stars are more generally disposed in groups than scattered equally over the celestial spaces." To secure a complete survey of any portion of the heavens, this or some similar method seems almost necessary.

III. An Explanation of the observed Irregularities in the Motion of *Uranus*, on the Hypothesis of Disturbance caused by a more distant Planet; with a Determination of the Mass, Orbit, and Position of the disturbing Body. By J. C. Adams, Esq., M.A. F.R.A.S. Fellow of St. John's College, Cambridge.*

The author introduces the subject by remarking, that when Bouvard constructed his *Tables of Uranus* (those now commonly in use), he found it impossible to reconcile the ancient observations, made before the discovery of *Uranus* as a planet, with the modern observations, and that, therefore, in the formation of his tables, he relied solely upon the latter; but that, in a very few years, the still more modern observations exhibited a departure from the tables nearly as great as the ancient ones, and, therefore, there seemed now to be no sufficient reason for rejecting the ancient observations. The author then states, that his attention was first directed to this subject by reading the Report on the recent progress of astronomy made to the British Association, at their meeting in Oxford; and that in July, 1841, he formed a design of investigating the yet unaccounted-for motions of *Uranus*, in order to discover whether they could be explained by an exterior disturbing planet. In 1843 he made a first attempt, supposing the orbit of the disturbing planet to be a circle, and its mean distance twice that of *Uranus*. This investigation was founded exclusively on the modern observations, using, as far as 1821, the errors given in the equations of condition in Bouvard's tables, and for subsequent years the errors given in the *Astronomische Nachrichten*, and the Cambridge and Greenwich Observations. The result shewed a good general agreement of the observed disturbance with the disturbance which would be produced by the action of such a planet. In February, 1844, the author received from the Astronomer Royal the results of the general reduction of the *Greenwich Planetary Observations*.

In the meantime the Göttingen Academy had proposed for the subject of a prize the theory of *Uranus*, and though the author had no hope of being able to complete an essay in time to compete for the prize, he was stimulated by the publication of this proposal again to enter on the investigation. He now took into account the possible eccentricity of the disturbing planet to the first order, retaining the same assumption for mean distance. For the modern observations, the tabular errors used, as far as 1830, were exclusively those of the Greenwich Observations, except one by Bessel in 1823; after 1830, the Cambridge and Greenwich determinations and those in the *Astronomische Nachrichten*, were used. Those for the observations anterior to the discovery of the planet were taken from Bouvard.

* This paper was presented to the Society on the evening of Nov. 13, 1846.

Results for the elements of the disturbing planet were obtained, which were communicated, in September 1845, to Professor Challis, and in October 1845 (slightly altered) to the Astronomer Royal. Afterwards the investigation was repeated, supposing the mean distance diminished by about $\frac{1}{30}$th part. The results were communicated to the Astronomer Royal in September 1846. They seemed to shew that the mean distance ought to be still further diminished.

The author after adverting to the dates of M. Le Verrier's papers, and shewing that his own calculations were earlier in date, says, "I mention these dates merely to shew that my results were arrived at independently and previously to the publication of M. Le Verrier, and not with the intention of interfering with his just claims to the honors of the discovery, for there is no doubt that his researches were first published to the world, and led to the actual discovery of the planet by Dr. Galle, so that the facts stated above cannot detract, in the slightest degree, from the credit due to M. Le Verrier.

The investigations proceeded as follows:—First, to diminish the number of equations, the results were collected in groups of three years each; and these were so arranged as to present results nearly independent of the error of radius vector. Thus twenty-one equations were obtained; and these, without extension for the two or three last years (which might subsequently have been included, but which would have disturbed the similarity of the calculations) were also used in the subsequent calculations for a different assumed mean distance. Then all the principal inequalities in the recognised theory of *Uranus* were verified, and corrections for an error pointed out by Bessel, and for the altered mass of *Jupiter*, were applied, as well as for some terms of the second order of masses pointed out by Hansen. Other inequalities of higher orders were neglected; as their effects may be represented, either by a very slow alteration of the epoch and mean motion, or by a very slow alteration of the perihelion and eccentricity; both which may, without sensible error, be assumed as constant, during the comparatively short period through which *Uranus* has been observed. The author then gives a table of the differences between the theoretical longitudes (thus corrected), and the observed longitudes; the maximum values are as follows:—

In 1712	$+92''.7$	In 1804	$+24''.2$
1750	$-47''.6$	1840	$-66''.6$

These are then converted into corresponding errors of mean longitude, which the author finds more convenient.

Then, formulæ are investigated for the effects of small corrections of the elements of the orbit of *Uranus*, and for the perturbations of mean longitude produced by a disturbing planet, expressed in the notation of Pontécoulant. These are expanded as far as the second order of eccentricities (involving only the first power of the eccentricity of the unknown planet), and the whole is re-

duced to numbers, with no symbols remaining, except for functions of the corrections of the elements of *Uranus*, and functions of the epoch, longitude of perihelion, eccentricity, and mass, of the disturbing planet. All the numerical quantities are computed on the supposition that the mean distance is double that of *Uranus*. Any one of these expressions, adapted to a certain time, being made equal to the error in the tabular place of *Uranus* for the same time, furnishes an equation of condition.

These equations of condition are treated by the method of least squares; and the successive steps of elimination are given. The author considers that the modern observations are scarcely sufficient to give the eccentricity and longitude of perihelion of the disturbing planet; but when the ancient observations (always omitting that of 1690, as uncertain) are combined, there are ample means for determining these elements. The equations, after the elimination had proceeded to a certain degree, were solved by successive substitution. The results thus obtained were

Hypothesis I.

Assumed Mean Distance = 2 × that of *Uranus*.

Mean Longitude, 6 October, 1846	325° 7′
Longitude of Perihelion	315 57
Eccentricity of the Orbit	0·16103
Mass (that of the Sun being 1)	0·0001656

Which were communicated to the Astronomer Royal in October, 1845.

The author then states that he made a second investigation, on the supposition that the mean distance of the disturbing planet = mean distance of *Uranus* × $\frac{1}{0{\cdot}515}$. The process, with very little difference, is the same as that for the former assumption of mean distance. The formulæ, the equations, &c., are given in the same manner as before. The elements obtained thus are as follows:—

Hypothesis II.

Assumed mean distance = 1·942 × that of *Uranus*.

Mean Longitude, 6th October, 1846.....	323° 2′
Longitude of Perihelion	299 11
Eccentricity of the Orbit	0·12 0615
Mass (that of the Sun being 1)	0·00 015003

The corrections to the elements of the orbit of *Uranus* are investigated on both hypotheses. Then on substituting the effects of the corrections, and the effects of the perturbations, the residual errors are obtained, of which the following are the maximum values:—

	Hypoth. I.	Hypoth. II.	
1712	+ 6·″7	+ 6·″3	
1715	− 6·8	− 6·6	
1753	+ 5·7	+ 5·2	
1764	− 5·1	− 4·1	
1771	+ 11·8	+ 12·8	Single Observation.

After this time, to the year 1840, the largest error is 2″·35.

After 1840, the errors increase on both hypotheses. They are,—

	Hypoth. I.	Hypoth. II.
1843	+ 7″·11	+ 5″·77
1844	+ 8·79	+ 7·05
1845	+ 12·40	+ 10·18

It appears from this extremely probable that the mean distance of the disturbing planet ought to be assumed nearly = mean distance of *Uranus* $\times \frac{1}{0{\cdot}574}$.

The residual errors for the single observation of 1690 are,—

Hypoth. I.	Hypoth. II.
+44″·5	+50″·0

It seems probable that these errors would be increased by still further diminishing the mean distance.

Expressions are then investigated for the correction of radius vector produced by the correction of elliptic elements, and by the effects of perturbation. The numerical values are as follows:—

	Hypoth. I.	Hypoth. II.
1834	+0·00505	+0·00492
1840	+0·00722	+0·00696
1846	+0·00868	+0·00825

The author states that no satisfactory results could be found for the node and inclination of the planet's orbit, as deduced from the irregularities in the latitude of *Uranus*.

The author then remarks that the perturbations of *Saturn* produced by the new planet will be undoubtedly sensible; and he suggests that it would be interesting to examine anew the theory of *Saturn*, and to ascertain whether the masses of *Jupiter* and *Uranus* deduced from it are consistent with those obtained by other methods. He remarks that the published Reductions of the Greenwich Observations now make such an inquiry comparatively easy.