observations. On the other hand, if the observed polarisation were attributed to secondary scattering in the atmosphere of light coming from the landscape or air in the neighbourhood of the edge of the shadow of the Moon, we should expect to have a change in the phenomena as the shadow passed over the observer. Observation showed that no such change was detected at Guelma. Any resultant polarisation would in this explanation depend also on dissymmetry in configuration of the contributing sources of illumination.

I refrain from dwelling upon some of the apparent difficulties arising in an acceptance of the idea that light pressure may be the agency by which just those particles of dust that are most active in scattering polarised light are driven out into streamers. It would appear that the particles most actively driven outwards would be those whose diameter was about one-third of the wave-length of the scattered polarised light. To adopt the view that they were driven out by the radiation pressure due to radiation of greater wave-length—let us say, ultra red—would mean that we are ready to accept the idea that the active pressure was connected with radiation known to be considerably feebler than the maximum components in the solar radiation. The particles driven out by the maximum components are already too big to scatter much polarised light, unless they simultaneously scatter unpolarised light in far larger quantities than there is evidence of in the photographic records. But it is unprofitable to try and solve what is essentially a quantitative problem by qualitative methods. I hope to revert to the subject later.

Solar Parallax Papers, No. 4. The Magnitude Equation in Right Ascension of the Etoiles de Repère. By Arthur R. Hinks, M.A.

1. The *étoiles de repère* for the reduction of the *Eros* plates were observed on the meridian at a large number of observatories, and the results were published in Paris Circulars Nos. 8 and 9.

M. Loewy published no definite catalogue of the places to be adopted as standard, preferring to leave to each observer the formation of a system for himself, at his own discretion as to the system of weights for the various observatories, and the allowance, if any, to be made for magnitude equation. As a result of this decision the published photographic places of *Eros* and the comparison stars are referred to a number of systems. The places adopted by M. Loewy for the reduction of the Paris photographs have been used also at Algiers, and in the reduction of the Catania plates at Paris. We will call this system L.

2. After a considerable part of the photographic reductions were completed, Professor Tucker published in Paris Circular No. 11 a system of places derived from a discussion of all the

meridian circle results, which he proposed for definitive adoption. He took no account of magnitude equation in the formation of this system, but published later (*Lick Observatory Bulletin*, No. 72) some reasons for thinking that the first part was free from magnitude equation, the second affected by an equation of $0^s{\cdot}012$ per magnitude. We will call this system T.

In a letter to *The Observatory* (1905 July) the writer gave brief reasons for believing that the conclusion with regard to the first part was incorrect.

3. More recently Dr. Fritz Cohn has published in *Astronomische Nachrichten*, 4059–60 (1905 December) a second fundamental system, derived from the same material as Professor Tucker's, with considerable additions, but reduced to a Königsberg system of right ascensions made with the Repsold clockwork transit micrometer. Dr. Cohn claims that these observations are necessarily free from magnitude equation, and that by his reductions he has freed the other series also from its effects. We will call this system C.

4. The position at the present time is therefore this : Of the photographic observations already published Paris, Algiers, and Catania are based on system L ; Toulouse on a system differing little from it ; Bordeaux, San Fernando, and Northfield have used each an independent system.

Of the unpublished observations communicated for use in this investigation by the kindness of the Astronomer Royal, Dr. Backlund, and Professor Donner, the Greenwich results are referred to system L, the Pulkowa to system T, and the Helsingfors to an independent system.

5. Before the observations of the planet can be used for a discussion of the Solar Parallax and the mass of the Moon they all must be referred to one system. It has become now of immediate importance to inquire whether either system T (to which at present only the Pulkowa results are reduced) or system C (to which no photographic observations are as yet reduced) possesses advantages sufficient to warrant a reduction to it of all the observations ; whether it would, on the contrary, be better to reduce all outstanding series to system L, on which very much is already founded ; or whether still another fundamental system should be made from a photographic revision of one of these systems.

The present paper presents as a contribution to this inquiry the results of a determination of the magnitude equation in systems L, T, and C, derived by comparison with the deduced photographic places.

6. There are good reasons for believing that the most complete final test of freedom from magnitude equation in a series of meridian places is comparison with the results of a photographic revision of those places.*

* See papers on the magnitude equation in A. G. Zone Catalogues by the writer, *M.N.* lvii. 473 (1897 April), and by Professor Turner, *M.N.* lx. 3 (1899 November) and lxii. 3 (1901 November).

Provided that the magnitudes of the standard stars are fairly well distributed, the deduced photographic places of those stars are on the whole free from magnitude equation; at least none is caused by the existence of magnitude equation in the adopted standard places. Hence, if we can assure ourselves that the photographic processes have introduced no magnitude equation on their own account, we may use the photographic results to determine the magnitude equation of the visual.

7. The mean photographic places of the *étoiles de repère* are found in Paris Circulars Nos. 10 and 11 (Table I. for each observatory). Each series has been compared with Paris, with the following results :—

Table I.

Paris minus Other Observatories, Photographic R.A.

	Magnitudes. –6·2	6·3–6·9	7·0–7·4	7·5–7·9	8·0–8·4	8·5–8·8	8·9–9·2	9·3–.
Paris – Algiers.								
List I.	(5) + 41	(6) + 31	(15) + 23	(27) + 21	(53) + 6	(64) – 1	(43) – 23	(11) – 24
List II.	(5) + 51	(7) + 37	(8) + 11	(17) – 3	(18) – 6	(24) – 9	(6) – 19	...
Mean	(5) + 46	(13) + 34	(23) + 19	(44) + 12	(71) + 3	(88) – 3	(43) – 23	(11) – 24
Paris – Bordeaux.								
List I.	(2) – 17	...	(7) – 7	(12) – 2	(24) + 10	(41) + 7	(24) + 2	(9) – 1
List II.	(2) + 22	(4) – 26	(4) – 10	(6) – 1	(13) + 1	(15) – 4	(3) + 11	...
Mean	(4) + 3	(4) – 26	(11) – 8	(18) – 2	(37) + 7	(56) + 4	(27) + 3	(9) – 1
Paris – Catania.								
List I.	(3) + 11	(2) – 16	(9) + 3	(21) + 11	(39) + 7	(43) + 3	(31) + 11	(8) – 4
List II.	(4) + 22	(7) + 16	(5) – 15	(12) + 8	(15) 0	(16) + 5	(3) – 13	...
Mean	(7) + 17	(9) + 9	(14) – 3	(33) + 10	(54) + 5	(59) + 3	(34) + 9	(8) – 4
Paris – Greenwich.								
List I.	(3) – 16	(5) – 2	(14) 0	(20) + 3	(47) + 2	(60) 0	(35) – 3	(8) + 1
List II.	(3) + 11	(4) – 2	(8) – 10	(12) + 1	(16) + 3	(16) – 5	(3) – 1	...
Mean	(6) – 3	(9) – 2	(22) – 3	(32) + 2	(63) + 2	(76) – 1	(38) – 3	(8) + 1
Paris – Helsingfors.								
List I.	(3) + 8	(4) + 38	(9) + 19	(15) + 22	(22) + 11	(38) + 11	(20) + 5	(10) + 25
List II.	(5) + 12	(3) + 5	(8) + 13	(12) + 4	(16) + 8	(18) – 9	(3) – 27	...
Mean	(8) + 11	(7) + 24	(17) + 16	(27) + 14	(38) + 10	(56) + 4	(23) + 2	(10) + 25
Paris – Northfield.								
List I.	(3) – 3	(5) 0	(12) + 26	(19) + 8	(33) + 5	(46) + 5	(33) 0	(7) 0
List II.	(3) + 14	(1) + 2	(2) + 7	(5) + 2	(9) 0	(8) – 8	(1) – 17	...
Mean	(6) + 5	(6) + 1	(14) + 24	(24) + 7	(42) + 4	(54) + 4	(34) – 1	(7) 0

	Magnitudes. -6·2	6·3–6·9	7·0–7·4	7·5–7·9	8·0–8·4	8·5–8·8	8·9–9·2	9·3–.
Paris – San Fernando								
List I.	(5)+12	(6)+14	(15)+17	(27)+16	(53)+15	(66)+10	(44)– 1	(13)–11
List II.	(5)+15	(7)+ 9	(8)– 6	(17)+ 8	(19)+ 1	(24)– 3	(3)–12	...
Mean	(10)+13	(13)+11	(23)+ 9	(44)+13	(72)+11	(90)+ 7	(47)– 2	(13)–11
Paris – Toulouse.								
List I.	(5)–29	(6)+ 3	(15)+ 4	(27)– 3	(53)+10	(66)+ 3	(44)+ 3	(13)+ 4
List II.	(3)+17	(5)–25	(7)– 5	(11)+ 2	(18)– 5	(17)+ 6	(3)– 6	...
Mean	(8)–12	(11)–10	(22)+ 1	(38)– 2	(71)+ 6	(83)+ 4	(47)+ 3	(13)+ 4

The unit is $0^s{\cdot}001$.

The number of separate results contributing to each mean difference is given in brackets before it.

The magnitudes are according to Tucker.

8. The comparison with Algiers shows very clearly the now well known but still unexplained systematic error, depending at any rate partly upon magnitude, which affects the whole series from that observatory. It may, perhaps, be called for convenience "objective magnitude equation." Owing to the existence of this error no use can be made of the Algiers results in this paper.

The comparison with San Fernando shows small traces of a magnitude equation which may probably be due to the fact that these plates were not reversed during measurement.

With these exceptions there is little evidence of relative magnitude equation between the different series of photographic results. And it is hard to suggest any reason why all should be affected by a similar absolute magnitude equation. We may conclude that there is in general no sensible magnitude equation in the photographic right ascensions.

9. We may, therefore, deduce the magnitude equation in system T by a similar series of comparisons. The results are given in Table II.

Table II.

System T minus Photographic R.A.

	Magnitudes. -6·2.	6·3–6·9.	7·0–7·4.	7·5–7·9.	8·0–8·4.	8·5–8·8.	8·9–9·2.	9·3–.
T – Bordeaux.								
List I.	(2)–25	...	(7)–38	(12)–19	(24)– 4	(41)+ 3	(24)+12	(9)+ 7
List II.	(2)–22	(4)–41	(4)–24	(6)– 2	(13)+ 4	(15)+ 1	(3)+40	...
T – Catania.								
List I.	(5)–45	(2)–30	(13)–23	(25)–12	(44)– 2	(51)+ 3	(38)+17	(9)– 2
List II.	(4)–12	(7)+ 5	(6)–31	(12)+ 4	(15)+ 1	(16)+12	(3)+15	...

	Magnitudes. −6·2	6·3−6·9	7·0−7·4	7·5−7·9	8·0−8·4	8·5−8·8	8·9−9·2	9·3−.
T − Greenwich.								
List I.	(4) − 19	(5) − 18	(18) − 26	(22) − 18	(49) − 6	(60) + 1	(38) + 11	(8) + 12
List II.	(3) − 3	(2) − 35	(11) − 23	(12) − 3	(16) + 4	(16) + 5	(3) + 29	...
T − Helsingfors.								
List I.	(4) − 29	(4) + 23	(12) − 4	(17) + 3	(26) + 8	(39) + 11	(23) + 14	(10) + 34
List II.	(5) − 18	(3) − 15	(8) + 3	(12) + 2	(16) + 5	(19) + 3	(3) − 5	...
T − Northfield.								
List I.	(5) − 12	(5) − 16	(16) + 3	(25) − 2	(36) + 1	(47) + 7	(35) + 11	(8) + 11
List II.	(3) − 12	(1) − 21	(2) − 14	(7) 0	(9) + 1	(8) + 1	(1) + 3	...
T − Paris.								
List I.	(5) − 24	(6) − 21	(15) − 26	(27) − 13	(53) − 10	(66) 0	(44) + 11	(13) + 14
List II.	(5) − 26	(7) − 21	(8) − 12	(17) − 5	(19) − 1	(24) + 4	(3) + 28	...
T − San Fernando.								
List I.	(7) − 21	(7) − 10	(20) − 7	(35) − 1	(58) + 5	(77) + 7	(55) + 10	(14) + 3
List II.	(5) − 11	(7) − 12	(9) − 19	(17) + 3	(19) + 1	(24) + 2	(3) + 16	...
T − Toulouse.								
List I.	(7) − 66	(7) − 21	(20) − 18	(35) − 17	(57) − 1	(77) + 4	(55) + 13	(14) + 17
List II.	(3) − 20	(5) − 50	(7) − 16	(11) + 2	(18) − 5	(17) + 13	(3) + 23	...

The magnitudes are according to Tucker.

10. The results are somewhat irregular, especially for List II. But all agree in showing that system T has a considerable magnitude equation; that it is nearly the same in the two lists; and that it amounts to about $+0^{s}\cdot015$ per magnitude, the plus sign indicating that the right ascensions of faint stars are too great, or that they are observed relatively late.

11. There is some difficulty in determining the numerical value of the magnitude equation, because there are relatively few stars in the brighter groups, and accidental errors there affect the means disproportionately. Moreover it is fairly certain that the magnitude equation is not a linear function of the magnitude, even down to $9^{m}\cdot0$; but there is not sufficient material to show its true form.

All the numerical values given in this paper have been obtained by plotting the tabular results, and finding the general slope of the curve by laying a ruler on it, having regard to the numbers of stars which contribute to the different parts.

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12. A similar comparison between Systems L and T shows that L has a magnitude equation in list I. about $+0^{s}{\cdot}005$ greater than that of T, thus confirming the result noticed by Professor Turner (*Nature*, 1904 December 15), and referred to by Professor Tucker (*L.O. Bulletin*, No. 72, last paragraph). For list II. there is practically no such difference.

13. Very similar results are found for the independent systems adopted at Bordeaux, Northfield, and Helsingfors. The magnitude equation between System T and San Fernando is of the same order, but is apparently not linear.

It follows that the published photographic results are based upon adopted systems which, though differing systematically, have nearly the same magnitude equation, about $+0^{s}{\cdot}020$ for list I. and $+0^{s}{\cdot}015$ for list II.

14. The comparison between System C and the photographic places from each observatory was made in the same way, and the results are given in Table III.

Table III.

System C minus Photographic R.A.

	Magnitudes.							
	−6·2.	6·3−6·9.	7·0−7·4.	7·5−7·9.	8·0−8·4.	8·5−8·8.	8·9−9·2	9·3−.
C − Bordeaux.								
List I.	(2) −62	(4) −53	(4) −46	(12) −59	(29) −52	(45) −52	(30) −55	(10) −75
List II.	(3) −48	(1) −50	(4) −54	(11) −36	(13) −31	(15) −41	(6) −31	...
C − Catania.								
List I.	(4) −54	(10) −70	(6) −73	(24) −48	(45) −48	(65) −53	(37) −55	(9) −82
List II.	(6) −16	(4) −32	(6) −38	(16) −35	(18) −26	(21) −42	(10) −41	...
C − Greenwich.								
List I.	(6) −36	(11) −47	(7) −48	(20) −48	(53) −53	(70) −54	(38) −58	(9) −51
List II.	(5) −24	(2) −29	(8) −40	(19) −44	(25) −36	(19) −36	(10) −40	...
C − Helsingfors.								
List I.	(5) −23	(7) − 7	(5) −25	(17) −27	(31) −39	(38) −42	(24) −48	(11) −42
List II.	(4) −24	(5) −23	(3) −36	(15) −21	(17) −40	(14) −45	(8) −46	...
C − Northfield.								
List I.	(6) −29	(10) −27	(8) −31	(23) −35	(42) −49	(51) −47	(29) −61	(10) −45
List II.	(4) −27	(2) −38	(1) −52	(10) −29	(15) −41	(12) −44	(8) −37	...
C − Paris.								
List I.	(5) −50	(13) −36	(8) −43	(23) −46	(59) −52	(78) −57	(46) −61	(14) −67
List II.	(8) −39	(6) −41	(7) −41	(27) −37	(30) −42	(28) −42	(14) −41	...

Magnitudes.

	−6·2	6·3–6·9	7·0–7·4	7·5–7·9	8·0–8·4	8·5–8·8	8·9–9·2	9·3–.

C – San Fernando.

List I.	(9) − 40	(17) − 35	(11) − 37	(34) − 38	(65) − 41	(95) − 50	(57) − 61	(14) − 75
List II.	(8) − 27	(6) − 44	(9) − 28	(27) − 34	(30) − 36	(32) − 40	(13) − 47	–

C – Toulouse.

List I.	(7) − 64	(16) − 52	(10) − 54	(33) − 54	(62) − 44	(92) − 54	(55) − 57	(14) − 65
List II.	(5) − 44	(5) − 34	(5) − 44	(22) − 42	(25) − 39	(25) − 44	(12) − 41	...

The magnitudes are according to Cohn.

Contrary to expectation, it appeared that the clockwork micrometer system for list I. showed a considerable magnitude equation, of sign opposite to that of system T, faint stars being observed early.

A result so unfavourable to the hopes which have been centred in the new method of meridian observing demanded close examination.

15. In the first place a new comparison was made with a provisional system of mean photographic right ascensions, formed by taking a simple mean of the results published by the eight observatories. The means were unweighted, since weighting would have introduced in an irregular manner the systematic differences in zero of the different series, which for our present purpose do no damage to the simple means, so long as each list is fairly completely observed at each observatory.

The results of this new comparison are, with the same magnitude groups as before :—

Table IV.

System T – Mean Photographic R.A.

List I.	(9) − 34	(16) − 26	(11) − 13	(34) − 8	(65) − 2	(96) + 4	(57) + 12	(14) + 14
List II.	(8) − 19	(6) − 17	(9) − 20	(28) − 4	(30) 0	(32) − 5	(15) + 15	...

System C – Mean Photographic R.A.

List I.	(9) − 45	(16) − 43	(11) − 44	(34) − 47	(65) − 47	(96) − 52	(57) − 60	(14) − 60
List II.	(8) − 28	(6) − 35	(9) − 39	(28) − 36	(30) − 39	(32) − 44	(15) − 42	...

System C – System T.

List I.	(9) − 11	(16) − 18	(11) − 32	(34) − 39	(65) − 45	(96) − 56	(57) − 72	(14) − 74
List II.	(8) − 9	(6) − 18	(9) − 19	(28) − 33	(30) − 38	(32) − 38	(15) − 57	...

The magnitude equation in system C for list I. is confirmed.

16. We must now introduce a distinction which has so far been neglected. List I. covers a range of declination from $+55°$ to $+37°$; list II. from $+39°$ to $+11°$. There seems to be no

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a priori reason why magnitude equation should be independent of declination—rather the reverse—but there was the fact that the general magnitude equation of system T for lists I. and II. came out almost the same; and further, it was shown by Dr. Cohn that the differences between his final system C and his clockwork micrometer results are also independent of declination (*loc. cit.* p. 45).

But in searching for a reason why system C should have a magnitude equation for list I., and not for list II., the difference of declination suggested itself as a possible cause.

The differences which were combined to form the preceding table were therefore rearranged into three groups.

Group I. corresponds approximately to the portion of list I. north of Decl. +49°; these stars belong to that portion of the track of the planet pursued from 1900 October 10 to December 6.

Group II. comprises the rest of list I. between Decl. +49° and +39°, covering the periods 1900 September 19–October 10, and December 6–29.

Group III. comprises the first part of list II. between Decl. +38° and +28°, covering 1900 December 29 to 1901 January 25.

The mean results are as follows:—

Table V.

System T – Mean Photographic R.A.

Group I.	(3) − 48	(7) − 15	(5) − 11	(15) − 6	(36) − 2	(52) − 1	(29) + 8	(12) + 14
Group II.	(6) − 27	(9) − 34	(6) − 13	(19) − 9	(29) − 2	(44) + 10	(28) + 17	(2) + 13
Group III.	(8) − 19	(6) − 17	(9) − 20	(28) − 4	(30) 0	(32) − 5	(15) + 15	...

System C – Mean Photographic R.A.

Group I.	(3) − 58	(7) − 29	(5) − 37	(15) − 37	(36) − 42	(52) − 52	(29) − 62	(12) − 61
Group II.	(6) − 39	(9) − 54	(6) − 50	(19) − 55	(29) − 53	(44) − 52	(28) − 59	(2) − 58
Group III.	(8) − 28	(6) − 35	(9) − 39	(28) − 36	(30) − 39	(32) − 44	(15) − 42	...

A graphical determination of the magnitude equation gave

	System T.	System C.
	s	s
Group I.	+0·010	−0·017
Group II.	+0·016	−0·002
Group III.	+0·008	−0·002

The magnitude equation in T is irregular, but not apparently related to the declination; that of C appears strongly in the group of highest declination, scarcely at all in the others.

17. One can scarcely conclude that it really depends on the declination, since the difference of declination between groups I. and II. is not very great. But it appears certain that in group I.,

covering the period between October 10 and December 6, the system C has a magnitude equation larger than the average magnitude equation of system T, of the opposite sign. And the period in question is precisely the most important part of the whole, including opposition (October 31) and by far the greater part of the complete series east and west of the meridian.

To test this result further the differences were broken up into other groups, of which two covered between them a little more than the extent of group I.

Group A covered from October 10 to November 11, or roughly, from Decl. $+48°$ up to Decl. $+55°$.

Group B covered from November 11 to December 11; or, roughly from Decl. $+55°$ down the other side of the loop which the planet described as far as $+47°$.

System C – Mean Photographic R.A.

Group A	(1) −64	(4) −32	(3) −38	(7) −35	(21) −48	(32) −54	(21) −64	(4) −75
Group B	(3) −50	(4) −27	(2) −37	(10) −37	(17) −35	(23) −48	(11) −55	(14) −60

The magnitude equation is somewhat more strongly shown in group A than in B, which is some evidence against its intimate connection with declination.

18. As a final test the differences C minus Phot. R.A. were taken out respectively for each observatory and grouped as above. The same result came out again. With considerable variations in detail all showed more or less clearly that System C has a magnitude equation in the part covered by group I., and that it is somewhat more pronounced in the first half of that group than in the second.

19. Before accepting as final the conclusion that System C is affected in large part by a magnitude equation as great as that of System T, but of opposite sign, we must reconsider from every point of view our original proposition, that photographic places should show no magnitude equation on the mean of a long series.

Guiding error may produce photographic magnitude equation on individual plates, but we can hardly suppose it persistent with the same sign for six weeks at eight observatories.

Objective error may make the whole results of one observatory abnormal, but can hardly be supposed to exist of the same sign in eight telescopes for six weeks and then disappear.

For the same reason personality in measurement is excluded from the possibilities, even if it had not already been eliminated by reversal, except in one case.

20. There remains for consideration one qualification to our original proposition—the words, "provided that the magnitudes of the standard stars are fairly well distributed." Is it possible that there is anything so perverse in the distribution of magnitudes in the present case that the photographic methods break

down? A thorough examination of this point has been made, and the answer is decidedly in the negative.

The results of a single plate give the *slope* of the magnitude-equation curve; the zero line depends on the mean magnitude of stars on the plate. If the magnitude-equation of a system is uniform each plate should give the same slope, but each may have a different zero. A rigorous treatment of this problem would require the determination of the slope from each plate separately, and the deduction of a mean slope. In the present case this was impracticable because, unfortunately, the concluded results from each plate have not been published separately.

It is not hard to construct an arrangement of magnitudes along the path of the planet so systematically peculiar that the method followed in this paper would give a false result. The following experiment was therefore made: It was supposed that a whole series of meridian places was affected by a uniform magnitude-equation of $0^{s}\cdot017$ per magnitude, and that the stars were distributed as in the actual system. By a method which is obvious, but tedious to explain, it was found that the magnitude-equation which would be derived from the different blocks of this system would be practically identical in all, viz. that originally introduced. There is, therefore, no peculiarity in the distribution of magnitudes which can affect the result we have obtained.

21. The method withstands all the tests which have been applied to it; but it gives results which are somewhat rough, because of the accidental errors in the standard systems, apart from the systematic magnitude-equation, and because of the accidental errors in the photographic places, which may become rather large for the brighter stars. Is it possible that accidental errors have combined to produce our result?

This question has been examined by a laborious method which can be but briefly described here. The stars which would fall on a series of plates taken on centres every 2° along the path of the planet have been taken out in the usual magnitude groups for each centre and the photographic places compared with Systems T and C. The mean difference for a given magnitude group was plotted for each centre, and these points were connected by lines of a different colour for each group. For example, the group 8·0–8·4 was represented by a green line, and 8·5–8·8 by a blue. If the green line runs parallel to the blue the magnitude-equation is consistent; if the lines of different colours cross, it is not consistent.

22. First a diagram was drawn for System C—mean photographic R.A. Throughout the parts of the curves belonging to October and November the magnitude-equation was shown roughly but fairly consistently. The same thing was done for C minus Paris, Greenwich, and Toulouse, separately. The results were rougher, but the same effect could be traced.

Finally, similar curves were drawn for System T—mean photographic R.A., and the results were decidedly more consistent than for C.

23. From these investigations, which have been tested and varied in every way that can be thought of, the following results emerge :—

System T has a fairly uniform magnitude-equation of about $+0^{s}\cdot015$ per magnitude throughout list I. and the first half of list II. ; for the second half there is little material.

System L and the independent adopted systems have a magnitude-equation of about $+0^{s}\cdot020$ for list I. and $+0^{s}\cdot015$ for list II.

System C has a magnitude-equation of about $-0^{s}\cdot017$ for about half list I. and of $-0^{s}\cdot002$ for the remainder and for list II.

It seems to me, therefore, that so far as magnitude-equation is concerned System T is little better than System L, while System C is worse, because it changes so suddenly. From this point of view there would be little advantage in adopting either T or C as the standard system for the reduction of all the photographs.

My sincere acknowledgments are due to Miss Julia Bell, who has carried out very skilfully the numerical work summarised in this paper. A large portion of the expense has been borne by a grant from the Royal Society Government Grant Fund.

Cambridge Observatory:
1906 *June* 7.

On some Points connected with the Determination of Orbits.
By H. C. Plummer, M.A.

1. The difference between the circular measure and the sine of an angle is an expression which occurs in several of the most important formulæ relating to elliptic motion. When the angle is large there is no difficulty in calculating this difference accurately, but when it is small ordinary logarithmic tables will not give the required accuracy without the help of some special device. Hence auxiliary tables have been published in a variety of forms, according to the purpose for which they are designed. It is possible to restrict the compass of such tables by making them merely supplementary to the ordinary logarithms by the use of some artifice. Of this nature is Tietjen's * formula, which may be expressed in the form

$$\epsilon - \sin\epsilon = \tfrac{4}{3}\,\mathrm{B}\,\sin^{3}\tfrac{1}{2}\epsilon\,(\sec\tfrac{1}{4}\epsilon)^{2\cdot4} \quad \ldots \quad \ldots \quad (1)$$

* *A.N.* 1463 ; also Watson's *Theoretical Astronomy*, p. 343.