960	1200
$\bar{3}.04$	$\bar{1},78266$ $,00228,6$		0229	0457	0686	0914	1143	$\bar{3}.29$	$\bar{1},84121$ $,00240,5$		0241	0481	0722	0962	1203
$\bar{3}.05$	$\bar{1},78495$ $,00229,2$		0229	0458	0688	0917	1146	$\bar{3}.30$	$\bar{1},84361$ $,00240,9$		0241	0482	0723	0964	1205
$\bar{3}.06$	$\bar{1},78724$ $,00229,6$		0230	0460	0689	0918	1148	$\bar{3}.31$	$\bar{1},84602$ $,00241,5$		0242	0483	0725	0966	1208
$\bar{3}.07$	$\bar{1},78954$ $,00230,0$		0230	0460	0690	0920	1150	$\bar{3}.32$	$\bar{1},84844$ $,00241,9$		0242	0484	0726	0968	1210
$\bar{3}.08$	$\bar{1},79184$ $,00230,5$		0231	0461	0692	0922	1153	$\bar{3}.33$	$\bar{1},85086$ $,00242,5$		0243	0485	0728	0970	1213
$\bar{3}.09$	$\bar{1},79414$ $,00230,8$		0231	0462	0692	0923	1154	$\bar{3}.34$	$\bar{1},85328$ $,00243,2$		0243	0486	0730	0973	1216
$\bar{3}.10$	$\bar{1},79645$ $,00231,4$		0231	0463	0694	0926	1157	$\bar{3}.35$	$\bar{1},85571$ $,00243,6$		0244	0487	0731	0974	1218
$\bar{3}.11$	$\bar{1},79877$ $,00231,8$		0232	0464	0695	0927	1159	$\bar{3}.36$	$\bar{1},85815$ $,00244,1$		0244	0488	0732	0976	1221
$\bar{3}.12$	$\bar{1},80108$ $,00232,2$		0232	0464	0697	0929	1161	$\bar{3}.37$	$\bar{1},86059$ $,00244,7$		0245	0489	0734	0979	1224
$\bar{3}.13$	$\bar{1},80341$ $,00232,7$		0233	0465	0698	0931	1164	$\bar{3}.38$	$\bar{1},86304$ $,00245,2$		0245	0490	0736	0981	1226
$\bar{3}.14$	$\bar{1},80573$ $,00233,2$		0233	0466	0700	0933	1166	$\bar{3}.39$	$\bar{1},86549$ $,00245,8$		0246	0492	0737	0983	1229
$\bar{3}.15$	$\bar{1},80806$ $,00233,6$		0234	0467	0701	0934	1168	$\bar{3}.40$	$\bar{1},86795$ $,00246,2$		0246	0492	0739	0985	1231
$\bar{3}.16$	$\bar{1},81040$ $,00234,2$		0234	0468	0703	0937	1171	$\bar{3}.41$	$\bar{1},87041$ $,00246,9$		0247	0494	0741	0988	1235
$\bar{3}.17$	$\bar{1},81274$ $,00234,5$		0235	0469	0704	0938	1173	$\bar{3}.42$	$\bar{1},87288$ $,00247,4$		0247	0495	0742	0990	1237
$\bar{3}.18$	$\bar{1},81509$ $,00235,0$		0235	0470	0705	0940	1175	$\bar{3}.43$	$\bar{1},87535$ $,00247,9$		0248	0496	0744	0992	1240
$\bar{3}.19$	$\bar{1},81744$ $,00235,5$		0236	0471	0707	0942	1178	$\bar{3}.44$	$\bar{1},87783$ $,00248,5$		0249	0497	0746	0994	1243
$\bar{3}.20$	$\bar{1},81979$ $,00236,0$		0236	0472	0708	0944	1180	$\bar{3}.45$	$\bar{1},88032$ $,00249,1$		0249	0498	0747	0996	1246
$\bar{3}.21$	$\bar{1},82215$ $,00236,5$		0237	0473	0710	0946	1183	$\bar{3}.46$	$\bar{1},88281$ $,00249,7$		0250	0499	0749	0999	1248
$\bar{3}.22$	$\bar{1},82452$ $,00237,0$		0237	0474	0711	0948	1185	$\bar{3}.47$	$\bar{1},88530$ $,00250,2$		0250	0500	0751	1001	1251
$\bar{3}.23$	$\bar{1},82689$ $,00237,4$		0237	0475	0712	0950	1187	$\bar{3}.48$	$\bar{1},88780$ $,00250,8$		0251	0502	0752	1003	1254
$\bar{3}.24$	$\bar{1},82926$ $,00238,0$		0238	0476	0714	0952	1190	$\bar{3}.49$	$\bar{1},89031$ $,00251,4$		0251	0503	0754	1006	1257
			2142	1904	1666	1428					2263	2011	1760	1508	

General Table II. $\lambda(a^7), \lambda\left(\frac{1-a^7}{a^7-1}\right)$.

$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^7-1}\right)$		1 9	2 8	3 7	4 6	5	$\lambda(a^7)$	$\lambda\left(\frac{1-a^7}{a^7-1}\right)$		1 9	2 8	3 7	4 6	5
3.50	1.89283 ,00252,0	,00	0252 2268	0504 2016	0756 1764	1008 1512	1260	3.75	1.95767 ,00268,0	,00	0268 2412	0536 2144	0804 1876	1072 1608	1340
3.51	1.89535 ,00252,6		0253 2273	0505 2021	0758 1768	1010 1516	1263	3.76	1.96035 ,00268,5		0269 2417	0537 2148	0806 1880	1074 1611	1343
3.52	1.89787 ,00253,1		0253 2278	0506 2025	0759 1772	1012 1519	1266	3.77	1.96303 ,00269,3		0269 2424	0539 2154	0808 1885	1077 1616	1347
3.53	1.90040 ,00253,7		0254 2283	0507 2030	0761 1776	1015 1522	1269	3.78	1.96573 ,00270,0		0270 2430	0540 2160	0810 1890	1080 1620	1350
3.54	1.90294 ,00254,3		0254 2289	0509 2034	0763 1780	1017 1526	1272	3.79	1.96843 ,00270,7		0271 2436	0541 2166	0812 1895	1083 1624	1354
3.55	1.90548 ,00255,0		0255 2295	0510 2040	0765 1785	1020 1530	1275	3.80	1.97113 ,00271,3		0271 2442	0543 2170	0814 1899	1085 1628	1357
3.56	1.90803 ,00255,5		0256 2300	0511 2044	0767 1789	1022 1533	1278	3.81	1.97385 ,00272,1		0272 2449	0544 2177	0816 1905	1088 1633	1361
3.57	1.91059 ,00256,2		0256 2306	0512 2050	0769 1793	1025 1537	1281	3.82	1.97657 ,00272,9		0273 2456	0546 2183	0819 1910	1092 1637	1365
3.58	1.91315 ,00256,8		0257 2311	0514 2054	0770 1798	1027 1541	1284	3.83	1.97930 ,00273,4		0273 2461	0547 2187	0820 1914	1094 1640	1367
3.59	1.91572 ,00257,4		0257 2317	0515 2059	0772 1802	1030 1544	1287	3.84	1.98203 ,00274,3		0274 2469	0549 2194	0823 1920	1097 1646	1372
3.60	1.91829 ,00258,0		0258 2322	0516 2064	0774 1806	1032 1548	1290	3.85	1.98477 ,00275,0		0275 2475	0550 2200	0825 1925	1100 1650	1375
3.61	1.92087 ,00258,7		0259 2328	0517 2070	0776 1811	1035 1552	1294	3.86	1.98752 ,00275,8		0276 2482	0552 2206	0827 1931	1103 1655	1379
3.62	1.92346 ,00259,2		0259 2333	0518 2074	0778 1814	1037 1555	1296	3.87	1.99028 ,00276,5		0277 2489	0553 2212	0830 1936	1106 1659	1383
3.63	1.92605 ,00259,9		0260 2339	0520 2079	0780 1819	1040 1559	1300	3.88	1.99305 ,00277,3		0277 2496	0555 2218	0832 1941	1109 1664	1387
3.64	1.92865 ,00260,6		0261 2345	0521 2085	0782 1824	1042 1564	1303	3.89	1.99582 ,00278,0		0278 2502	0556 2224	0834 1946	1112 1668	1390
3.65	1.93125 ,00261,2		0261 2351	0522 2090	0784 1828	1045 1567	1306	3.90	1.99860 ,00278,8		0279 2509	0558 2230	0836 1952	1115 1673	1394
3.66	1.93387 ,00261,8		0262 2356	0524 2094	0785 1833	1047 1571	1309	3.91	1.00139 ,00279,5		0280 2516	0559 2236	0839 1957	1118 1677	1398
3.67	1.93648 ,00262,5		0263 2363	0525 2100	0788 1838	1050 1575	1313	3.92	1.00418 ,00280,3		0280 2523	0561 2242	0841 1962	1121 1682	1402
3.68	1.93911 ,00263,2		0263 2369	0526 2106	0790 1842	1053 1579	1316	3.93	1.00699 ,00281,1		0281 2530	0562 2249	0843 1968	1124 1687	1406
3.69	1.94174 ,00263,8		0264 2374	0528 2110	0791 1847	1055 1583	1319	3.94	1.00980 ,00281,9		0282 2537	0564 2255	0846 1973	1128 1691	1410
3.70	1.94438 ,00264,4		0264 2380	0529 2115	0793 1851	1058 1586	1322	3.95	1.01262 ,00282,7		0283 2544	0565 2262	0848 1979	1131 1696	1414
3.71	1.94702 ,00265,1		0265 2386	0530 2121	0795 1856	1060 1591	1326	3.96	1.01544 ,00283,4		0283 2551	0567 2267	0850 1984	1134 1700	1417
3.72	1.94967 ,00265,8		0266 2392	0532 2126	0797 1861	1063 1595	1329	3.97	1.01828 ,00284,2		0284 2558	0568 2274	0853 1989	1137 1705	1421
3.73	1.95233 ,00266,5		0267 2399	0533 2132	0800 1866	1066 1599	1333	3.98	1.02112 ,00285,0		0285 2565	0570 2280	0855 1995	1140 1710	1425
3.74	1.95500 ,00267,2		0267 2405	0534 2138	0802 1870	1069 1603	1336	3.99	1.02397 ,00285,9		0286 2573	0572 2287	0858 2001	1144 1715	1430

General Table II. $\lambda (a^t), \lambda \left(\frac{1-a^t}{a^{-1}-1}\right)$.

$\lambda (a^t)$	$\lambda \left(\frac{1-a^t}{a^{-1}-1}\right)$	1 9	2 8	3 7	4 6	5	$\lambda (a^t)$	$\lambda \left(\frac{1-a^t}{a^{-1}-1}\right)$	1 9	2 8	3 7	4 6	5
$\bar{1}.00$,36374 ,00399,3	,00 0399	0799	1198	1597	1997	$\bar{1}.25$,48811 ,00438,0	,00 0438	0876	1314	1752	2190
$\bar{1}.01$,36773 ,00400,7	0401 3606	0801	1202	1603	2004	$\bar{1}.26$,47249 ,00439,4	0439 3955	0879	1318	1758	2197
$\bar{1}.02$,37174 ,00402,2	0402 3620	0804	1207	1609	2011	$\bar{1}.27$,47689 ,00441,2	0441 3971	0882	1324	1765	2206
$\bar{1}.03$,37576 ,00403,7	0404 3633	0807	1211	1615	2019	$\bar{1}.28$,48130 ,00442,9	0443 3986	0886	1329	1772	2215
$\bar{1}.04$,37980 ,00405,4	0405 3649	0811	1216	1622	2027	$\bar{1}.29$,48573 ,00444,5	0445 4001	0889	1334	1778	2223
$\bar{1}.05$,38385 ,00406,4	0406 3658	0813	1219	1626	2032	$\bar{1}.30$,49017 ,00446,2	0446 4016	0892	1339	1785	2231
$\bar{1}.06$,38792 ,00408,2	0408 3674	0816	1225	1633	2041	$\bar{1}.31$,49463 ,00447,8	0448 4030	0896	1343	1791	2239
$\bar{1}.07$,39200 ,00409,7	0410 3687	0819	1229	1639	2050	$\bar{1}.32$,49911 ,00449,5	0450 4046	0899	1349	1798	2248
$\bar{1}.08$,39610 ,00411,2	0411 3701	0822	1234	1645	2056	$\bar{1}.33$,50361 ,00451,1	0451 4060	0902	1353	1804	2256
$\bar{1}.09$,40021 ,00412,8	0413 3715	0826	1238	1651	2064	$\bar{1}.34$,50812 ,00452,9	0453 4076	0909	1359	1812	2265
$\bar{1}.10$,40434 ,00414,2	0414 3728	0828	1243	1657	2071	$\bar{1}.35$,51265 ,00454,5	0455 4090	0909	1364	1818	2273
$\bar{1}.11$,40848 ,00415,8	0416 3742	0832	1247	1663	2079	$\bar{1}.36$,51719 ,00456,2	0456 4106	0912	1369	1825	2281
$\bar{1}.12$,41264 ,00417,3	0417 3756	0835	1252	1669	2087	$\bar{1}.37$,52175 ,00457,8	0458 4120	0916	1373	1831	2289
$\bar{1}.13$,41681 ,00419,1	0419 3772	0838	1257	1676	2096	$\bar{1}.38$,52633 ,00459,7	0460 4137	0919	1379	1839	2299
$\bar{1}.14$,42100 ,00420,2	0420 3782	0840	1261	1681	2101	$\bar{1}.39$,53093 ,00461,4	0461 4153	0923	1384	1846	2307
$\bar{1}.15$,42520 ,00421,9	0422 3797	0844	1266	1688	2110	$\bar{1}.40$,53554 ,00462,9	0463 4166	0926	1389	1852	2315
$\bar{1}.16$,42942 ,00423,6	0424 3812	0847	1271	1694	2118	$\bar{1}.41$,54017 ,00464,8	0465 4183	0930	1394	1859	2324
$\bar{1}.17$,43366 ,00425,1	0425 3826	0850	1275	1700	2126	$\bar{1}.42$,54482 ,00466,5	0467 4199	0933	1400	1866	2333
$\bar{1}.18$,43791 ,00426,8	0427 3841	0854	1280	1707	2134	$\bar{1}.43$,54948 ,00468,1	0468 4213	0936	1404	1872	2341
$\bar{1}.19$,44218 ,00428,1	0428 3853	0856	1284	1712	2141	$\bar{1}.44$,55417 ,00469,9	0470 4229	0940	1410	1880	2350
$\bar{1}.20$,44646 ,00429,9	0430 3869	0860	1290	1720	2150	$\bar{1}.45$,55886 ,00471,7	0472 4245	0943	1415	1887	2359
$\bar{1}.21$,45076 ,00431,5	0432 3884	0863	1295	1726	2158	$\bar{1}.46$,56358 ,00473,6	0474 4262	0947	1421	1894	2368
$\bar{1}.22$,45507 ,00433,1	0433 3898	0866	1299	1732	2166	$\bar{1}.47$,56832 ,00475,1	0475 4276	0950	1425	1900	2376
$\bar{1}.23$,45940 ,00434,7	0435 3912	0869	1304	1739	2174	$\bar{1}.48$,57307 ,00476,9	0477 4292	0954	1431	1901	2385
$\bar{1}.24$,46375 ,00436,3	0436 3927	0873	1309	1745	2182	$\bar{1}.49$,57784 ,00478,7	0479 4308	0957	1436	1915	2394

General Table III. $\lambda (a^5), \lambda \left(\frac{1-a^5}{a^1-1} \right)$.

$\lambda(a^5)$	$\lambda \left(\frac{1-a^5}{a^1-1} \right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^5)$	$\lambda \left(\frac{1-a^5}{a^1-1} \right)$	1 9	2 8	3 7	4 6	5		
$\bar{z}.00$	$\bar{1},81612$, $00322,5$,00	0323	0645	0968	1290	1613	$\bar{z}.25$	$\bar{1},89923$, $00343,8$,00	0344	0688	1031	1375	1719
$\bar{z}.01$	$\bar{1},81934$, $00323,4$		2903	2580	2258	1935	1617	$\bar{z}.26$	$\bar{1},90267$, $00344,7$		3094	2750	2407	2063	1724
$\bar{z}.02$	$\bar{1},82257$, $00324,2$		0323	0647	0970	1294	1617	$\bar{z}.27$	$\bar{1},90612$, $00345,7$		3102	2758	2413	2068	1729
$\bar{z}.03$	$\bar{1},82582$, $00324,9$		0324	0648	0973	1297	1621	$\bar{z}.28$	$\bar{1},90958$, $00346,6$		3111	2766	2420	2074	1733
$\bar{z}.04$	$\bar{1},82907$, $00325,8$		2918	2594	2269	1945	1625	$\bar{z}.29$	$\bar{1},91304$, $00347,6$		3128	2781	2433	2086	1738
$\bar{z}.05$	$\bar{1},83232$, $00326,5$		0325	0650	0975	1300	1625	$\bar{z}.30$	$\bar{1},91652$, $00348,5$		0349	0697	1046	1394	1743
$\bar{z}.06$	$\bar{1},83559$, $00327,3$		2924	2599	2274	1949	1629	$\bar{z}.31$	$\bar{1},92000$, $00349,3$		3137	2788	2440	2091	1747
$\bar{z}.07$	$\bar{1},83886$, $00328,2$		0326	0652	0977	1303	1629	$\bar{z}.32$	$\bar{1},92349$, $00350,7$		0349	0699	1048	1397	1747
$\bar{z}.08$	$\bar{1},84214$, $00329,0$		2946	2618	2291	1964	1641	$\bar{z}.33$	$\bar{1},92700$, $00351,4$		3144	2794	2445	2096	1754
$\bar{z}.09$	$\bar{1},84543$, $00329,8$		0328	0656	0985	1313	1641	$\bar{z}.34$	$\bar{1},93052$, $00352,4$		0351	0701	1052	1403	1754
$\bar{z}.10$	$\bar{1},84873$, $00330,6$		2954	2626	2297	1969	1645	$\bar{z}.35$	$\bar{1},93404$, $00353,3$		3156	2806	2455	2104	1757
$\bar{z}.11$	$\bar{1},85204$, $00331,5$		0329	0658	0987	1316	1645	$\bar{z}.36$	$\bar{1},93757$, $00354,3$		0351	0703	1054	1406	1757
$\bar{z}.12$	$\bar{1},85553$, $00332,4$		2961	2632	2303	1974	1649	$\bar{z}.37$	$\bar{1},94112$, $00355,4$		3163	2811	2460	2108	1762
$\bar{z}.13$	$\bar{1},85868$, $00333,2$		0330	0660	0989	1319	1649	$\bar{z}.38$	$\bar{1},94467$, $00356,4$		0352	0705	1057	1410	1762
$\bar{z}.14$	$\bar{1},86201$, $00334,0$		2968	2638	2309	1979	1653	$\bar{z}.39$	$\bar{1},94823$, $00357,3$		3172	2819	2467	2114	1767
$\bar{z}.15$	$\bar{1},86535$, $00334,9$		0331	0661	0992	1322	1653	$\bar{z}.40$	$\bar{1},95181$, $00358,4$		0353	0707	1060	1413	1767
$\bar{z}.16$	$\bar{1},86870$, $00335,7$		2975	2645	2314	1984	1658	$\bar{z}.41$	$\bar{1},95539$, $00359,4$		3180	2826	2473	2120	1771
$\bar{z}.17$	$\bar{1},87205$, $00336,6$		0332	0663	0995	1326	1658	$\bar{z}.42$	$\bar{1},95898$, $00360,4$		0354	0709	1063	1417	1771
$\bar{z}.18$	$\bar{1},87542$, $00337,5$		2983	2652	2320	1989	1662	$\bar{z}.43$	$\bar{1},96259$, $00361,4$		3189	2834	2480	2126	1777
$\bar{z}.19$	$\bar{1},87880$, $00338,4$		0332	0665	0997	1330	1662	$\bar{z}.44$	$\bar{1},96620$, $00362,4$		0355	0711	1066	1422	1777
$\bar{z}.20$	$\bar{1},88218$, $00339,3$		2992	2659	2327	1994	1666	$\bar{z}.45$	$\bar{1},96983$, $00363,4$		3199	2843	2488	2132	1782
$\bar{z}.21$	$\bar{1},88557$, $00340,2$		0333	0666	1000	1333	1666	$\bar{z}.46$	$\bar{1},97346$, $00364,6$		0356	0713	1069	1426	1782
$\bar{z}.22$	$\bar{1},88897$, $00341,0$		2999	2666	2332	1999	1670	$\bar{z}.47$	$\bar{1},97711$, $00365,6$		3208	2851	2495	2138	1787
$\bar{z}.23$	$\bar{1},89238$, $00342,1$		0334	0668	1002	1336	1670	$\bar{z}.48$	$\bar{1},98076$, $00366,7$		3216	2858	2501	2144	1787
$\bar{z}.24$	$\bar{1},89581$, $00342,8$		3006	2672	2338	2004	1675	$\bar{z}.49$	$\bar{1},98443$, $00367,8$		0357	0715	1072	1429	1787
											0358	0717	1075	1434	1792
											3226	2867	2509	2150	1792
											0359	0719	1078	1438	1797
											3235	2875	2516	2156	1802
											0360	0721	1081	1442	1802
											3244	2883	2523	2162	1807
											0361	0723	1084	1446	1807
											3253	2891	2530	2168	1812
											0362	0725	1087	1450	1812
											3262	2899	2537	2174	1817
											0363	0727	1090	1454	1817
											3271	2907	2544	2180	1823
											0365	0729	1094	1458	1823
											3281	2917	2552	2188	1828
											0366	0731	1097	1462	1828
											3290	2925	2559	2194	1834
											0367	0733	1100	1467	1834
											3300	2934	2567	2200	1839
											0368	0735	1103	1471	1839
											3310	2942	2575	2207	1839

General Table III. $\lambda(a^x), \lambda\left(\frac{1-a^x}{a^x-1}\right)$.

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$		1	2	3	4	5	$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$		1	2	3	4	5
			9	8	7	6	5				9	8	7	6	5
$\bar{z}.50$.98811	.00	0369	0738	1106	1475	1844	$\bar{z}.75$.08373	.00	0398	0796	1194	1592	1990
	.00368,8		3319	2950	2582	2213			.00397,9		3581	3183	2785	2387	
$\bar{z}.51$.99179		0370	0740	1109	1480	1850	$\bar{z}.76$.08771		0399	0799	1198	1598	1997
	.00369,9		3329	2959	2589	2219			.00399,4		3595	3195	2796	2396	
$\bar{z}.52$.99549		0371	0742	1113	1484	1855	$\bar{z}.77$.09170		0401	0801	1202	1602	2003
	.00371,0		3339	2968	2597	2226			.00400,6		3605	3205	2804	2404	
$\bar{z}.53$.99920		0372	0744	1116	1488	1861	$\bar{z}.78$.09571		0402	0803	1205	1607	2009
	.00372,1		3349	2977	2605	2233			.00401,7		3615	3214	2812	2410	
$\bar{z}.54$.00293		0373	0747	1120	1493	1867	$\bar{z}.79$.09972		0403	0806	1209	1612	2016
	.00373,3		3360	2986	2613	2240			.00403,1		3628	3225	2822	2419	
$\bar{z}.55$.00666		0374	0749	1123	1497	1872	$\bar{z}.80$.10376		0404	0809	1213	1618	2022
	.00374,3		3369	2994	2620	2246			.00404,4		3640	3235	2831	2426	
$\bar{z}.56$.01040		0376	0751	1127	1502	1878	$\bar{z}.81$.10780		0406	0811	1217	1623	2029
	.00375,5		3380	3004	2629	2253			.00405,7		3651	3246	2840	2434	
$\bar{z}.57$.01416		0376	0753	1129	1506	1882	$\bar{z}.82$.11186		0407	0814	1221	1628	2035
	.00376,4		3388	3011	2635	2258			.00407,0		3663	3256	2849	2442	
$\bar{z}.58$.00792		0378	0756	1134	1512	1890	$\bar{z}.83$.11593		0408	0817	1225	1633	2042
	.00377,9		3401	3023	2645	2267			.00408,3		3675	3266	2858	2450	
$\bar{z}.59$.02170		0379	0758	1136	1515	1894	$\bar{z}.84$.12001		0410	0819	1229	1638	2048
	.00378,3		3409	3030	2652	2273			.00409,5		3686	3276	2867	2457	
$\bar{z}.60$.02549		0380	0760	1140	1520	1900	$\bar{z}.85$.12410		0411	0822	1233	1644	2055
	.00380,0		3420	3040	2660	2280			.00410,9		3698	3287	2876	2465	
$\bar{z}.61$.02929		0381	0762	1144	1525	1906	$\bar{z}.86$.12821		0412	0824	1237	1649	2061
	.00381,2		3431	3050	2668	2287			.00412,2		3710	3298	2885	2473	
$\bar{z}.62$.03310		0382	0765	1147	1530	1912	$\bar{z}.87$.13234		0414	0827	1241	1654	2068
	.00382,4		3442	3059	2677	2294			.00413,6		3722	3309	2895	2482	
$\bar{z}.63$.03692		0383	0767	1150	1534	1917	$\bar{z}.88$.13647		0415	0830	1245	1660	2075
	.00383,4		3451	3067	2684	2300			.00414,9		3734	3319	2904	2489	
$\bar{z}.64$.04076		0385	0769	1154	1539	1924	$\bar{z}.89$.14062		0416	0833	1249	1665	2082
	.00384,7		3462	3078	2693	2308			.00416,3		3747	3330	2914	2498	
$\bar{z}.65$.04460		0386	0772	1157	1543	1929	$\bar{z}.90$.14478		0418	0835	1253	1670	2088
	.00385,8		3472	3086	2701	2315			.00417,6		3758	3341	2923	2506	
$\bar{z}.66$.04846		0387	0774	1161	1548	1936	$\bar{z}.91$.14896		0419	0838	1257	1676	2095
	.00387,1		3484	3097	2710	2323			.00419,0		3771	3352	2933	2514	
$\bar{z}.67$.05233		0388	0776	1165	1553	1941	$\bar{z}.92$.15315		0420	0841	1261	1682	2102
	.00388,2		3494	3106	2717	2329			.00420,4		3784	3363	2943	2522	
$\bar{z}.68$.05621		0389	0779	1168	1558	1947	$\bar{z}.93$.15735		0422	0843	1265	1687	2109
	.00389,4		3505	3115	2726	2336			.00421,7		3795	3344	2952	2530	
$\bar{z}.69$.06010		0391	0781	1172	1562	1953	$\bar{z}.94$.16157		0423	0846	1269	1692	2116
	.00390,6		3515	3125	2734	2344			.00423,1		3808	3385	2962	2539	
$\bar{z}.70$.06401		0392	0784	1176	1568	1960	$\bar{z}.95$.16580		0425	0849	1274	1698	2123
	.00391,9		3527	3135	2743	2351			.00424,6		3821	3397	2972	2548	
$\bar{z}.71$.06793		0393	0786	1179	1572	1965	$\bar{z}.96$.17005		0426	0852	1277	1703	2129
	.00393,0		3537	3144	2751	2358			.00425,8		3832	3406	2981	2555	
$\bar{z}.72$.07186		0394	0789	1183	1577	1972	$\bar{z}.97$.17430		0427	0855	1282	1719	2137
	.00394,3		3549	3154	2760	2366			.00427,3		3846	3418	2991	2564	
$\bar{z}.73$.07581		0396	0791	1187	1582	1978	$\bar{z}.98$.17858		0429	0857	1286	1715	2144
	.00395,5		3560	3164	2769	2373			.00428,7		3858	3430	3001	2572	
$\bar{z}.74$.07976		0397	0794	1190	1587	1984	$\bar{z}.99$.18286		0430	0860	1290	1720	2151
	.00396,8		3571	3174	2778	2381			.00430,1		3871	3441	3011	2581	

General Table III. $\lambda(a^5), \lambda\left(\frac{1-a^5}{a^x-1}\right)$.

$\lambda(a^5)$	$\lambda\left(\frac{1-a^5}{a^x-1}\right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^5)$	$\lambda\left(\frac{1-a^5}{a^x-1}\right)$	1 9	2 8	3 7	4 6	5
$\bar{1}.00$,18717 ,00431,6	,00 0432 3884	0863 3453	1295 3021	1726 2590	2158	$\bar{1}.25$,29950 ,00469,3	,00 0469 4224	0939 3754	1408 3285	1877 2816	2347
$\bar{1}.01$,19148 ,00433,0	0433 3897	0866 3464	1299 3031	1732 2598	2165	$\bar{1}.26$,30419 ,00470,9	0471 4238	0942 3767	1413 3206	1884 2825	2355
$\bar{1}.02$,19581 ,00434,5	0434 3911	0869 3476	1304 3042	1738 2607	2175	$\bar{1}.27$,30890 ,00472,5	0473 4253	0945 3780	1418 3308	1890 2835	2363
$\bar{1}.03$,20016 ,00435,4	0435 3919	0871 3483	1306 3048	1742 2612	2177	$\bar{1}.28$,31363 ,00474,1	0474 4267	0948 3793	1422 3319	1896 2845	2371
$\bar{1}.04$,20451 ,00437,6	0438 3938	0875 3401	1313 3063	1750 2626	2188	$\bar{1}.29$,31837 ,00475,7	0476 4281	0951 3806	1427 3330	1903 2854	2379
$\bar{1}.05$,20889 ,00438,7	0439 3948	0877 3510	1316 3071	1755 2632	2194	$\bar{1}.30$,32312 ,00477,3	0477 4296	0955 3818	1432 3341	1909 2864	2387
$\bar{1}.06$,21328 ,00440,2	0440 3962	0880 3522	1321 3081	1761 2641	2201	$\bar{1}.31$,32790 ,00479,0	0479 4310	0958 3831	1437 3352	1916 2873	2395
$\bar{1}.07$,21768 ,00441,7	0442 3975	0883 3534	1325 3092	1767 2650	2209	$\bar{1}.32$,33269 ,00480,6	0481 4325	0961 3845	1442 3364	1922 2884	2403
$\bar{1}.08$,22209 ,00443,2	0443 3989	0886 3546	1330 3102	1773 2659	2216	$\bar{1}.33$,33749 ,00482,2	0482 4340	0964 3858	1447 3375	1929 2893	2411
$\bar{1}.09$,22653 ,00444,7	0445 4002	0889 3558	1334 3113	1779 2668	2224	$\bar{1}.34$,34232 ,00483,9	0484 4355	0968 3871	1452 3387	1936 2903	2420
$\bar{1}.10$,23097 ,00446,1	0446 4015	0892 3569	1338 3123	1784 2677	2231	$\bar{1}.35$,34715 ,00485,5	0486 4370	0971 3884	1457 3399	1942 2913	2428
$\bar{1}.11$,23543 ,00447,7	0448 4029	0895 3582	1343 3134	1791 2686	2240	$\bar{1}.36$,35201 ,00487,2	0487 4385	0974 3898	1462 3410	1949 2923	2436
$\bar{1}.12$,23991 ,00449,2	0449 4043	0898 3594	1348 3144	1797 2695	2246	$\bar{1}.37$,35688 ,00488,8	0489 4399	0978 3910	1466 3422	1955 2933	2444
$\bar{1}.13$,24440 ,00450,9	0451 4058	0902 3607	1353 3156	1804 2705	2255	$\bar{1}.38$,36177 ,00490,4	0490 4414	0981 3923	1471 3433	1962 2942	2452
$\bar{1}.14$,24891 ,00452,0	0452 4068	0904 3616	1356 3164	1808 2712	2260	$\bar{1}.39$,36676 ,00492,1	0492 4429	0984 3937	1476 3445	1968 2953	2461
$\bar{1}.15$,25343 ,00453,7	0454 4083	0907 3630	1361 3176	1815 2722	2269	$\bar{1}.40$,37159 ,00493,8	0494 4444	0988 3950	1481 3457	1975 2963	2469
$\bar{1}.16$,25797 ,00455,3	0455 4098	091 3642	1366 3187	1821 2732	2277	$\bar{1}.41$,37653 ,00495,5	0496 4460	0991 3964	1487 3469	1982 2973	2477
$\bar{1}.17$,26252 ,00456,8	0457 4111	0914 3654	1370 3198	1827 2741	2284	$\bar{1}.42$,38149 ,00497,2	0497 4475	0994 3988	1492 3480	1989 2983	2486
$\bar{1}.18$,26709 ,00458,3	0458 4125	0917 3666	1375 3208	1833 2750	2292	$\bar{1}.43$,38646 ,00498,9	0499 4490	0998 3991	1497 3492	1996 2993	2495
$\bar{1}.19$,27167 ,00459,9	0460 4139	0920 3679	1380 3219	1840 2759	2300	$\bar{1}.44$,39145 ,00500,6	0501 4505	1001 4005	1502 3504	2002 3004	2503
$\bar{1}.20$,27627 ,00461,4	0461 4153	0923 3691	1384 3230	1846 2768	2307	$\bar{1}.45$,39645 ,00502,2	0502 4520	1004 4018	1507 3515	2009 3013	2511
$\bar{1}.21$,28089 ,00463,0	0463 4167	0926 3704	1389 3241	1852 2778	2315	$\bar{1}.46$,40148 ,00504,0	0504 4536	1008 4032	1512 3528	2016 3024	2520
$\bar{1}.22$,28551 ,00464,6	0465 4181	0929 3717	1394 3252	1858 2788	2323	$\bar{1}.47$,40652 ,00505,7	0506 4551	1011 4046	1517 3540	2023 3034	2529
$\bar{1}.23$,29016 ,00466,1	0466 4194	0932 3729	1398 3263	1864 2797	2331	$\bar{1}.48$,41157 ,00507,4	0507 4567	1015 4059	1522 3552	2030 3044	2537
$\bar{1}.24$,29482 ,00467,7	0468 4209	0935 3742	1403 3274	1871 2806	2339	$\bar{1}.49$,41665 ,00509,0	0509 4582	1018 4073	1527 3564	2036 3055	2546

General Table III. $\lambda(a^x), \lambda\left(\frac{1-a^x}{a^x-1}\right)$.

$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$	1 9	2 8	3 7	4 6	5	$\lambda(a^x)$	$\lambda\left(\frac{1-a^x}{a^x-1}\right)$	$\lambda(a^x)$	1 9	2 8	3 7	4 6	5	
$\bar{1}.50$.42174 .00510,8	.00	0511 4597	1022 4086	1532 3576	2043 3065	2554	$\bar{1}.75$.55471 .00555,2	.00	0555 4997	1110 4442	1666 3886	2221 3331	2776
$\bar{1}.51$.42684 .00512,5		0513 4613	1025 4100	1538 3588	2050 3075	2563	$\bar{1}.76$.56026 .00556,9		0557 5012	1114 4455	1671 3898	2228 3341	2785
$\bar{1}.52$.43197 .00514,2		0514 4628	1028 4114	1543 3599	2057 3085	2571	$\bar{1}.77$.56583 .00558,6		0559 5027	1117 4469	1676 3910	2234 3352	2793
$\bar{1}.53$.43711 .00516,1		0516 4645	1032 4129	1548 3613	2064 3097	2581	$\bar{1}.78$.57142 .00560,7		0561 5046	1121 4486	1682 3925	2243 3364	2804
$\bar{1}.54$.44227 .00517,8		0518 4660	1036 4142	1553 3625	2071 3107	2589	$\bar{1}.79$.57702 .00562,4		0562 5062	1125 4499	1687 3937	2250 3374	2812
$\bar{1}.55$.44745 .00519,5		0520 4676	1039 4156	1559 3637	2078 3117	2598	$\bar{1}.80$.58265 .00564,2		0564 5078	1128 4514	1693 3949	2257 3385	2821
$\bar{1}.56$.45264 .00521,2		0521 4691	1042 4170	1564 3648	2085 3127	2606	$\bar{1}.81$.58829 .00565,9		0566 5093	1132 4527	1698 3961	2264 3395	2830
$\bar{1}.57$.45786 .00523,0		0523 4707	1046 4184	1569 3661	2092 3138	2615	$\bar{1}.82$.59395 .00567,8		0568 5110	1136 4542	1703 3975	2271 3407	2839
$\bar{1}.58$.46309 .00524,7		0525 4722	1049 4198	1574 3673	2099 3148	2624	$\bar{1}.83$.59963 .00569,7		0570 5127	1139 4558	1709 3988	2279 3418	2849
$\bar{1}.59$.46833 .00526,5		0527 4739	1053 4212	1580 3686	2106 3159	2633	$\bar{1}.84$.60532 .00571,5		0572 5144	1143 4572	1715 4001	2286 3429	2858
$\bar{1}.60$.47350 .00528,3		0528 4755	1057 4226	1585 3698	2113 3170	2642	$\bar{1}.85$.61104 .00573,3		0573 5160	1147 4586	1720 4013	2293 3440	2867
$\bar{1}.61$.47888 .00530,0		0530 4770	1060 4240	1590 3710	2120 3180	2650	$\bar{1}.86$.61677 .00575,1		0575 4176	1150 4601	1725 4026	2300 3451	2876
$\bar{1}.62$.48418 .00531,8		0532 4786	1064 4254	1595 3723	2127 3191	2659	$\bar{1}.87$.62252 .00577,0		0577 5193	1154 4616	1731 4039	2308 3462	2885
$\bar{1}.63$.48950 .00533,6		0534 4802	1067 4269	1601 3735	2134 3202	2668	$\bar{1}.88$.62829 .00578,8		0579 5210	1158 4630	1736 4052	2315 3473	2894
$\bar{1}.64$.49484 .00535,3		0535 4818	1071 4282	1606 3747	2141 3212	2677	$\bar{1}.89$.63408 .00580,8		0581 5227	1162 4646	1742 4066	2323 3485	2904
$\bar{1}.65$.50019 .00537,1		0537 4834	1074 4297	1611 3760	2148 3223	2688	$\bar{1}.90$.63989 .00582,5		0583 5243	1165 4660	1748 4078	2330 3495	2913
$\bar{1}.66$.50556 .00538,9		0539 4850	1078 4311	1617 3772	2156 3233	2695	$\bar{1}.91$.64572 .00584,4		0584 5260	1169 4675	1753 4091	2338 3506	2922
$\bar{1}.67$.51095 .00540,7		0541 4866	1081 4326	1622 3785	2163 3244	2704	$\bar{1}.92$.65156 .00586,2		0586 5276	1172 4690	1759 4103	2345 3517	2931
$\bar{1}.68$.51636 .00542,5		0543 4883	1085 4340	1628 3798	2170 3255	2713	$\bar{1}.93$.65742 .00587,9		0588 5291	1176 4713	1764 4115	2352 3527	2940
$\bar{1}.69$.52178 .00544,3		0544 4899	1089 4354	1633 3810	2177 3266	2722	$\bar{1}.94$.66330 .00590,0		0590 5310	1180 4720	1770 4130	2360 3540	2950
$\bar{1}.70$.52722 .00546,1		0546 4915	1092 4369	1638 3823	2184 3277	2731	$\bar{1}.95$.66920 .00591,7		0592 5325	1183 4734	1775 4142	2367 3550	2959
$\bar{1}.71$.53269 .00547,8		0548 4920	1096 4382	1643 3835	2191 3287	2739	$\bar{1}.96$.67512 .00593,5		0594 5342	1187 4748	1781 4155	2374 3561	2968
$\bar{1}.72$.53816 .00549,7		0550 4947	1099 4398	1649 3848	2199 3298	2749	$\bar{1}.97$.68105 .00595,4		0595 5359	1191 4763	1786 4168	2382 3572	2977
$\bar{1}.73$.54366 .00551,5		0552 4964	1103 4412	1655 3861	2206 3309	2758	$\bar{1}.98$.68701 .00597,3		0597 5376	1195 4778	1791 4181	2389 3584	2987
$\bar{1}.74$.54918 .00553,3		0553 4980	1107 4426	1660 3873	2213 3320	2767	$\bar{1}.99$.69298						

General Table IV. For the whole of life. — $\lambda(a^{-1}1)$.

$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$
$\bar{1}.700$,00206 ,00201	$\bar{1}.725$,05372 ,00213	$\bar{1}.750$,10886 ,00229	$\bar{1}.775$,16826 ,00247	$\bar{1}.800$,23292 ,00271	$\bar{1}.825$,30431 ,00302
$\bar{1}.701$,00407 ,00201	$\bar{1}.726$,05585 ,00214	$\bar{1}.751$,11115 ,00230	$\bar{1}.776$,17073 ,00248	$\bar{1}.801$,23564 ,00272	$\bar{1}.826$,30733 ,00304
$\bar{1}.702$,00608 ,00202	$\bar{1}.727$,05799 ,00215	$\bar{1}.752$,11345 ,00230	$\bar{1}.777$,17322 ,00249	$\bar{1}.802$,23836 ,00274	$\bar{1}.827$,31037 ,00305
$\bar{1}.703$,00810 ,00202	$\bar{1}.728$,06014 ,00215	$\bar{1}.753$,11575 ,00231	$\bar{1}.778$,17571 ,00250	$\bar{1}.803$,24110 ,00275	$\bar{1}.828$,31342 ,00307
$\bar{1}.704$,01012 ,00203	$\bar{1}.729$,06229 ,00216	$\bar{1}.754$,11806 ,00232	$\bar{1}.779$,17822 ,00251	$\bar{1}.804$,24385 ,00270	$\bar{1}.829$,31649 ,00308
$\bar{1}.705$,01214 ,00203	$\bar{1}.730$,06445 ,00216	$\bar{1}.755$,12037 ,00233	$\bar{1}.780$,18073 ,00252	$\bar{1}.805$,24661 ,00277	$\bar{1}.830$,31957 ,00309
$\bar{1}.706$,01418 ,00203	$\bar{1}.731$,06661 ,00217	$\bar{1}.756$,12270 ,00233	$\bar{1}.781$,18325 ,00253	$\bar{1}.806$,24938 ,00278	$\bar{1}.831$,32266 ,00311
$\bar{1}.707$,01621 ,00204	$\bar{1}.732$,06878 ,00217	$\bar{1}.757$,12503 ,00234	$\bar{1}.782$,18578 ,00254	$\bar{1}.807$,25216 ,00279	$\bar{1}.832$,32577 ,00313
$\bar{1}.708$,01825 ,00205	$\bar{1}.733$,07095 ,00219	$\bar{1}.758$,12736 ,00235	$\bar{1}.783$,18832 ,00254	$\bar{1}.808$,25495 ,00281	$\bar{1}.833$,32890 ,00314
$\bar{1}.709$,02030 ,00205	$\bar{1}.734$,07314 ,00218	$\bar{1}.759$,12971 ,00235	$\bar{1}.784$,19086 ,00256	$\bar{1}.809$,25776 ,00281	$\bar{1}.834$,33204 ,00315
$\bar{1}.710$,02235 ,00205	$\bar{1}.735$,07532 ,00219	$\bar{1}.760$,13206 ,00236	$\bar{1}.785$,19342 ,00257	$\bar{1}.810$,26057 ,00283	$\bar{1}.835$,33519 ,00317
$\bar{1}.711$,02440 ,00206	$\bar{1}.736$,07751 ,00220	$\bar{1}.761$,13442 ,00237	$\bar{1}.786$,19599 ,00258	$\bar{1}.811$,26340 ,00284	$\bar{1}.836$,33836 ,00319
$\bar{1}.712$,02646 ,00207	$\bar{1}.737$,07971 ,00221	$\bar{1}.762$,13679 ,00237	$\bar{1}.787$,19856 ,00259	$\bar{1}.812$,26624 ,00285	$\bar{1}.837$,34155 ,00320
$\bar{1}.713$,02853 ,00207	$\bar{1}.738$,08192 ,00221	$\bar{1}.763$,13916 ,00238	$\bar{1}.788$,20115 ,00259	$\bar{1}.813$,26909 ,00287	$\bar{1}.838$,34475 ,00322
$\bar{1}.714$,03060 ,00207	$\bar{1}.739$,08413 ,00221	$\bar{1}.764$,14154 ,00239	$\bar{1}.789$,20374 ,00260	$\bar{1}.814$,27196 ,00287	$\bar{1}.839$,34797 ,00324
$\bar{1}.715$,03267 ,00208	$\bar{1}.740$,08634 ,00223	$\bar{1}.765$,14393 ,00240	$\bar{1}.790$,20634 ,00261	$\bar{1}.815$,27483 ,00289	$\bar{1}.840$,35121 ,00325
$\bar{1}.716$,03476 ,00208	$\bar{1}.741$,08857 ,00223	$\bar{1}.766$,14633 ,00240	$\bar{1}.791$,20896 ,00262	$\bar{1}.816$,27772 ,00291	$\bar{1}.841$,35446 ,00327
$\bar{1}.717$,03684 ,00209	$\bar{1}.742$,09080 ,00223	$\bar{1}.767$,14873 ,00241	$\bar{1}.792$,21158 ,00263	$\bar{1}.817$,28063 ,00291	$\bar{1}.842$,35773 ,00329
$\bar{1}.718$,03893 ,00210	$\bar{1}.743$,09303 ,00224	$\bar{1}.768$,15114 ,00242	$\bar{1}.793$,21421 ,00264	$\bar{1}.818$,28354 ,00293	$\bar{1}.843$,36102 ,00331
$\bar{1}.719$,04103 ,00210	$\bar{1}.744$,09527 ,00225	$\bar{1}.769$,15356 ,00243	$\bar{1}.794$,21685 ,00265	$\bar{1}.819$,28647 ,00294	$\bar{1}.844$,36433 ,00332
$\bar{1}.720$,04313 ,00211	$\bar{1}.745$,09752 ,00226	$\bar{1}.770$,15599 ,00244	$\bar{1}.795$,21951 ,00266	$\bar{1}.820$,28941 ,00295	$\bar{1}.845$,36765 ,00334
$\bar{1}.721$,04524 ,00211	$\bar{1}.746$,09978 ,00226	$\bar{1}.771$,15843 ,00244	$\bar{1}.796$,22217 ,00267	$\bar{1}.821$,29236 ,00297	$\bar{1}.846$,37099 ,00336
$\bar{1}.722$,04735 ,00212	$\bar{1}.747$,10204 ,00227	$\bar{1}.772$,16087 ,00245	$\bar{1}.797$,22484 ,00268	$\bar{1}.822$,29533 ,00298	$\bar{1}.847$,37435 ,00338
$\bar{1}.723$,04947 ,00212	$\bar{1}.748$,10431 ,00227	$\bar{1}.773$,16333 ,00246	$\bar{1}.798$,22753 ,00269	$\bar{1}.823$,29831 ,00299	$\bar{1}.848$,37773 ,00339
$\bar{1}.724$,05159 ,00213	$\bar{1}.749$,10658 ,00228	$\bar{1}.774$,16579 ,00247	$\bar{1}.799$,23022 ,00270	$\bar{1}.824$,30130 ,00301	$\bar{1}.849$,38112 ,00342

General Table IV. For the whole of life. — $\lambda(a^{-1}1)$.

$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$	$\lambda(a)$	$-\lambda(a^{-1}1)$
$\bar{1}.850$,38454 ,00343	$\bar{1}.875$,47688 ,00401	$\bar{1}.900$,58683 ,00488	$\bar{1}.925$,72468 ,00635	$\bar{1}.950$,91357 ,00929	$\bar{1}.975$	1,22728 ,01824
$\bar{1}.851$,38797 ,00345	$\bar{1}.876$,48088 ,00405	$\bar{1}.901$,59171 ,00493	$\bar{1}.926$,73103 ,00642	$\bar{1}.951$,92286 ,00946	$\bar{1}.976$	1,24552 ,01899
$\bar{1}.852$,39142 ,00348	$\bar{1}.877$,48493 ,00407	$\bar{1}.902$,59664 ,00497	$\bar{1}.927$,73745 ,00650	$\bar{1}.952$,93232 ,00966	$\bar{1}.977$	1,26451 ,01980
$\bar{1}.853$,39490 ,00349	$\bar{1}.878$,48900 ,00410	$\bar{1}.903$,60161 ,00502	$\bar{1}.928$,74395 ,00659	$\bar{1}.953$,94198 ,00984	$\bar{1}.978$	1,28431 ,02071
$\bar{1}.854$,39839 ,00351	$\bar{1}.879$,49310 ,00412	$\bar{1}.904$,60663 ,00507	$\bar{1}.929$,75054 ,00668	$\bar{1}.954$,95182 ,01006	$\bar{1}.979$	1,30502 ,02170
$\bar{1}.855$,40190 ,00353	$\bar{1}.880$,49722 ,00416	$\bar{1}.905$,61170 ,00511	$\bar{1}.930$,75722 ,00676	$\bar{1}.955$,96188 ,01027	$\bar{1}.980$	1,32672 ,02278
$\bar{1}.856$,40543 ,00355	$\bar{1}.881$,50138 ,00419	$\bar{1}.906$,61681 ,00516	$\bar{1}.931$,76398 ,00685	$\bar{1}.956$,97215 ,01049	$\bar{1}.981$	1,34950 ,02398
$\bar{1}.857$,40899 ,00357	$\bar{1}.882$,50557 ,00422	$\bar{1}.907$,62197 ,00521	$\bar{1}.932$,77083 ,00695	$\bar{1}.957$,98264 ,01073	$\bar{1}.982$	1,37348 ,02583
$\bar{1}.858$,41256 ,00360	$\bar{1}.883$,50979 ,00425	$\bar{1}.908$,62718 ,00527	$\bar{1}.933$,77778 ,00704	$\bar{1}.958$,99337 ,01097	$\bar{1}.983$	1,39881 ,02683
$\bar{1}.859$,41616 ,00362	$\bar{1}.884$,51404 ,00428	$\bar{1}.909$,63245 ,00531	$\bar{1}.934$,78482 ,00715	$\bar{1}.959$	1,00434 ,01123	$\bar{1}.984$	1,42564 ,02853
$\bar{1}.860$,41978 ,00364	$\bar{1}.885$,51832 ,00431	$\bar{1}.910$,63776 ,00537	$\bar{1}.935$,79197 ,00724	$\bar{1}.960$	1,01557 ,01150	$\bar{1}.985$	1,45417 ,03047
$\bar{1}.861$,42342 ,00366	$\bar{1}.886$,52263 ,00435	$\bar{1}.911$,64313 ,00543	$\bar{1}.936$,79921 ,00735	$\bar{1}.961$	1,02707 ,01179	$\bar{1}.986$	1,48464 ,03269
$\bar{1}.862$,42708 ,00368	$\bar{1}.887$,52698 ,00438	$\bar{1}.912$,64856 ,00548	$\bar{1}.937$,80656 ,00746	$\bar{1}.962$	1,03886 ,01209	$\bar{1}.987$	1,51733 ,03526
$\bar{1}.863$,44076 ,00371	$\bar{1}.888$,53136 ,00442	$\bar{1}.913$,65404 ,00554	$\bar{1}.938$,81402 ,00758	$\bar{1}.963$	1,05095 ,01241	$\bar{1}.988$	1,55259 ,03829
$\bar{1}.864$,43447 ,00373	$\bar{1}.889$,53578 ,00445	$\bar{1}.914$,65958 ,00559	$\bar{1}.939$,82160 ,00769	$\bar{1}.964$	1,06336 ,01274	$\bar{1}.989$	1,59088 ,04189
$\bar{1}.865$,43820 ,00376	$\bar{1}.890$,54023 ,00449	$\bar{1}.915$,66517 ,00566	$\bar{1}.940$,82929 ,00781	$\bar{1}.965$	1,07610 ,01309	$\bar{1}.990$	1,63277 ,04626
$\bar{1}.866$,44196 ,00378	$\bar{1}.891$,54472 ,00452	$\bar{1}.916$,67083 ,00572	$\bar{1}.941$,83710 ,00793	$\bar{1}.966$	1,08919 ,01348	$\bar{1}.991$	1,67903 ,05163
$\bar{1}.867$,44574 ,00380	$\bar{1}.892$,54924 ,00456	$\bar{1}.917$,67655 ,00578	$\bar{1}.942$,84503 ,00807	$\bar{1}.967$	1,10267 ,01387	$\bar{1}.992$	1,73069 ,05849
$\bar{1}.868$,44954 ,00383	$\bar{1}.893$,55380 ,00460	$\bar{1}.918$,68233 ,00584	$\bar{1}.943$,85310 ,00820	$\bar{1}.968$	1,11654 ,01429	$\bar{1}.993$	1,78918 ,06745
$\bar{1}.869$,45337 ,00385	$\bar{1}.894$,55840 ,00464	$\bar{1}.919$,68817 ,00591	$\bar{1}.944$,86130 ,00833	$\bar{1}.969$	1,13083 ,01475	$\bar{1}.994$	1,85663 ,07968
$\bar{1}.870$,45722 ,00388	$\bar{1}.895$,56304 ,00467	$\bar{1}.920$,69408 ,00598	$\bar{1}.945$,86963 ,00848	$\bar{1}.970$	1,14558 ,01523	$\bar{1}.995$	1,93631 ,09741
$\bar{1}.871$,46110 ,00390	$\bar{1}.896$,56771 ,00472	$\bar{1}.921$,70006 ,00695	$\bar{1}.946$,87811 ,00863	$\bar{1}.971$	1,16081 ,01574	$\bar{1}.996$	2,03372 ,12544
$\bar{1}.872$,46500 ,00393	$\bar{1}.897$,57243 ,00476	$\bar{1}.922$,70611 ,00611	$\bar{1}.947$,88674 ,00878	$\bar{1}.972$	1,17655 ,01530	$\bar{1}.997$	2,15916 ,17459
$\bar{1}.873$,46893 ,00396	$\bar{1}.898$,57719 ,00479	$\bar{1}.923$,71222 ,00620	$\bar{1}.948$,89552 ,00894	$\bar{1}.973$	1,19285 ,01690	$\bar{1}.998$	2,33575 ,30153
$\bar{1}.874$,47289 ,00399	$\bar{1}.899$,58198 ,00485	$\bar{1}.924$,71842 ,00626	$\bar{1}.949$,90446 ,00911	$\bar{1}.974$	1,20975 ,01753	$\bar{1}.999$	2,63728

TABLE V.—Logarithms of the accommodated chances of living 10 years, deduced from the value of an annuity for 10 years, at 5 per cent. from the actual tables of mortality, and considered equal to a geometrical series of ten terms, of which the common ratio is the same as the first term, and the tenth term the accommodated chance; and to find the accommodated chance for 5, 7 years, &c. without a table calculated for the purpose, it may be considered sufficient to multiply by .5; .7, &c. the accommodated ratio in this table when extreme accuracy be not required.

Age.	Carlisle.	Deparcieux.	Northampton.	Age.	Carlisle.	Deparcieux.	Northampton.
0	1.6892	—	—	52	1.9172	1.9006	1.8523
1	1.6763	—	1.7044	53	1.9098	1.8957	1.8471
2	1.8699	—	1.8356	54	1.9013	1.8901	1.8417
3	1.9159	1.9166	1.8790	55	1.8915	1.8853	1.8357
4	1.9401	1.9315	1.9081	56	1.8803	1.8799	1.8294
5	1.9586	1.9411	1.9220	57	1.8680	1.8732	1.8228
6	1.9686	1.9486	1.9369	58	1.8513	1.8673	1.8156
7	1.9737	1.9544	1.9476	59	1.8435	1.8601	1.8081
8	1.9764	1.9592	1.9550	60	1.8318	1.8511	1.7998
9	1.9773	1.9637	1.9586	61	1.8243	1.8398	1.7908
10	1.9768	1.9669	1.9592	62	1.8171	1.8264	1.7811
11	1.9754	1.9679	1.9582	63	1.8090	1.8120	1.7699
12	1.9742	1.9669	1.9566	64	1.7974	1.7946	1.7576
13	1.9729	1.9658	1.9546	65	1.7860	1.7735	1.7431
14	1.9716	1.9704	1.9521	66	1.7703	1.7510	1.7267
15	1.9704	1.9628	1.9490	67	1.7506	1.7270	1.7083
16	1.9698	1.9609	1.9455	68	1.7107	1.7017	1.6879
17	1.9694	1.9600	1.9419	69	1.7005	1.6754	1.6651
18	1.9693	1.9586	1.9388	70	1.6689	1.6480	1.6402
19	1.9690	1.9574	1.9358	71	1.6319	1.6167	1.6126
20	1.9685	1.9559	1.9337	72	1.5936	1.5841	1.5823
21	1.9679	1.9554	1.9321	73	1.5563	1.5500	1.5487
22	1.9670	1.9549	1.9311	74	1.5269	1.5119	1.5117
23	1.9659	1.9544	1.9298	75	1.4940	1.4711	1.4723
24	1.9644	1.9540	1.9289	76	1.4642	1.4218	1.4308
25	1.9628	1.9534	1.9277	77	1.4344	1.3684	1.3846
26	1.9573	1.9531	1.9264	78	1.4007	1.3134	1.3307
27	1.9591	1.9524	1.9257	79	1.3538	1.2497	1.2644
28	1.9570	1.9521	1.9238	80	1.3134	1.1876	1.1900
29	1.9556	1.9518	1.9226	81	1.2582	1.1214	1.1101
30	1.9552	1.9514	1.9211	82	1.2043	1.0609	1.0234
31	1.9548	1.9514	1.9196	83	1.1765	2.9688	2.9341
32	1.9540	1.9514	1.9180	84	1.0727	2.8536	2.8592
33	1.9528	1.9515	1.9164	85	2.9939	2.7199	2.7813
34	1.9513	1.9517	1.9146	86	2.9166	2.5736	2.7003
35	1.9485	1.9522	1.9126	87	2.8490	2.4254	2.6149
36	1.9477	1.9528	1.9104	88	2.8055	2.1943	2.5369
37	1.9452	1.9534	1.9083	89	2.7537	3.9129	2.4179
38	1.9437	1.9527	1.8057	90	2.6695	3.5265	2.2414
39	1.9406	1.9517	1.9031	91	2.6658	3.0266	3.9356
40	1.9383	1.9506	1.9001	92	2.7323	4.3694	3.5037
41	1.9372	1.9488	1.8973	93	2.8031	5.2971	4.7375
42	1.9365	1.9466	1.8943	94	2.8355	—	5.5769
43	1.9365	1.9438	1.8915	95	2.8107	—	7.0496
44	1.9366	1.9403	1.8882	96	2.8279	—	—
45	1.9367	1.9361	1.8848	97	2.7589	—	—
46	1.9366	1.9308	1.8810	98	2.6695	—	—
47	1.9358	1.9263	1.8767	99	2.5111	—	—
48	1.9351	1.9200	1.8740	100	2.1629	—	—
49	1.9328	1.9158	1.8631	101	3.5689	—	—
50	1.9292	1.9098	1.8621	102	4.3245	—	—
51	1.9233	1.9027	1.8571	103	6.0595	—	—

TABLE VI.—Accommodated annual ratio for an unlimited period for every age a

$$\lambda r = \lambda \overset{1.05^{-1}}{i} a - \lambda \overset{1.05^{-1}}{0} a + \lambda 1.05.$$

a	λr Carlisle.	λr Deparcieux.	λr Northampton.	a	λr Carlisle.	λr Deparcieux.	λr Northampton.
0	1.98665	—	—	52	1.98390	1.98216	1.97950
1	1.99121	—	1.98517	53	1.98305	1.98240	1.97878
2	1.99313	—	1.98997	54	1.98212	1.98156	1.97802
3	1.99458	1.99399	1.99151	55	1.98112	1.98073	1.97721
4	1.99528	1.99446	1.99244	56	1.98005	1.97982	1.97635
5	1.99577	1.99473	1.99284	57	1.97887	1.97882	1.97542
6	1.99599	1.99493	1.99324	58	1.97763	1.97780	1.97445
7	1.99606	1.99505	1.99346	59	1.97637	1.97668	1.97341
8	1.99600	1.99513	1.99357	60	1.97514	1.97545	1.97230
9	1.99600	1.99519	1.99354	61	1.97400	1.97408	1.97111
10	1.99529	1.99519	1.99341	62	1.97281	1.97254	1.96983
11	1.99576	1.99514	1.99323	63	1.97154	1.97093	1.96843
12	1.99563	1.99503	1.99304	64	1.97014	1.96912	1.96693
13	1.99549	1.99491	1.99284	65	1.96858	1.96707	1.96526
14	1.99535	1.99478	1.99262	66	1.96685	1.96489	1.96346
15	1.99522	1.99464	1.99240	67	1.96491	1.96250	1.96150
16	1.99509	1.99450	1.99213	68	1.96273	1.96009	1.95936
17	1.99487	1.99438	1.99190	69	1.96029	1.95750	1.95703
18	1.99484	1.99425	1.99167	70	1.95755	1.95480	1.95448
19	1.99471	1.99413	1.99145	71	1.95440	1.95181	1.95171
20	1.99455	1.99398	1.99124	72	1.95111	1.94870	1.94869
21	1.99442	1.99388	1.99106	73	1.94784	1.94542	1.94541
22	1.99425	1.99375	1.99088	74	1.94467	1.94180	1.94186
23	1.99408	1.99364	1.99070	75	1.94185	1.93790	1.93812
24	1.99390	1.99350	1.99051	76	1.93888	1.93330	1.93423
25	1.99370	1.99338	1.99030	77	1.93591	1.92833	1.92994
26	1.99349	1.99323	1.99009	78	1.93265	1.92314	1.92503
27	1.99328	1.99308	1.98988	79	1.92863	1.91715	1.91916
28	1.99306	1.99293	1.98965	80	1.92461	1.91124	1.91246
29	1.99290	1.99277	1.98943	81	1.92049	1.90493	1.90509
30	1.99265	1.99259	1.98917	82	1.91491	1.89856	1.89697
31	1.99245	1.99241	1.98892	83	1.90939	1.89069	1.88844
32	1.99224	1.99223	1.98865	84	1.90344	1.88005	1.88110
33	1.99201	1.99202	1.98837	85	1.89657	1.86782	1.87361
34	1.99176	1.99181	1.98808	86	1.88978	1.85416	1.86599
35	1.99149	1.99158	1.98777	87	1.88369	1.84011	1.85803
36	1.99110	1.99134	1.98745	88	1.87972	1.81788	1.85069
37	1.99090	1.99109	1.98811	89	1.87481	1.78941	1.85454
38	1.99058	1.99077	1.98675	90	1.86660	1.75223	1.82243
39	1.99025	1.99043	1.98637	91	1.86560	1.70265	1.79303
40	1.98991	1.99006	1.98597	92	1.87056	1.63694	1.74992
41	1.98958	1.98967	1.98556	93	1.87595	1.52971	1.67376
42	1.98924	1.98924	1.98513	94	1.87840	—	1.55753
43	1.98890	1.98878	1.98469	95	1.87967	—	1.30505
44	1.98853	1.98828	1.98423	96	1.77774	—	—
45	1.98814	1.98771	1.98375	97	1.77140	—	—
46	1.98771	1.98714	1.98323	98	1.76333	—	—
47	1.98725	1.98655	1.98270	99	1.84829	—	—
48	1.98673	1.98590	1.98209	100	1.81282	—	—
49	1.98612	1.98526	1.98148	101	1.75663	—	—
50	1.98546	1.98456	1.98083	102	1.65421	—	—
51	1.98471	1.98386	1.98017	103	1.40266	—	—

TABLE VII.—Logarithm of Carlisle chance of living 5 years at every age *a*.

<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.
0	$\bar{1},83232$	20	$\bar{1},98469$	40	$\bar{1},96915$	60	$\bar{1},91826$	80	$\bar{1},66927$
1	$\bar{1},89709$	21	$\bar{1},98457$	41	$\bar{1},96836$	61	$\bar{1},91483$	81	$\bar{1},64194$
2	$\bar{1},92823$	22	$\bar{1},98439$	42	$\bar{1},96790$	62	$\bar{1},91180$	82	$\bar{1},61095$
3	$\bar{1},95354$	23	$\bar{1},98405$	43	$\bar{1},96780$	63	$\bar{1},90864$	83	$\bar{1},57100$
4	$\bar{1},96747$	24	$\bar{1},98333$	44	$\bar{1},96808$	64	$\bar{1},90492$	84	$\bar{1},53422$
5	$\bar{1},97792$	25	$\bar{1},98213$	45	$\bar{1},96857$	65	$\bar{1},90067$	85	$\bar{1},50393$
6	$\bar{1},98376$	26	$\bar{1},98091$	46	$\bar{1},96918$	66	$\bar{1},89586$	86	$\bar{1},45652$
7	$\bar{1},98703$	27	$\bar{1},97967$	47	$\bar{1},96941$	67	$\bar{1},88838$	87	$\bar{1},40377$
8	$\bar{1},98869$	28	$\bar{1},97863$	48	$\bar{1},96915$	68	$\bar{1},87746$	88	$\bar{1},36691$
9	$\bar{1},98930$	29	$\bar{1},97804$	49	$\bar{1},96818$	69	$\bar{1},86279$	89	$\bar{1},34438$
10	$\bar{1},98911$	30	$\bar{1},97789$	50	$\bar{1},96676$	70	$\bar{1},84362$	90	$\bar{1},32483$
11	$\bar{1},98836$	31	$\bar{1},97783$	51	$\bar{1},96477$	71	$\bar{1},82305$	91	$\bar{1},34054$
12	$\bar{1},98754$	32	$\bar{1},97767$	52	$\bar{1},96269$	72	$\bar{1},80220$	92	$\bar{1},38021$
13	$\bar{1},98670$	33	$\bar{1},97736$	53	$\bar{1},96017$	73	$\bar{1},78348$	93	$\bar{1},41373$
14	$\bar{1},98593$	34	$\bar{1},97687$	54	$\bar{1},95660$	74	$\bar{1},76877$	94	$\bar{1},43933$
15	$\bar{1},98528$	35	$\bar{1},97611$	55	$\bar{1},95155$	75	$\bar{1},75508$	95	$\bar{1},47712$
16	$\bar{1},98490$	36	$\bar{1},97490$	56	$\bar{1},94461$	76	$\bar{1},74231$	96	$\bar{1},48337$
17	$\bar{1},98479$	37	$\bar{1},97349$	57	$\bar{1},93711$	77	$\bar{1},72712$	97	$\bar{1},44370$
18	$\bar{1},98476$	38	$\bar{1},97194$	58	$\bar{1},92973$	78	$\bar{1},71062$	98	$\bar{1},33099$
19	$\bar{1},98472$	39	$\bar{1},97044$	59	$\bar{1},92343$	79	$\bar{1},68963$		

Logarithm of the Carlisle chance of living 10 years at every age *a*.

0	$\bar{1},81023$	19	$\bar{1},96805$	38	$\bar{1},93973$	57	$\bar{1},84891$	76	$\bar{1},38425$
1	$\bar{1},88086$	20	$\bar{1},96682$	39	$\bar{1},93851$	58	$\bar{1},83836$	77	$\bar{1},33807$
2	$\bar{1},91526$	21	$\bar{1},96548$	40	$\bar{1},93772$	59	$\bar{1},82835$	78	$\bar{1},28163$
3	$\bar{1},94223$	22	$\bar{1},96406$	41	$\bar{1},93754$	60	$\bar{1},81893$	79	$\bar{1},22385$
4	$\bar{1},95677$	23	$\bar{1},96268$	42	$\bar{1},93731$	61	$\bar{1},81070$	80	$\bar{1},17320$
5	$\bar{1},96702$	24	$\bar{1},96136$	43	$\bar{1},93694$	62	$\bar{1},80018$	81	$\bar{1},09846$
6	$\bar{1},97213$	25	$\bar{1},96002$	44	$\bar{1},93626$	63	$\bar{1},78610$	82	$\bar{1},01472$
7	$\bar{1},97457$	26	$\bar{1},95873$	45	$\bar{1},93533$	64	$\bar{1},76771$	83	$\bar{2},93791$
8	$\bar{1},97540$	27	$\bar{1},95734$	46	$\bar{1},93395$	65	$\bar{1},74430$	84	$\bar{2},87860$
9	$\bar{1},97523$	28	$\bar{1},95598$	47	$\bar{1},93211$	66	$\bar{1},71891$	85	$\bar{2},82876$
10	$\bar{1},97438$	29	$\bar{1},95490$	48	$\bar{1},92932$	67	$\bar{1},69058$	86	$\bar{2},79706$
11	$\bar{1},97326$	30	$\bar{1},95400$	49	$\bar{1},92478$	68	$\bar{1},66094$	87	$\bar{2},78398$
12	$\bar{1},97233$	31	$\bar{1},95273$	50	$\bar{1},91830$	69	$\bar{1},63157$	88	$\bar{2},78064$
13	$\bar{1},97146$	32	$\bar{1},95116$	51	$\bar{1},90938$	70	$\bar{1},59870$	89	$\bar{2},78371$
14	$\bar{1},07065$	33	$\bar{1},94929$	52	$\bar{1},89980$	71	$\bar{1},56536$	90	$\bar{2},80195$
15	$\bar{1},96996$	34	$\bar{1},94730$	53	$\bar{1},88990$	72	$\bar{1},52932$	91	$\bar{2},82391$
16	$\bar{1},96947$	35	$\bar{1},94526$	54	$\bar{1},88003$	73	$\bar{1},49411$	92	$\bar{2},82391$
17	$\bar{1},96918$	36	$\bar{1},94326$	55	$\bar{1},86981$	74	$\bar{1},45840$	93	$\bar{2},74473$
18	$\bar{1},96881$	37	$\bar{1},94138$	56	$\bar{1},85944$	75	$\bar{1},42434$		

TABLE VII.—continued.

Logarithm of the Carlisle chance of living 15 years for every age *a*.

<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.
0	$\bar{1},79934$	18	$\bar{1},94744$	36	$\bar{1},91244$	54	$\bar{1},78495$	72	$\bar{1},14027$
1	$\bar{1},86922$	19	$\bar{1},94609$	37	$\bar{1},91080$	55	$\bar{1},77048$	73	$\bar{1},06511$
2	$\bar{1},90280$	20	$\bar{1},94471$	38	$\bar{1},90888$	56	$\bar{1},75530$	74	$\bar{2},99263$
3	$\bar{1},92893$	21	$\bar{1},94331$	39	$\bar{1},90669$	57	$\bar{1},73729$	75	$\bar{2},92827$
4	$\bar{1},94270$	22	$\bar{1},94173$	40	$\bar{1},90448$	58	$\bar{1},71582$	76	$\bar{2},84078$
5	$\bar{1},95230$	23	$\bar{1},94004$	41	$\bar{1},90231$	59	$\bar{1},69114$	77	$\bar{2},74184$
6	$\bar{1},95702$	24	$\bar{1},93823$	42	$\bar{1},90000$	60	$\bar{1},66256$	78	$\bar{2},64853$
7	$\bar{1},95936$	25	$\bar{1},93613$	43	$\bar{1},89712$	61	$\bar{1},63375$	79	$\bar{2},56823$
8	$\bar{1},96015$	26	$\bar{1},93364$	44	$\bar{1},89284$	62	$\bar{1},60238$	80	$\bar{2},49803$
9	$\bar{1},95995$	27	$\bar{1},93082$	45	$\bar{1},88687$	63	$\bar{1},56958$	81	$\bar{2},43900$
10	$\bar{1},95907$	28	$\bar{1},92792$	46	$\bar{1},87856$	64	$\bar{1},53648$	82	$\bar{2},39494$
11	$\bar{1},95784$	29	$\bar{1},92534$	47	$\bar{1},86922$	65	$\bar{1},49937$	83	$\bar{2},35164$
12	$\bar{1},95672$	30	$\bar{1},92316$	48	$\bar{1},85905$	66	$\bar{1},46123$	84	$\bar{2},31794$
13	$\bar{1},95551$	31	$\bar{1},92108$	49	$\bar{1},84820$	67	$\bar{1},41770$	85	$\bar{2},30588$
14	$\bar{1},95397$	32	$\bar{1},91905$	50	$\bar{1},83656$	68	$\bar{1},37157$	86	$\bar{2},28043$
15	$\bar{1},95209$	33	$\bar{1},91709$	51	$\bar{1},82421$	69	$\bar{1},32120$	87	$\bar{2},22768$
16	$\bar{1},95038$	34	$\bar{1},91538$	52	$\bar{1},81160$	70	$\bar{1},26797$	88	$\bar{2},11163$
17	$\bar{1},94885$	35	$\bar{1},91383$	53	$\bar{1},79853$	71	$\bar{1},20730$		

Logarithm of the Carlisle chance of living 20 years for every age *a*.

0	$\bar{1},78462$	17	$\bar{1},92652$	34	$\bar{1},88356$	51	$\bar{1},72007$	68	$\bar{2},94257$
1	$\bar{1},85412$	18	$\bar{1},92479$	35	$\bar{1},88059$	52	$\bar{1},69998$	69	$\bar{2},85542$
2	$\bar{1},88759$	19	$\bar{1},92295$	36	$\bar{1},87721$	53	$\bar{1},67599$	70	$\bar{2},77190$
3	$\bar{1},91369$	20	$\bar{1},92082$	37	$\bar{1},87349$	54	$\bar{1},64744$	71	$\bar{2},66383$
4	$\bar{1},92742$	21	$\bar{1},91821$	38	$\bar{1},86906$	55	$\bar{1},61410$	72	$\bar{2},54404$
5	$\bar{1},93699$	22	$\bar{1},91521$	39	$\bar{1},86329$	56	$\bar{1},57835$	73	$\bar{2},43202$
6	$\bar{1},94160$	23	$\bar{1},91197$	40	$\bar{1},85602$	57	$\bar{1},53949$	74	$\bar{2},33701$
7	$\bar{1},94376$	24	$\bar{1},90866$	41	$\bar{1},84692$	58	$\bar{1},49930$	75	$\bar{2},25311$
8	$\bar{1},94421$	25	$\bar{1},90528$	42	$\bar{1},83711$	59	$\bar{1},45991$	76	$\bar{2},18132$
9	$\bar{1},94328$	26	$\bar{1},90199$	43	$\bar{1},82684$	60	$\bar{1},41763$	77	$\bar{2},12205$
10	$\bar{1},94120$	27	$\bar{1},89872$	44	$\bar{1},81628$	61	$\bar{1},37606$	78	$\bar{2},06227$
11	$\bar{1},93874$	28	$\bar{1},89572$	45	$\bar{1},80513$	62	$\bar{1},32950$	79	$\bar{2},00757$
12	$\bar{1},93639$	29	$\bar{1},89342$	46	$\bar{1},79339$	63	$\bar{1},28021$	80	$\bar{3},97515$
13	$\bar{1},93414$	30	$\bar{1},89172$	47	$\bar{1},78101$	64	$\bar{1},22611$	81	$\bar{3},92237$
14	$\bar{1},93201$	31	$\bar{1},89027$	48	$\bar{1},76768$	65	$\bar{1},16864$	82	$\bar{3},83863$
15	$\bar{1},92999$	32	$\bar{1},88847$	49	$\bar{1},75312$	66	$\bar{1},10317$	83	$\bar{3},68263$
16	$\bar{1},92821$	33	$\bar{1},88624$	50	$\bar{1},73723$	67	$\bar{1},02866$		

TABLE VII.—continued.

Logarithm of the Carlisle chance of living 25 years at every age *a*.

<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.
0	1,76930	16	1,90311	32	1,85116	48	1,64514	64	2,76034
1	1,83969	17	1,90001	33	1,84641	49	1,61591	65	2,67257
2	1,87198	18	1,89673	34	1,84016	50	1,58086	66	2,55969
3	1,89774	19	1,89339	35	1,83213	51	1,5431	67	2,43242
4	1,91075	20	1,88997	36	1,82182	52	1,50218	68	2,30948
5	1,91912	21	1,88657	37	1,81060	53	1,45948	69	2,19980
6	1,92251	22	1,88311	38	1,79878	54	1,41651	70	2,09673
7	1,92342	23	1,87977	39	1,78672	55	1,36918	71	2,00437
8	1,92283	24	1,87674	40	1,77428	56	1,32067	72	1,92425
9	1,92131	25	1,87385	41	1,76175	57	1,26661	73	1,84575
10	1,91909	26	1,87117	42	1,74891	58	1,20993	74	1,77634
11	1,91657	27	1,86813	43	1,73548	59	1,14954	75	1,71323
12	1,91406	28	1,86487	44	1,72120	60	1,08690	76	1,66469
13	1,91150	29	1,86159	45	1,70581	61	1,01800	77	1,56575
14	1,90888	30	1,85848	46	1,68926	62	1,94045	78	1,39326
15	1,90610	31	1,85504	47	1,66940	63	2,85121		

Logarithm of the Carlisle chance of living 30 years at every age *a*.

0	1,75143	15	1,87525	30	1,81003	45	1,54943	60	2,59083
1	1,81960	16	1,87146	31	1,79964	46	1,51231	61	2,47452
2	1,85164	17	1,86790	32	1,78827	47	1,47160	62	2,34422
3	1,87637	18	1,86453	33	1,77614	48	1,42863	63	2,21811
4	1,88879	19	1,86147	34	1,76358	49	1,38467	64	2,10472
5	1,89701	20	1,85854	35	1,75039	50	1,33594	65	1,99740
6	1,90033	21	1,85575	36	1,73665	51	1,28544	66	1,90023
7	1,90109	22	1,85253	37	1,72240	52	1,22930	67	1,81264
8	1,90019	23	1,84892	38	1,70742	53	1,17010	68	1,72321
9	1,89818	24	1,84492	39	1,69164	54	1,10614	69	1,63913
10	1,89520	25	1,84061	40	1,67496	55	1,03845	70	1,55385
11	1,89147	26	1,83594	41	1,65761	56	1,96261	71	1,48774
12	1,88755	27	1,83083	42	1,63729	57	2,87756	72	1,36795
13	1,88344	28	1,82504	43	1,61294	58	2,78093	73	1,17674
14	1,87931	29	1,81819	44	1,58399	59	2,68376		

Logarithm of the Carlisle chance of living 35 years for every age *a*.

0	1,72933	14	1,84739	28	1,75476	42	1,43949	56	2,41913
1	1,79743	15	1,84382	29	1,74162	43	1,39642	57	2,28133
2	1,82932	16	1,84065	30	1,72829	44	1,35277	58	2,14784
3	1,85373	17	1,83732	31	1,71448	45	1,30451	59	2,02815
4	1,86565	18	1,83368	32	1,70007	46	1,25462	60	1,91566
5	1,87312	19	1,82964	33	1,68477	47	1,19872	61	1,81506
6	1,87523	20	1,82530	34	1,66850	48	1,13925	62	1,72443
7	1,87458	21	1,82052	35	1,65106	49	1,07432	63	1,63185
8	1,87213	22	1,81522	36	1,63251	50	1,00520	64	1,54405
9	1,86861	23	1,80909	37	1,61078	51	1,92738	65	1,47452
10	1,86435	24	1,80152	38	1,58488	52	1,84025	66	1,38360
11	1,85983	25	1,79216	39	1,55443	53	1,74110	67	1,25633
12	1,85544	26	1,78055	40	1,51858	54	1,64036	68	1,05420
13	1,85123	27	1,76794	41	1,48066	55	1,54237		

TABLE VII.—continued.

Logarithm of the Carlisle chance of living 40 years for every age a .

a	λ chance.	a	λ chance.	a	λ chance.	a	λ chance.	a	λ chance.
0	$\bar{1},70544$	13	$\bar{1},82038$	26	$\bar{1},69538$	39	$\bar{1},32320$	52	$\bar{2},24402$
1	$\bar{1},77233$	14	$\bar{1},81557$	27	$\bar{1},67973$	40	$\bar{1},27366$	53	$\bar{2},10801$
2	$\bar{1},80280$	15	$\bar{1},81057$	28	$\bar{1},66340$	41	$\bar{1},22298$	54	$\bar{3},98474$
3	$\bar{1},82567$	16	$\bar{1},80542$	29	$\bar{1},64654$	42	$\bar{1},16661$	55	$\bar{3},86721$
4	$\bar{1},83609$	17	$\bar{1},80001$	30	$\bar{1},62896$	43	$\bar{1},10705$	56	$\bar{3},75967$
5	$\bar{1},84227$	18	$\bar{1},79385$	31	$\bar{1},61034$	44	$\bar{1},04240$	57	$\bar{3},66154$
6	$\bar{1},84359$	19	$\bar{1},78624$	32	$\bar{1},58845$	45	$\bar{2},97377$	58	$\bar{3},56157$
7	$\bar{1},84247$	20	$\bar{1},77684$	33	$\bar{1},56223$	46	$\bar{2},89656$	59	$\bar{3},46748$
8	$\bar{1},83992$	21	$\bar{1},76513$	34	$\bar{1},53130$	47	$\bar{2},80967$	60	$\bar{3},39278$
9	$\bar{1},83669$	22	$\bar{1},75233$	35	$\bar{1},49469$	48	$\bar{2},71025$	61	$\bar{3},29843$
10	$\bar{1},83292$	23	$\bar{1},73882$	36	$\bar{1},45556$	49	$\bar{2},60854$	62	$\bar{3},16813$
11	$\bar{1},82901$	24	$\bar{1},72494$	37	$\bar{1},41298$	50	$\bar{2},50913$	63	$\bar{4},96284$
12	$\bar{1},82486$	25	$\bar{1},71042$	38	$\bar{1},36836$	51	$\bar{2},38390$		

Logarithm of the Carlisle chance of living 45 years for every age a .

0	$\bar{1},67459$	12	$\bar{1},78755$	24	$\bar{1},62987$	36	$\bar{1},19787$	48	$\bar{2},07716$
1	$\bar{1},74068$	13	$\bar{1},78055$	25	$\bar{1},61109$	37	$\bar{1},14010$	49	$\bar{3},95292$
2	$\bar{1},77070$	14	$\bar{1},77217$	26	$\bar{1},59125$	38	$\bar{1},07899$	50	$\bar{3},83396$
3	$\bar{1},79346$	15	$\bar{1},76212$	27	$\bar{1},56812$	39	$\bar{1},01283$	51	$\bar{3},72444$
4	$\bar{1},80417$	16	$\bar{1},75002$	28	$\bar{1},54086$	40	$\bar{2},94292$	52	$\bar{3},62423$
5	$\bar{1},81084$	17	$\bar{1},73712$	29	$\bar{1},50933$	41	$\bar{2},86492$	53	$\bar{3},52174$
6	$\bar{1},81277$	18	$\bar{1},72357$	30	$\bar{1},47258$	42	$\bar{2},77756$	54	$\bar{3},42408$
7	$\bar{1},81190$	19	$\bar{1},70967$	31	$\bar{1},43339$	43	$\bar{2},67805$	55	$\bar{3},34433$
8	$\bar{1},80907$	20	$\bar{1},69510$	32	$\bar{1},39065$	44	$\bar{2},57662$	56	$\bar{3},24304$
9	$\bar{1},80487$	21	$\bar{1},67996$	33	$\bar{1},34571$	45	$\bar{2},47770$	57	$\bar{3},10524$
10	$\bar{1},79968$	22	$\bar{1},66412$	34	$\bar{1},30007$	46	$\bar{2},35308$	58	$\bar{4},89256$
11	$\bar{1},79378$	23	$\bar{1},64745$	35	$\bar{1},24977$	47	$\bar{2},21344$		

Logarithm of the Carlisle chance of living 50 years for every age a .

0	$\bar{1},64316$	11	$\bar{1},73839$	22	$\bar{1},55251$	33	$\bar{1},05634$	44	$\bar{3},92100$
1	$\bar{1},70987$	12	$\bar{1},72466$	23	$\bar{1},52491$	34	$\bar{2},93970$	45	$\bar{3},80253$
2	$\bar{1},74011$	13	$\bar{1},71028$	24	$\bar{1},49266$	35	$\bar{2},91903$	46	$\bar{3},69362$
3	$\bar{1},76261$	14	$\bar{1},69560$	25	$\bar{1},45471$	36	$\bar{2},83982$	47	$\bar{3},59365$
4	$\bar{1},77234$	15	$\bar{1},68038$	26	$\bar{1},41430$	37	$\bar{2},75105$	48	$\bar{3},49089$
5	$\bar{1},77760$	16	$\bar{1},66486$	27	$\bar{1},37032$	38	$\bar{2},65000$	49	$\bar{3},39225$
6	$\bar{1},77754$	17	$\bar{1},64892$	28	$\bar{1},32434$	39	$\bar{2},54705$	50	$\bar{3},31109$
7	$\bar{1},77458$	18	$\bar{1},63222$	29	$\bar{1},27810$	40	$\bar{2},44685$	51	$\bar{3},20781$
8	$\bar{1},76925$	19	$\bar{1},61459$	30	$\bar{1},22766$	41	$\bar{2},32144$	52	$\bar{3},06793$
9	$\bar{1},76147$	20	$\bar{1},59577$	31	$\bar{1},17570$	42	$\bar{2},18133$	53	$\bar{4},85274$
10	$\bar{1},75123$	21	$\bar{1},57582$	32	$\bar{1},11777$	43	$\bar{2},04495$		

TABLE VII.—continued.

Logarithm of the Carlisle chance of living 55 years for every age *a*.

<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.	<i>a</i>	λ chance.
0	$\bar{1},60991$	10	$\bar{1},66949$	20	$\bar{1},43940$	30	$\bar{2},89693$	40	$\bar{3},77168$
1	$\bar{1},67464$	11	$\bar{1},65322$	21	$\bar{1},39887$	31	$\bar{2},81764$	41	$\bar{3},66198$
2	$\bar{1},70281$	12	$\bar{1},63646$	22	$\bar{1},35471$	32	$\bar{2},72872$	42	$\bar{3},56155$
3	$\bar{1},72278$	13	$\bar{1},61892$	23	$\bar{1},30840$	33	$\bar{2},62734$	43	$\bar{3},45869$
4	$\bar{1},72894$	14	$\bar{1},60051$	24	$\bar{1},26143$	34	$\bar{2},52392$	44	$\bar{3},36033$
5	$\bar{1},72914$	15	$\bar{1},58105$	25	$\bar{1},20979$	35	$\bar{2},42296$	45	$\bar{3},27966$
6	$\bar{1},72215$	16	$\bar{1},56072$	26	$\bar{1},15661$	36	$\bar{2},29634$	46	$\bar{3},17699$
7	$\bar{1},71169$	17	$\bar{1},53730$	27	$\bar{1},09743$	37	$\bar{2},15482$	47	$\bar{3},03735$
8	$\bar{1},69897$	18	$\bar{1},50967$	28	$\bar{1},03497$	38	$\bar{2},01689$	48	$\bar{4},82189$
9	$\bar{1},68490$	19	$\bar{1},47738$	29	$\bar{2},96773$	39	$\bar{3},89144$		

Logarithm of the Carlisle chance of living 60 years for every age *a*.

0	$\bar{1},56146$	9	$\bar{1},58982$	18	$\bar{1},29315$	27	$\bar{2},70839$	36	$\bar{3},63689$
1	$\bar{1},61925$	10	$\bar{1},57016$	19	$\bar{1},24615$	28	$\bar{2},60597$	37	$\bar{3},53503$
2	$\bar{1},63992$	11	$\bar{1},54908$	20	$\bar{1},19448$	29	$\bar{2},50196$	38	$\bar{3},43063$
3	$\bar{1},65251$	12	$\bar{1},52484$	21	$\bar{1},14119$	30	$\bar{2},40086$	39	$\bar{3},33077$
4	$\bar{1},65237$	13	$\bar{1},49638$	22	$\bar{1},08183$	31	$\bar{2},27417$	40	$\bar{3},24881$
5	$\bar{1},64740$	14	$\bar{1},46331$	23	$\bar{1},01902$	32	$\bar{2},13249$	41	$\bar{3},14535$
6	$\bar{1},63698$	15	$\bar{1},42467$	24	$\bar{2},95106$	33	$\bar{3},99425$	42	$\bar{3},00524$
7	$\bar{1},62349$	16	$\bar{1},38377$	25	$\bar{2},87906$	34	$\bar{3},86830$	43	$\bar{4},78568$
8	$\bar{1},60761$	17	$\bar{1},33950$	26	$\bar{2},79855$	35	$\bar{3},74779$		

Logarithm of the Carlisle chance of living 65 years for every age *a*.

0	$\bar{1},47972$	8	$\bar{1},48507$	16	$\bar{1},12608$	24	$\bar{2},48528$	32	$\bar{3},51270$
1	$\bar{1},53408$	9	$\bar{1},45261$	17	$\bar{1},06562$	25	$\bar{2},38298$	33	$\bar{3},40798$
2	$\bar{1},55171$	10	$\bar{1},41378$	18	$\bar{1},00378$	26	$\bar{2},25507$	34	$\bar{3},30763$
3	$\bar{1},56115$	11	$\bar{1},37213$	19	$\bar{2},93578$	27	$\bar{2},11216$	35	$\bar{3},22492$
4	$\bar{1},55729$	12	$\bar{1},32704$	20	$\bar{2},86374$	28	$\bar{3},97288$	36	$\bar{3},12025$
5	$\bar{1},54807$	13	$\bar{1},27986$	21	$\bar{2},78313$	29	$\bar{3},84634$	37	$\bar{4},97873$
6	$\bar{1},53285$	14	$\bar{1},23208$	22	$\bar{2},69278$	30	$\bar{3},72569$	38	$\bar{4},76162$
7	$\bar{1},51187$	15	$\bar{1},17975$	23	$\bar{2},59002$	31	$\bar{3},61471$		

Logarithm of the Carlisle chance of living 70 years for every age *a*.

0	$\bar{1},38039$	7	$\bar{1},31407$	14	$\bar{2},92171$	21	$\bar{2},23965$	28	$\bar{3},38661$
1	$\bar{1},42994$	8	$\bar{1},26855$	15	$\bar{2},84902$	22	$\bar{2},09655$	29	$\bar{3},28567$
2	$\bar{1},44010$	9	$\bar{1},22138$	16	$\bar{2},76802$	23	$\bar{3},95693$	30	$\bar{3},20281$
3	$\bar{1},43861$	10	$\bar{1},16886$	17	$\bar{2},67757$	24	$\bar{3},82967$	31	$\bar{3},09808$
4	$\bar{1},42008$	11	$\bar{1},11445$	18	$\bar{2},57478$	25	$\bar{3},70782$	32	$\bar{4},95640$
5	$\bar{1},39170$	12	$\bar{1},05416$	19	$\bar{2},47001$	26	$\bar{3},59561$	33	$\bar{4},73897$
6	$\bar{1},35590$	13	$\bar{2},99049$	20	$\bar{2},36767$	27	$\bar{3},49237$		

TABLE VII.—continued.

Logarithm of the Carlisle chance of living 75 years for every age a .

a	λ chance.	a	λ chance.	a	λ chance.	a	λ chance.	a	λ chance.
0	$\bar{1},22402$	6	$\bar{1},09821$	12	$\bar{2},66511$	18	$\bar{3},94169$	24	$\bar{3},26900$
1	$\bar{1},25299$	7	$\bar{1},04119$	13	$\bar{2},56149$	19	$\bar{3},81439$	25	$\bar{3},18494$
2	$\bar{1},24230$	8	$\bar{2},97918$	14	$\bar{2},45593$	20	$\bar{3},69250$	26	$\bar{3},07898$
3	$\bar{1},22209$	9	$\bar{2},91101$	15	$\bar{2},35295$	21	$\bar{3},58019$	27	$\bar{2},93607$
4	$\bar{1},18885$	10	$\bar{2},83813$	16	$\bar{2},22455$	22	$\bar{3},47676$	28	$\bar{2},71760$
5	$\bar{1},14678$	11	$\bar{2},75639$	17	$\bar{2},08134$	23	$\bar{3},37066$		

Logarithm of the Carlisle chance of living 80 years for every age a .

0	$\bar{2},97909$	5	$\bar{2},81604$	10	$\bar{2},34206$	15	$\bar{3},67778$	20	$\bar{3},16963$
1	$\bar{2},99530$	6	$\bar{2},74015$	11	$\bar{2},21291$	16	$\bar{3},56508$	21	$\bar{3},06356$
2	$\bar{2},96941$	7	$\bar{2},65214$	12	$\bar{2},06888$	17	$\bar{3},46155$	22	$\bar{2},92046$
3	$\bar{2},93272$	8	$\bar{2},55018$	13	$\bar{3},92839$	18	$\bar{3},35542$	23	$\bar{2},70166$
4	$\bar{2},87848$	9	$\bar{2},44523$	14	$\bar{3},80031$	19	$\bar{3},25372$		

Logarithm of the Carlisle chance of living 85 years for every age a .

0	$\bar{2},64836$	4	$\bar{2},41271$	8	$\bar{3},91708$	12	$\bar{3},44909$	16	$\bar{3},04845$
1	$\bar{2},63724$	5	$\bar{2},31997$	9	$\bar{3},78962$	13	$\bar{3},34213$	17	$\bar{2},90525$
2	$\bar{2},58037$	6	$\bar{2},19667$	10	$\bar{3},66689$	14	$\bar{3},23965$	18	$\bar{2},68641$
3	$\bar{2},50372$	7	$\bar{2},05591$	11	$\bar{3},55345$	15	$\bar{3},15490$		

Logarithm of the Carlisle chance of living 90 years for every age a .

0	$\bar{2},15229$	3	$\bar{3},87062$	6	$\bar{3},53721$	9	$\bar{3},22895$	12	$\bar{2},89279$
1	$\bar{2},09377$	4	$\bar{3},75709$	7	$\bar{3},43612$	10	$\bar{3},14401$	13	$\bar{2},67312$
2	$\bar{3},98414$	5	$\bar{3},64480$	8	$\bar{3},33082$	11	$\bar{3},03682$		

Logarithm of the Carlisle chance of living 95 years for every age a .

0	$\bar{3},47712$	2	$\bar{3},36435$	4	$\bar{3},19642$	6	$\bar{3},02058$	8	$\bar{2},66181$
1	$\bar{3},43431$	3	$\bar{3},28436$	5	$\bar{3},12193$	7	$\bar{2},87982$		

Logarithm of the Carlisle chance of living 100 years for every age a .

0	$\bar{4},95424$	1	$\bar{4},91768$	2	$\bar{4},80805$	3	$\bar{4},61535$
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TABLE VIII.—Logarithm of Deparcieux chance of living for every age *a*.

<i>a</i>	10 years.	20 years.	30 years.	40 years.	50 years.	60 years.	70 years.	80 years.	90 years.
0									
1									
2									
3	1.93450	1.89763	1.85126	1.80346	1.73957	1.62634	1.39967	2.85126	3.30103
4	1.94469	1.90644	1.85957	1.81188	1.74401	1.62495	1.37684	2.78408	3.01323
5	1.95159	1.91193	1.86455	1.81698	1.74418	1.61979	1.34747	2.70443	
6	1.95683	1.91575	1.86784	1.82040	1.74248	1.61130	1.31482	2.61130	
7	1.96027	1.91825	1.86981	1.82177	1.73928	1.59968	1.27663	2.50098	
8	1.96282	1.91985	1.87151	1.82222	1.73410	1.58512	1.23231	2.38721	
9	1.96495	1.92101	1.87278	1.82146	1.72821	1.56781	1.18415	2.25473	
10	1.96614	1.92122	1.87309	1.81970	1.72110	1.54688	1.12740	2.09691	
11	1.96582	1.92042	1.87239	1.81612	1.71269	1.52337	1.06380	3.90458	
12	1.96448	1.91860	1.87069	1.81067	1.70296	1.49545	2.99190	3.66454	
13	1.96313	1.91676	1.86896	1.80507	1.69184	1.46517	2.91676	3.36653	
14	1.96175	1.91488	1.86719	1.79932	1.68026	1.43215	2.83939	3.06854	
15	1.96034	1.91296	1.86539	1.79259	1.66820	1.39588	2.75284		
16	1.95892	1.91101	1.86357	1.78565	1.65447	1.35799	2.65447		
17	1.95798	1.90954	1.86150	1.77901	1.63941	1.31636	2.54071		
18	1.95703	1.90869	1.85940	1.77128	1.62230	1.26949	2.42439		
19	1.95606	1.90783	1.85651	1.76326	1.60286	1.21920	2.28978		
20	1.95508	1.90695	1.85356	1.75496	1.58074	1.16126	2.13077		
21	1.95460	1.90657	1.85030	1.74687	1.55755	1.09798	3.93876		
22	1.95412	1.90621	1.84619	1.73848	1.53097	1.02742	3.70006		
23	1.95363	1.90583	1.84194	1.72871	1.50204	2.95363	3.40340		
24	1.95313	1.90544	1.83757	1.71851	1.47040	2.87764	3.10679		
25	1.95262	1.90505	1.83225	1.70786	1.43554	2.79250			
26	1.95209	1.90465	1.82673	1.69555	1.39907	2.69555			
27	1.95156	1.90352	1.82103	1.68143	1.35838	2.58273			
28	1.95166	1.90237	1.81425	1.66527	1.31246	2.46736			
29	1.95177	1.90045	1.80720	1.64180	1.26314	2.33372			
30	1.95187	1.89848	1.79928	1.62566	1.20618	2.17569			
31	1.95197	1.89570	1.79227	1.60295	1.14338	3.98416			
32	1.95209	1.89207	1.78436	1.57685	1.07330	3.74594			
33	1.95220	1.88831	1.77508	1.54841	1.00000	3.44977			
34	1.95231	1.88444	1.76538	1.51727	2.92451	3.15366			
35	1.95243	1.87963	1.75524	1.48292	2.83988				
36	1.95256	1.87464	1.74346	1.44698	2.74346				
37	1.95196	1.86947	1.72937	1.40682	2.63117				
38	1.95071	1.86259	1.71361	1.36080	2.51570				
39	1.94868	1.85543	1.69503	1.31137	2.38195				
40	1.94661	1.84801	1.67379	1.25431	2.22382				
41	1.94373	1.84030	1.65098	1.19141	2.03219				
42	1.93998	1.83227	1.62476	1.12121	3.79385				
43	1.93611	1.82288	1.59621	1.04780	3.49757				
44	1.93213	1.81307	1.56496	2.97220	3.20135				
45	1.92720	1.80281	1.53049	2.88745					
46	1.92208	1.79090	1.49442	2.79090					
47	1.91751	1.77791	1.45486	2.67921					
48	1.91188	1.76290	1.41009	2.56439					
49	1.90675	1.74635	1.36269	2.43327					
50	1.90140	1.72718	1.30770	2.27721					
51	1.89657	1.70725	1.24768	2.08846					
52	1.89229	1.68478	1.18123	3.85387					

[continued.]

TABLE IX.—Logarithm of the Northampton chance for living at every age *a*.

<i>a</i>	10 years.	20 years.	30 years.	40 years.	50 years.	60 years.	70 years.	80 years.	90 years.
0	1̄,68764	1̄,64396	1̄,57564	1̄,49417	1̄,38958	1̄,24287	1̄,02428	2̄,60484	3̄,59643
1	1̄,81295	1̄,76713	1̄,69746	1̄,61431	1̄,50640	1̄,35435	1̄,12443	2̄,67151	3̄,59446
2	1̄,88378	1̄,83536	1̄,76454	1̄,67952	1̄,56809	1̄,41046	1̄,16788	2̄,67677	3̄,51790
3	1̄,91089	1̄,85979	1̄,78780	1̄,70070	1̄,58568	1̄,42229	1̄,16522	2̄,62961	3̄,37283
4	1̄,92894	1̄,87511	1̄,80190	1̄,71263	1̄,59383	1̄,42421	1̄,15070	2̄,55993	3̄,14495
5	1̄,93843	1̄,88180	1̄,80733	1̄,71581	1̄,59300	1̄,41691	1̄,12431	2̄,47370	4,80625
6	1̄,94739	1̄,88788	1̄,81211	1̄,71823	1̄,59118	1̄,40806	1̄,09339	2̄,37854	
7	1̄,95322	1̄,89101	1̄,81390	1̄,71755	1̄,58601	1̄,39522	1̄,05661	2̄,27263	
8	1̄,95660	1̄,89203	1̄,81352	1̄,71459	1̄,57827	1̄,37909	1̄,01505	2̄,15453	
9	1̄,95739	1̄,89080	1̄,81084	1̄,70923	1̄,56781	1̄,35940	2̄,99001	2,03386	
10	1̄,95632	1̄,88800	1̄,80653	1̄,70194	1̄,55523	1̄,33664	2,91720	3,90879	
11	1̄,95418	1̄,88451	1̄,80136	1̄,69345	1̄,54140	1̄,31148	2,85856	3,78151	
12	1̄,95158	1̄,88076	1̄,79574	1̄,68431	1̄,52668	1̄,28410	2,79299	3,63412	
13	1̄,94890	1̄,87691	1̄,78981	1̄,67479	1̄,51140	1̄,25433	2,71872	3,46194	
14	1̄,94617	1̄,87296	1̄,78369	1̄,66489	1̄,49527	1̄,22176	2,63099	3,21602	
15	1̄,94337	1̄,86890	1̄,77738	1̄,65457	1̄,47848	1̄,18588	2,53527	4,86782	
16	1̄,94049	1̄,86472	1̄,77084	1̄,64379	1̄,46067	1̄,14600	2,43115		
17	1̄,93779	1̄,86068	1̄,76433	1̄,63279	1̄,44200	1̄,10339	2,31941		
18	1̄,93543	1̄,85692	1̄,75799	1̄,62167	1̄,42249	1̄,05845	2,19793		
19	1̄,93341	1̄,85345	1̄,75184	1̄,61042	1̄,40201	1̄,01162	2,07647		
20	1̄,93168	1̄,85021	1̄,74562	1̄,59891	1̄,38032	2,96088	3,95247		
21	1̄,93033	1̄,84718	1̄,73927	1̄,58722	1̄,35730	2,90438	3,82733		
22	1̄,92918	1̄,84417	1̄,73274	1̄,57511	1̄,33253	2,84141	3,68255		
23	1̄,92801	1̄,84091	1̄,72589	1̄,56250	1̄,30543	2,76982	3,51304		
24	1̄,92679	1̄,83752	1̄,71872	1̄,54910	1̄,27559	2,68482	3,26984		
25	1̄,92553	1̄,83401	1̄,71120	1̄,53511	1̄,24251	2,59191	4,92445		
26	1̄,92423	1̄,83035	1̄,70330	1̄,52018	1̄,20551	2,49066			
27	1̄,92289	1̄,82654	1̄,69500	1̄,50421	1̄,16560	2,38162			
28	1̄,92149	1̄,82256	1̄,68624	1̄,48706	1̄,12302	2,26250			
29	1̄,92004	1̄,81843	1̄,67701	1̄,46860	1̄,07821	2,14306			
30	1̄,91853	1̄,81394	1̄,66723	1̄,44864	1̄,02920	2,02079			
31	1̄,91685	1̄,80894	1̄,65689	1̄,42697	2,97405	3,89700			
32	1̄,91498	1̄,80355	1̄,64592	1̄,40334	2,91223	3,75336			
33	1̄,91290	1̄,79788	1̄,63449	1̄,37742	2,84181	3,58503			
34	1̄,91073	1̄,79193	1̄,62231	1̄,34880	2,75802	3,34305			
35	1̄,90848	1̄,78567	1̄,60958	1̄,31698	2,66637	4,95892			
36	1̄,90612	1̄,77907	1̄,59595	1̄,28128	2,56643				
37	1̄,90365	1̄,77211	1̄,58132	1̄,24271	2,45873				
38	1̄,90107	1̄,76475	1̄,56557	1̄,20153	2,34101				
39	1̄,89839	1̄,75697	1̄,54856	1̄,15817	2,22302				
40	1̄,89541	1̄,74870	1̄,53011	1̄,11067	2,10226				
41	1̄,89209	1̄,74004	1̄,51012	1̄,05720	3,98015				
42	1̄,88857	1̄,73094	1̄,48836	2,99725	3,83838				
43	1̄,88498	1̄,72159	1̄,46452	2,92891	3,67213				
44	1̄,88120	1̄,71158	1̄,43807	2,84730	3,43232				
45	1̄,87719	1̄,70110	1̄,40850	2,75759	3,09044				
46	1̄,87295	1̄,68983	1̄,37516	2,66031					
47	1̄,86846	1̄,67767	1̄,33906	2,55508					
48	1̄,86368	1̄,66450	1̄,30046	2,43994					
49	1̄,85858	1̄,65017	1̄,25978	2,32463					
50	1̄,85329	1̄,63470	1̄,21526	2,20685					
51	1̄,84795	1̄,61803	1̄,16511	2,08806					
52	1̄,84237	1̄,59979	1,10868	3,94981					

[continued.]

How the value of particular assurances may be determined from the value of annuities, is shown in my Paper in the Philosophical Transactions for the year 1820, many of the cases of which are solved by methods essentially the same as those which have been long adopted; but when such assurances are but for terms, which are not of great extension, very near approximations may be had by using a geometrical progression, without confining the arithmetical operations to the same route, since the chance of extinction of the joint lives of the present age $a, b, c, \&c.$ taking place between the period commencing with the time $n+t-1$, and finishing with the time $n+t$, from the present, is $= (\overset{L}{n+t-1} : a, b, c, \&c. - \overset{L}{n+t} : a, b, c, \&c.) \div \overset{L}{a, b, c, \&c.}$; it follows that if r be the present value of unity, to be received certain in the time 1, and $\overset{L}{n+t-1} : a, b, c, \&c. = \overset{L}{n-1} : a, b, c, \&c. \times \pi^t$, whatever t

may be, that $\overset{r}{m} \left[\begin{smallmatrix} 1 \\ n \\ m \end{smallmatrix} \right] a, b, c, \&c.$ or the assurance of unity to be received at the first of the equal periods 1, from the commencement of the time $n-1$ to the expiration of the time m , which shall happen after the extinction of the joint lives, is equal to $\frac{\overset{L}{n-1} : a, b, c, \&c.}{\overset{L}{a, b, c, \&c.}} \times \left\{ r^n \times (1-\pi) + r^{n+1} \times (\pi - \pi^2) + r^{n+2} \times (\pi^2 - \pi^3) \dots r^m \times (\pi^{m-n-1} - \pi^{m-n}) \right\} = (1-\pi) \times \frac{\overset{L}{n-1} : a, b, c, \&c.}{\overset{L}{a, b, c, \&c.}} \times \left\{ r + \pi r^2 + \pi^2 r^3 + \pi^3 r^4 \dots r^m \pi^{m-n-1} \right\} = \frac{(1-\pi)r}{\overset{L}{a, b, c, \&c.}} \times \left\{ r^{n-1} \overset{L}{n-1} : a, b, c, \&c. + r^n \overset{L}{n} : a, b, c, \&c. + \dots + r^{m-1} \overset{L}{m-1} : a, b, c, \&c. \right\} = (1-\pi) \cdot r \times \overset{r}{m-1} \left[\begin{smallmatrix} 1 \\ n-1 \end{smallmatrix} \right] a, b, c, \&c.$

If the assurance be not deferred, n will be equal to 1, and we shall have, according to the hypothesis, $\overset{r}{m} \left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \right] a, b, c, \&c. = (1-\pi) \cdot r \times \overset{r}{m} \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix} \right] a, b, c, \&c.$; and also $= \frac{1-\pi}{\pi} \cdot \overset{r}{m} \left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \right] a, b, c, \&c.$ If t be

taken equal to 1, we shall have from the equation $L_{n+t-1:a,b,c,\&c.} =$
 $L_{n-1:a,b,c,\&c.} \times \pi^t$, $\pi = \frac{L_{n:a,b,c,\&c.}}{L_{n-1:a,b,c,\&c.}}$, and this would be the real
value which should be taken for π , if the geometrical pro-
gression coincided perfectly with the fact; and it would be
indifferent whether we made it equal to $\frac{L_{n+t:a,b,c,\&c.}}{L_{n-1+t:a,b,c,\&c.}}$, or
 $\frac{L_{n:a,b,c,\&c.}}{L_{n-1:a,b,c,\&c.}}$, as the two would be the same; but this not
being the case, there will be a preference; and generally, if
not always, π should be taken an intermediate value between
the two; and when the term is not very long, it will answer
a good purpose to take it about the middle between them,
inclining generally, though perhaps not always, rather
nearer the last than the first, as the first terms are generally
of more consequence than the last. If the said assurance be
not deferred, and instead of being paid for immediately, be
to be paid for by equal periodic payments, at an unite of
time from each other, up to the time $m-1$ inclusive, and the
first payment be to be made immediately, then will the
present value of such periodic payment be $\overset{r}{\underset{m-1}{\int}} a, b, c, \&c.$, and
consequently each payment, from what is shown above, is
equal to $\frac{\overset{r}{\underset{1}{\int}} a, b, c, \&c.}{\overset{r}{\underset{m-1}{\int}} a, b, c, \&c.} = (1-\pi) \cdot r$. From whence
we may draw an inference worthy of remark, namely; when
an assurance of joint lives is meant to commence immedi-
ately, and to continue for a term of t years, which is not
large, and to be paid for by t annual payments, that those
payments will not differ much with the increase of the time
 t , provided, as I have said, that t be not large, and the ages

be not at the extremes of life, a consequence which follows from the near agreement to a geometrical progression which takes place in the number of living at each small equal increment of time; that is to say, from the near coincidence of $\frac{L}{n : a, b, c, \&c.}$ with $\frac{L}{n+t : a, b, c, \&c.}$, or the small variation of π for the different values of t : and also, that when the number of years for which an assurance continues be not very long, and the ages be not at the extremes of life, the annual premiums will not differ widely from the premiums to be paid for an assurance of one year of a life older than the proposed life by about half the term: thus, according to the Northampton table, at three per cent. to assure 100 *l.* at the

Age	15	20	30	40	50	60	64
For 7 years, the annual premium by the common modes of calculation	£1..2..11	1.. 9.. 5	1..14..11	2.. 4.. 1	3.. 0.. 8	4.. 7.. 1	5.. 4..10
And the premium for one year assurance for an age 3 years older	1..3.. 3	1.. 9.. 8	1..15.. 0	2.. 4.. 6	3.. 1.. 0	4.. 7.. 8	5 5.. 6

the difference of which is very small.—As another example, let

Age	10	20	30	40	50	60
For 10 years, the annual premium will be, by common modes of calculation	£0..19.. 2	1..9.. 1	1..15.. 8	2.. 5.. 8	3.. 3.. 4	4..12.. 6
Premium for one year assurance, age 5 years older	0..17..11	1..10.. 7	1..16.. 4	2.. 6.. 8	3.. 5.. 1	4..15.. 2

Here, except at the age 10, the excess is rather more in the approximation than in the first set of examples; but it should be recollected, that we took the exact middle, instead of inclining to the early age.

According to the Carlisle table of mortality at 3 per cent. to assure 100*l.* at the

Age	10	20	30	40	50	60
For 7 years, the annual premium, by common modes of calculation .	£0 10 5	0 13 10	0 19 10	1 7 8	1 11 0	3 13 8
For one year, the premium	0 10 5	0 13 9	0 19 2	1 8 6	1 12 1	3 15 9
For 10 years, the annual premium, by common modes of calculation .	0 11 3	0 14 7	1 0 4	1 7 7	1 14 11	3 17 8
For one year, at an age 5 years older	0 12 0	0 14 2	0 19 11	1 9 0	1 14 10	3 19 9

Moreover, because $\overset{r}{\underset{m-1}{\overline{1}}}_{a, b, c, \&c.}$, or the single premium for the assurance of unity, on the joint lives *a, b, c, &c.* for *m* years, is $= \frac{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}}{m-1} \cdot r - \frac{\overset{r}{\underset{1}{\overline{1}}}_{a, b, c, \&c.}}{m} = \frac{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}}{m-1} \cdot r + 1 -$

$$\frac{L_{m: a, b, c, \&c.}}{L_{a, b, c, \&c.}} r^m - \frac{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}}{m-1} = 1 - \frac{L_{m: a, b, c, \&c.}}{L_{a, b, c, \&c.}} \cdot r^m - (1-r)^{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}};$$

if this be divided by $\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}$, we shall have the annual

premium for such assurance; that is, $\frac{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}}{\overset{r}{\underset{m-1}{\overline{1}}}_{a, b, c, \&c.}} = \frac{1 - \frac{L_{m: a, b, c, \&c.}}{L_{a, b, c, \&c.}} \cdot r^m}{\overset{r}{\underset{0}{\overline{1}}}_{a, b, c, \&c.}} - 1 + r$. The said annual premium may be expressed by

$$\left(1 - \frac{L_{m: a, b, c, \&c.}}{L_{a, b, c, \&c.}} \cdot r^m\right) \div \left(\overset{r}{\underset{m-1}{\overline{1}}}_{a-1, b-1, c-1, \&c.} \times \frac{L_{a-1, b-1, c-1, \&c.}}{r L_{a, b, c, \&c.}}\right) - 1 + r$$

This last mode is well adapted to logarithms in the use of our general tables; and this method, supposing the annuities were accurately determinable by our general tables, would be accurate. The last formula is derived from that immediately before, in consequence of $\overset{r}{\underset{m-1}{\overline{1}}}_{a, b, c, \&c.}$ being identical

with $\overset{r}{\underset{m}{\overline{1}}}_{a-1, b-1, c-1, \&c.} \times \frac{L_{a-1, b-1, c-1, \&c.}}{r L_{a, b, c, \&c.}}$

Example. To find the annual premium to assure a life, at the age a years, for 10 years, according to the Carlisle mortality, and three per cent. interest.

$a =$	20	30	40	50	60	70
Log. of the accommoda. chance for living 10 yrs. at the age $a-1$, Tab. V.	$\bar{1}.9690$	$\bar{1}.9556$	$\bar{1}.9406$	$\bar{1}.9328$	$\bar{1}.8435$	$\bar{1}.7005$
$\lambda 1,03^{-1} =$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$	$\bar{1}.8716$
Sum . . . =	$\bar{1}.8406$	$\bar{1}.8272$	$\bar{1}.8122$	$\bar{1}.8044$	$\bar{1}.7151$	$\bar{1}.5721$
Corresponding91443	.90407	.89892	.89379	.84846	.78092
To this we get from Ta. I.	31	362	103	205	248	94
$\lambda \frac{1}{10} a-1$91474	.90779	.90005	.89605	.85099	.78191
Therefore, $\lambda \frac{L+a+10}{L}$ (T.VII.)	$\bar{1}.96682$	$\bar{1}.95400$	$\bar{1}.93772$	$\bar{1}.91830$	$\bar{1}.81893$	$\bar{1}.59873$
$\lambda 1,03^{-10} =$	$\bar{1}.87163$	$\bar{1}.87163$	$\bar{1}.87163$	$\bar{1}.87163$	$\bar{1}.87163$	$\bar{1}.87163$
Sum = the log.	$\bar{1}.83845$	$\bar{1}.82563$	$\bar{1}.80935$	$\bar{1}.78993$	$\bar{1}.69056$	$\bar{1}.47036$
The N° corresponding =	.68937	.66932	.64469	.61650	.49041	.29536
Its complement to unity	.31063	.33068	.35531	.38350	.50959	.70464
The log. of the last . . . =	$\bar{1}.49224$	$\bar{1}.51941$	$\bar{1}.55061$	$\bar{1}.58377$	$\bar{1}.70722$	$\bar{1}.84797$
Complement of $\lambda \frac{1}{10} a-1$ =	$\bar{1}.08526$	$\bar{1}.09221$	$\bar{1}.09995$	$\bar{1}.10395$	$\bar{1}.14901$	$\bar{1}.21809$
$\lambda \frac{L}{L-a+1}$ =	$\bar{1}.99694$	$\bar{1}.99571$	$\bar{1}.99481$	$\bar{1}.99402$	$\bar{1}.98754$	$\bar{1}.97813$
λr^{-1} =	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$	$\bar{1}.98716$
Sum = logarithm	$\bar{2}.56160$	$\bar{2}.59449$	$\bar{2}.63253$	$\bar{2}.66890$	$\bar{2}.83093$	$\bar{1}.03135$
Number corresponding =	.03644	.03931	.04291	.04666	.06775	.10749
$1,03^{-1} 1$ =	-.02913	-.02913	-.02913	-.02913	-.02913	-.02913
Ann. premium for an assurance of 1 <i>l.</i> . . . =	.00732	.01018	.01378	.01753	.03862	.07836
Ditto for 100 <i>l.</i> =	£0..14..8	1.. 0.. 4	1.. 7.. 7	1..15.. 1	3..1 .. 3	7..16.. 9

} for annual premiums.

The reader has here an opportunity of comparing the results from my tables, with those above calculated by Mr. MILNE'S Carlisle tables.— I may probably be able at a future period to add examples, which I regret time will not at present permit.