Name.	Position 1900·0. α	δ	Mag.	Sp. Type D.C.	P.M. on Great Circle	Notes.
	h m	° ′			″	
1 Cygni	21 2·4	+38 15	5·11	H.	5·16	0″·36 Wilsing, with suspected periodic motion of 61₁; Davis finds parallax of two stars sensibly different.
Equulei... ...	21 9·6	+ 9 37	4·5	F.	0·30	Short period binary. 0·02 Flint; 0·02 Leavenworth; 0·07 Hussey (spectroscopic). Necessary to take plates through period of six years to eliminate effect of close binary.
Cygni	21 10·8	+37 36	4; 10	F.	0·48	Binary. 0·08 Beloposky.
alande 43492 ...	22 12·3	+12 24	7	A.?	0·83	
Kr. 60	22 24·5	+57 12	9; 11		0·95	Binary. 3″.
Cephei	22 25·5	+57 54	Var.	F.?	0·01	Sp. binary.
Pegasi	22 58·9	+27 32	Var.	M.?	0·22	Irregular.
alande 45755 ...	23 16·8	+48 33	7·5	A.	0·68	
Andromedæ ...	23 32·7	+45 55	4	K.	0·45	Sp. binary.
alande 46650 ...	23 44·0	+ 1 52	8·7		1·4	0·23 Flint.

Cambridge Observatory:
1905 *June* 2.

The Parallax of Lalande 21185 *and γ Virginis from Photographs taken at the Cambridge Observatory.* By Henry Norris Russell, Ph.D.

§ 1. The work upon which the writer has been engaged for the past two years as a research assistant of the Carnegie Institution has now progressed far enough to permit the publication of its first results. An outline of the methods employed, with the reasons which led to their adoption, is given in the preceding paper. The present communication deals with the numerical data obtained for the first two stars whose discussion has been completed.

§ 2. *Lalande* 21185.—

R.A. $10^{h}\ 57^{m}{\cdot}9$, Dec. 36° 37′ N. (1900·0), Mag. 7·3, P.M. 4″·77.

Previous investigations have shown that this is one of the nearest stars in the northern hemisphere, but they differ among themselves sufficiently to justify a fresh determination of its parallax.

The present discussion is based upon eight plates taken with the Sheepshanks telescope (the first five by the writer, and the rest by Mr. Hinks), the circumstances being as follows :

No. of Plate.	Date.	Sid. T.	Expo-sures.	No. of Plate	Date.	Sid. T.	Expo-sures.
		h m				h m	
191	1903 Dec. 9	11 10	4	268	1904 Apr. 25	10 57	4
194	13	11 11	4	397	Dec. 30	11 6	3
258	1904 Apr. 16	11 3	4	405	1905 Jan. 9	10 43	4
260	19	11 10	4	426	Apr. 15	10 56	4

The fourth column gives the number of measurable exposures on each plate, and the third the mean of the times of the middle of these exposures.

The plates are coated on "patent plate" glass, and are of the size used for the astrographic chart, but owing to the longer focal length of the Cambridge telescope the field is a little less than $1\frac{1}{2}°$ square. A standard Gautier réseau is impressed on all plates. The réseau interval of 5 mm. corresponds to $175''\cdot8$.

§ 3. There is a marked absence of stars in the N.E. part of this field, so that it was not possible to secure a perfectly symmetrical distribution of the comparison stars.

The following table shows the stars finally chosen, their B.D. numbers and magnitudes, the magnitudes given in the A.G. Catalogue (Lund) when they appear therein, and the approximate coordinates of the stars upon our plates the plate, centre being (20, 20). A denotes the "parallax star," Lal. 21185.

Star.	B.D.	Magnitude. B.D.	Magnitude. Lund.	x.	y.
1	+37 2142	8·3	8·2	9·87	29·37
2	36 2141	8·6	8·5	10·95	19·63
3	37 2145	6·8	7·5	13·04	32·13
4	37 2151	7·7	8·2	16·65	25·18
5	36 2144	9·1	—	17·25	11·87
6	36 2146	8·5	8·5	17·91	11·69
7	36 2150	8·9	8·8	25·99	14·56
8	36 2151	8·8	8·9	26·09	9·87
9	37 2153	8·5	8·4	32·70	32·54
A	36 2147	7·3	7·3	19·85	20·29
Centre of gravity of comparison stars ...			...	18·94	20·76

The centre of gravity falls very near the parallax star, but there is only one comparison star (No. 9) in the north-east quarter of the plate.

§ 4. On the first two plates all images were measured in both orientations, but on the others the first two were measured in the direct position and the last two in the reversed. The measures of individual images are carried to four decimal places (in terms of a réseau interval), the last place corresponding to estimated tenths of a division of the micrometer head. The means of the coordinates of the four images of each star are then taken, and carried to five decimal places to avoid errors of computation. The differences from this mean are then tabulated for each exposure.

The scale value for the four exposures must be sensibly the same; but the orientation may differ a little, owing to refraction and possible maladjustment of the polar axis of the telescope, and the centering for each exposure is of course different. If there were no accidental errors the differences from the mean should therefore be of the form $\triangle x = by + c$. The deviations from such a formula (which are easily obtained graphically) give a measure of the accuracy of the plate (though they will not show such things as "guiding error," which differs from star to star, but not from exposure to exposure). They also serve as a control of the numerical work, and to detect any errors that may have been made in recording the measures.

The y-coordinates were measured to three decimal places on one plate of each epoch.

§ 5. For the standard coordinates there were chosen the mean of the x's of Plates 191 and 194, with the y's of Plate 191. The approximate method of reduction may safely be applied in this case. It may be worth while to give an example of the method, say the case of Plate 258. Each comparison star gives us one equation of condition of the form

$$a\xi + b\eta + c = x - \xi$$

Taking the mean of the three equations in which ξ is greater than its mean value, and of the six in which it is less, we obtain

$$\begin{aligned} 28{\cdot}262a + 18{\cdot}993b + c &= +12531 \\ 14{\cdot}277a + 21{\cdot}646b + c &= +11066 \end{aligned}$$

where the absolute terms are expressed in units of the fifth place. Similarly, from the four equations in which y is greater than its mean value, and the five in which it is less, we obtain

$$\begin{aligned} 18{\cdot}066a + 29{\cdot}806b + c &= +\ 9316 \\ 19{\cdot}638a + 13{\cdot}526b + c &= +13346 \end{aligned}$$

From these two pairs of equations we find by subtraction

$$\begin{aligned} 13{\cdot}985a - \ 2{\cdot}653b &= +1465 \\ -\ 1{\cdot}572a + 16{\cdot}280b &= -4030 \end{aligned}$$

whence $\qquad a = +58{\cdot}88 \qquad b = -241{\cdot}84$

and from any one of the first four equations

$$c = +15460$$

By calculating c from all four of these equations we get a control for the numerical work.

We next calculate the residuals for each star in the sense $x-\xi-a\xi-b\eta-c$. The sum of the residuals for each set of comparison stars which was grouped together above must be zero. This gives a searching control on the reduction.

§ 6. The residuals for the parallax star are now converted into seconds of arc and reduced to the epoch 1904·0 with the star's proper motion taken from Bossert's catalogue, which in the present case is $-0^{s}\cdot044$ or $-0''\cdot53$.

We then obtain the following equations of condition, in which δx denotes the correction to the standard x-coordinate for 1904·0, $\delta\mu$ the correction to Bossert's proper motion in R.A., and π the parallax of our star relative to the mean of the comparison stars, while the absolute terms are given in thousandths of a second of arc.

				O−C.
$1\cdot000\delta x$	$-0\cdot061\delta\mu$	$+0\cdot907\pi$	$=-\ 18$	+20
1·000	−0·051	+0·900	= − 41	− 1
1·000	+0·291	−0·634	= −538	+34
1·000	+0·299	−0·668	= −589	− 5
1·000	+0·315	−0·736	= −650	−49
1·000	+0·999	+0·817	= −139	−66
1·000	+1·026	+0·733	= − 58	+44
1·000	+1·288	−0·621	= −549	+23

The influence of the parallax is very conspicuous in the absolute terms.

Our normal equations are as follows :

$$\begin{array}{llll} 8\cdot000\delta x & +4\cdot106\delta\mu & +0\cdot698\pi & =-2588 \\ 4\cdot106 & +3\cdot989 & +0\cdot051 & =-1442 \\ 0\cdot698 & +0\cdot051 & +4\cdot613 & =+1349 \end{array}$$

whence

$$\begin{array}{lll} \delta x=-351\cdot7 & \text{weight} & 3\cdot68 \\ \delta\mu=-\ \ 4\cdot0 & \text{,,} & 1\cdot86 \\ \pi=+345\cdot7 & \text{,,} & 4\cdot50 \end{array}$$

The residuals left on substituting these values in the equations of condition are given above under the heading O—C. The sum of these squares is 10,804, whence we derive

Probable error of one equation	... ...	±31·4
,, ,, δx ,,	... ...	±16·5
,, ,, $\delta\mu$,,	... ...	±23·0
,, ,, π ,,	... ...	±14·8

We have thus for the definitive result of the measures in x

$$\pi = +0''\cdot346 \pm 0''\cdot015$$

and for the probable error of one equation, *i.e.* of a coordinate derived from one plate, $\pm 0''\cdot031$.

§ 7. We may now investigate the parallaxes and proper motions of our comparison stars. In this case we are justified in an approximate but much shorter form of solution. If $\Delta_1 \ldots \Delta_8$ denote the absolute terms in the successive equations of condition for any star, we easily find by combining the equations in which the factors of π have the same sign

$$1\cdot000\delta x + 0\cdot548\delta\mu - 0\cdot665\pi = \tfrac{1}{4}(\Delta_3 + \Delta_4 + \Delta_5 + \Delta_8)$$
$$1\cdot000\delta x + 0\cdot548\delta\mu + 0\cdot831\pi = 0\cdot217(\Delta_1 + \Delta_2) + 0\cdot283(\Delta_6 + \Delta_7)$$

whence we obtain by subtracting and then dividing by 1·496

$$\pi = + 0\cdot145(\Delta_1 + \Delta_2) + 0\cdot189(\Delta_6 + \Delta_7) - 0\cdot167(\Delta_3 + \Delta_4 + \Delta_5 + \Delta_8)$$

Similarly by constructing two equations in which the coefficients of δx and π are the same, but those of $\delta\mu$ widely different, we find

$$\delta\mu = 0\cdot325(\Delta_6 + \Delta_7 + \Delta_8) - 0\cdot306(\Delta_1 + \Delta_2) - 0\cdot123(\Delta_3 + \Delta_4 + \Delta_5)$$

Applying these formulæ as a test to our parallax star, we find $\delta\mu = -5$, $\pi = +343$, in very good agreement with the least-square solution.

For our comparison stars we find in the same way, in thousandths of a second—

Star.	1.	2.	3.	4.	5.	6.	7.	8.	9.
μ	+21	+84	−26	−42	−9	−23	+9	−58	+48
π	−32	+18	0	−5	+7	+7	−19	−18	+35

The sum of all the proper motions or of all the parallaxes vanishes, as it ought to do, since they are all relative to the mean of the group.

If we assume that these values are wholly spurious, and due to errors of observation, we find for the probable errors of a proper motion or parallax for one comparison star the values ± 30 and ± 14 respectively. Comparing these with the values for the parallax star we see that the values of π are completely accounted for by accidental errors (supposing these to be the same for the comparison stars and parallax star), while those of μ are a little larger than the accidental errors would lead us to expect. The large value for star 2 may perhaps be real.

If we assume that our comparison stars have no parallax or proper motion (or, rather, that they all have the *same*), the differences of the residuals on different plates will be due to errors of observation. In this way we obtain for the probable error of a coordinate derived from one plate values which range from $\pm 0''.018$ to $\pm 0''.046$ for the different stars, the mean value being $\pm 0''.030$. As this has been derived from residuals left after the reduction of the plates to standard, in which we had to determine three unknowns from eight equations, we must multiply it by $\sqrt{\frac{8}{5}}$ in order to obtain a number comparable with the one previously found for the parallax star. We thus obtain $\pm 0''.038$ for the true probable error of an x-coordinate of a comparison star derived from one plate. This is somewhat larger than the value for the parallax star, perhaps because the comparison stars really have small proper motions of their own.

It is of interest to compare the agreement of the plates with one another with that of the different exposures on one plate, which can be found from the differences mentioned in § 4.

The average value (without regard to sign) of these discordances for all the stars measured on the eight plates is 3.12 units of the fourth place, or $0''.055$. To find the corresponding probable error of a single image we must multiply by the constant 0.845, and also by $\sqrt{\frac{4}{3}}$, since we are considering deviations from the mean of four quantities, and $\sqrt{\frac{10}{8}}$ because we have tried to represent ten quantities for each exposure by a formula with two constants. This gives for the probable error of one image $\pm 0''.060$. That of the mean of four images would then be $\pm 0''.030$, which is close to that found from the agreement of different plates. We may therefore conclude that for these plates the "plate errors" are very small.

The reduction of the approximate values of y for the four epochs gives residuals for the comparison stars that lie within the errors of the measures, showing that their proper motions in declination, like those in R.A., are all small.

§ 8. We pass now to the discussion of the y's. For this purpose three of the comparison stars were chosen—Nos. 2, 6, and 9—whose centre of gravity falls within one réseau-interval of the parallax star, and whose parallaxes all appear to be very small. The y's of these four stars were measured accurately on all the plates. The reduction to standard is in this case very simple. If ξ_2, η_2, denote the standard coordinates of star 2, and so on, we determine three auxiliary constants, α, β, γ, by the equations

$$\alpha\xi_2+\beta\xi_6+\gamma\xi_9=\xi_A$$
$$\alpha\eta_2+\beta\eta_6+\gamma\eta_9=\eta_A$$
$$\alpha+\beta+\gamma=1$$

Then if we denote any expression of the form $a\xi+b\eta+c$ by f, we will have

$$f_A=\alpha f_2+\beta f_6+\gamma f_9$$

The correction to reduce the place of the parallax star to standard may thus be derived immediately from the differences from standard for the three comparison stars.

The results obtained in this case are interesting as showing how conspicuous a large proper motion is, even on photographs taken at short intervals. In the table below the first line gives the residuals in thousandths of a second of arc; the second, the correction necessary to reduce them to 1904·0 with Bossert's proper motion, $-4''\cdot74$; and the third, the corrected values:

Plate ...	191	194	258	260	268	397	405	426
Residual...	+ 33	− 33	−1419	−1436	−1486	−4984	−5102	−6035
Correction	−289	−242	+1379	+1417	+1493	+4735	+4863	+6105
Corrected value	−256	−275	− 40	− 19	+ 7	− 249	− 239	+ 70

Our equations of condition are:

					O−C.
$1\cdot000\delta y$	$-0\cdot061\delta\mu$	$-0\cdot294\pi$	=	−256	+40
1·000	−0·051	−0·256	=	−275	+ 9
1·000	+0·291	+0·595	=	− 40	−41
1·000	+0·299	+0·588	=	− 19	−18
1·000	+0·315	+0·571	=	+ 7	+13
1·000	+0·999	−0·078	=	−249	−24
1·000	+1·026	+0·027	=	−239	−49
1·000	+1·288	+0·596	=	+ 70	+70

The influence of the parallax is again conspicuous.

The normal equations are:

$$+8\cdot000\delta y \quad +4\cdot106\delta\mu \quad +1\cdot749\pi = -1001$$
$$+4\cdot106 \quad +3\cdot989 \quad +1\cdot277 = -\ 389$$
$$+1\cdot749 \quad +1\cdot277 \quad +1\cdot540 = +\ 169$$

Whence we find

	Weight.
$\delta y = -197\cdot8$	3·61
$\delta\mu = -\ \ 1\cdot5$	1·76
$\pi = +335\cdot5$	1·08

The residuals in the equations of condition are given above. From them we derive:

Probable error of δy	± 17
$\delta\mu$	± 25
π	± 31
One equation	± 33

The definition solution from the y's gives therefore

$$\pi = +0''\cdot335 \pm 0''\cdot031$$

The probable error of a y-coordinate derived from one plate is almost exactly the same as that of an x-coordinate, but the latter gives a determination of the parallax with four times as much weight as the former. The agreement of the two values is very satisfactory. Combining them with regard to these probable errors, we have for our final value, relative to the nine comparison stars—

Parallax of Lalande 21185 = 0″·344±0″·013

§ 9. The following table gives in summary form the result of previous investigations of this star's parallax :

Observer.	Date.	Method.	Number of Comp. Stars.	Number of Obs.	Result.
(1) Winnecke	... 1857–58	Heliometer	2	12	+0″·511 ± 0″·015
(2) Kapteyn	... 1885–87	Transits	2	46–47	+0·434 ± 0·028
(3) Flint ...	... 1893–95	Transits	2	18	+0·36 ± 0·047
		or, including a systematic correction,			+0·37

References: (1) *A.N.* 1147. (2) *A.N.* 2935. (3) *Publications of the Washburn Observatory*, vol. xi. pp. 219, 437.

The present investigation supports the most recent ones in showing that the parallax is smaller than at first supposed, so that this star is not the nearest in the heavens after *α Centauri*, but is more remote than *Sirius*, and probably 61 *Cygni* as well.

§ 10. We have still to consider the effect of atmospheric dispersion on our results. The displacement of a star-image on the plate by refraction is given by the equations

$$\Delta x = \beta X + \text{small terms}$$
$$\Delta y = \beta Y + \text{small terms}$$

where β is the constant of refraction, and X, Y the coordinates of the zenith projected on the plane of the plate, expressed in terms of the focal length as unit.

If the effective mean wave-length of the light of the parallax star differs from that of the comparison stars, the refraction constant will also differ, say by $d\beta$, and the parallax star will be displaced on the plate relatively to the others by $Xd\beta$ and $Yd\beta$ in the two coordinates.

For plates taken near the meridian we have (neglecting terms involving the cube of the hour-angle)

$$X = \frac{t \cos \phi}{\cos (\phi - \delta)}, \quad Y = \tan (\phi - \delta) + \tfrac{1}{4} t^2 \sin 2\phi \sec^2 (\phi - \delta)$$

where ϕ is the observer's latitude, δ the declination of the plate centre, and t the hour-angle expressed in circular measure. The dispersion in x is therefore proportional to the hour-angle, and vanishes at the meridian, while that in y is practically constant for each field.

Computing thus the effect of refraction for each of our plates, and introducing the results into our equations of condition and normal equations, we find for the effect on our unknowns :

Measures in x.	Measures in y.
$d\delta x = +0{\cdot}028 d\beta$	$d\delta y = +0{\cdot}280 d\beta$
$d\delta\mu = -0{\cdot}034 d\beta$	$d\delta\mu = +0{\cdot}000 d\beta$
$d\pi = +0{\cdot}002 d\beta$	$d\pi = +0{\cdot}000 d\beta$

Here $d\beta$ denotes the change in the refraction constant expressed in seconds of arc. As the whole difference between the refraction constants for the visual and photographic rays is less than $1''$, it is clear that our results must be free from any sensible error arising from this source, except as regards δy, whose exact value is quite immaterial.

It should, however, be noticed that we have been regarding $d\beta$ as constant, whereas it really varies with the meteorological conditions proportionately to the total refraction. This cannot affect our x-equations, where the coefficients of $d\beta$ are all very small ; but as the change is a seasonal one it may produce some effect on the value of the parallax derived from the y's. The refraction averages greater in winter than in summer ; for our star Y is positive ; therefore the star will appear farther north in winter than in summer, if $d\beta$ is positive. But the effect of annual parallax is to displace a star to the southward in winter and northward in summer.

Consequently if $\delta\beta$ is positive—that is, if the star is bluer than the comparison stars—the effect of seasonal variations in the dispersion will be to make the value of the parallax found from the y's too small. This effect is, however, a small quantity of the second order, and is probably quite insensible.

§ 11. We have finally to consider what is the probable parallax of our comparison stars. We have already found that their relative proper motions and parallaxes are very small. The very small values of the corrections found to the catalogued motion of the parallax star, which is very well determined, show that our comparison stars have no common drift. Their proper motions as computed from our plates are probably largely due to accidental error. If we assume that the true motions and the errors of observation contribute equally to the observed results, the observed proper motions in one coordinate will on the average be equal to the true proper motions in the plane of reference.

We may then apply Professor Kapteyn's formulæ for the mean parallax of a group of stars of given proper motion and magnitude given in No. 8 of the Publications of the Astronomical Laboratory of Groningen. The average magnitude of our comparison stars is 8·3, and their average observed proper motion in x, without regard to sign, is $0''{\cdot}036$. With these arguments

Kapteyn's table [*loc. cit.* p. 31, Table G, headed "All the Stars"] gives mean parallax $= 0''\cdot 0071$.

If we discard all hypotheses concerning the proper motions and use the magnitude alone as the criterion of distance, Kapteyn's Table C [*loc. cit.* p. 28] gives mean parallax $0''\cdot 0074$.

We may therefore assume with some confidence for our comparison stars

$$\textit{Mean Parallax} = 0''\cdot 007$$

From Kapteyn's researches it appears that it is more likely than not that the parallax of a single star will be within 50 per cent. of the value given by his table for a star of its magnitude and proper motion. For the mean of nine stars we should have a much closer agreement, so that the value just found is not likely to be in error by more than a very few thousandths of a second, especially as we have already seen that none of the stars has a large parallax.

By adding this to the value already found for the parallax of *Lalande* 21185 relative to the comparison stars, we may obtain a very close approximation to its absolute parallax, and this should be used rather than the relative parallax in computing the star's distance, light, and the like.

§ 12. γ *Virginis* R.A. $12^{h}\ 36^{m}\cdot 3$. Dec. $0°\ 55'$ S. (1900·0). Binary. Components equal: joint magnitude 2·91. P.M. $0''\cdot 57$. Pos. 327°. Dist. $5''\cdot 7$ (1904).

This bright star was photographed through the colour screen, and eight plates at three epochs were secured before the failure of the latter.

Except on very unsteady nights the images of the two components are well separated; but to ensure this the exposures had to be short, and, as the field is a very poor one, it was found impossible to get the ordinary number of measurable comparison stars. If we had had a series of colour screens of varying densities this could have been remedied by using a denser screen and longer exposures; but, as things were, it was necessary to get along with only six comparison stars—the smallest number for any of our fields. It also appeared early in the course of measurement that these plates were below the average in quality, owing perhaps to the relatively low altitude of the star, which is one of the southernmost on our list. One of the plates was shown by the discordance of the four exposures to be particularly bad, and it was given half weight, a decision confirmed later by the large residuals which it gave in the final solutions.

The present discussion may therefore be taken as an example of our photographs at their worst, and it is gratifying to find that even then they give results of some apparent value.

The general plan of the work was exactly similar to that for the previous star, so that only the points of difference need be mentioned here.

§ 13. Having only six comparison stars the method of reduction was somewhat altered. The stars were divided into three pairs, and the means of the equations of condition for each pair were taken, thus giving three equations for the three plate constants. As the centre of gravity of the six stars fell within a réseau interval of the parallax stars, the use of this approximate method is justifiable.

Solutions were made for the two components separately, the parallaxes of the comparison stars were approximately determined, three of them were chosen and the y's measured, with the results given below. A denotes the southern and B the northern component of the binary, and the assumed proper motions are $-0^s.038$ ($= -0''.57$) in x and $+0''.015$ in y.

From measures in x.

	Star A.	Star B.	Weight.
$\delta x =$	$-0''.029 \pm 0''.030$	$-0''.019 \pm 0''.038$	2.66
$\delta\mu =$	$+0.110 \pm 0.046$	$+0.089 \pm 0.059$	1.14
$\pi =$	$+0.072 \pm 0.027$	$+0.054 \pm 0.034$	3.34
Probable error of unit weight	± 0.049	± 0.063	

From measures in y.

	Star A.	Star B.	Weight.
$\delta y =$	$+0''.028 \pm 0''.037$	$+0''.026 \pm 0''.079$	2.64
$\delta\mu =$	-0.088 ± 0.057	-0.107 ± 0.121	1.12
$\pi =$	$+0.070 \pm 0.074$	$+0.068 \pm 0.157$	0.67
Probable error of unit weight	± 0.061	± 0.128	

The weight of the parallax derived from the y's is but one fifth of that from the x's (and even this is more than it would be for the average star). It would not ordinarily pay to measure them; but as the present series cannot be continued, it seemed worth while to get all possible information out of the plates.

The large probable errors found for the y coordinates of star B are due to one very large residual for the plate which had previously, for quite other reasons, been given half-weight.

If we combine the results from the x's and y's with regard to their probable errors, we have

Parallax of A	$+0''.072 \pm 0''.024$
B	$+0.054 \pm 0.033$

The two values agree within their probable errors. Taking the mean with equal weights, we have for the parallax of γ *Virginis* relative to the six comparison stars

$$\pi = +0''.063 \pm 0''.022$$

There is, however, something unsatisfactory about this solution. The proper motion of γ *Virginis* (which is in the Fundamental Catalogue) is very well determined, and the large corrections found above are almost certainly not real. It is indeed barely possible that the comparison stars have a "group motion" which accounts for the discrepancy; but this is exceedingly improbable, and the large probable errors of the calculated values of $\delta\mu$ suggest that these values themselves are due to errors of observation. It therefore seemed advisable to repeat the least-square solutions, rejecting the terms in $\delta\mu$. The results were

From measures in x.

	Star A.	Star B.
$\delta x =$	$+0''.025 \pm 0''.023$	$+0''.024 \pm 0''.026$
$\pi =$	$+0.094 \pm 0.029$	$+0.072 \pm 0.032$
Probable error of unit weight	± 0.056	± 0.063

From measures in y.

	Star A.	Star B.
$\delta y =$	$-0''.018 \pm 0''.023$	$-0''.028 \pm 0''.047$
$\pi =$	$+0.106 \pm 0.070$	$+0.117 \pm 0.139$
Probable error of unit weight	± 0.061	± 0.122

The representation of the observations is about as good as before, so that the idea that the large values of $\delta\mu$ are due to accidental error is confirmed. Combining these new values of the parallax with regard to their probable errors we have

Parallax of A	$+0''.096 \pm 0''.027$
B	$+0.074 \pm 0.031$

and for the mean of the two, with equal weights,

$$\pi = +0''.085 \pm 0''.021$$

This result differs from the one previously found by less than the probable error of either one. In the absence of certainty which of the two solutions is to be preferred we may perhaps best take the mean of the two, which gives

Parallax of γ Virginis = $+0''.074 \pm 0''.022$

as the best value, relative to the mean of the six comparison stars, which can be derived from our plates.

§ 14. The approximate discussion of the residuals for the comparison stars gives values for their parallaxes and proper motions whose means (without regard to sign) are $0''\cdot037$ and $0''\cdot051$ respectively. These values appear to be due to errors of observation. If we assume that the comparison stars have no sensible parallax or proper motion, the probable error of a measured coordinate for one of them derived from one plate comes out $\pm 0''\cdot080$. This is larger than the value previously found for the parallax star, so that it would appear that in this case the images taken through the gelatine patch of our colour-screen are better than those taken through the clear glass outside.

The probable error of a single image deduced from the comparison of the exposures on each plate with one another is $\pm 0''\cdot084$, which would lead us to expect a probable error of $\pm 0''\cdot042$ for a plate with four exposures. This is much less than the value given by comparison of different plates, so that it seems that in this series there is some sort of "plate error" which is nearly the same for all the images of one star on a plate.

Calculation of the effect of atmosphere dispersion on our results gives the following (when the seasonal variations of $d\beta$ are disregarded):

Results from x.	Results from y.
$d\pi = -0\cdot005 d\beta$	$d\pi = +0\cdot004 d\beta$

so that we need fear no error from this source.

The average magnitude of our comparison stars is $8\cdot9$, corresponding to which Kapteyn gives the mean parallax $0''\cdot006$.

§ 15. The only previous determination of the parallax of γ *Virginis* known to the writer is a spectroscopic one by Belopolsky. He finds (*A.N.* 3510) that the relative velocity of the two components is $0\cdot278$ geographical miles per second, with a probable error of about $\pm 0\cdot1$ g.m. With Doberck's elements of 1881 this gives $\pi = 0''\cdot051$. Owing to the uncertainty of the inclination of the orbit of the binary (given by different computers as from 31° to 37°) and that of the observed radial velocities of the two stars the probable error of the above value must be considerable. The agreement with the results of the present investigation is as good as there is any reason to expect.

§ 16. We may conclude by deriving from our parallaxes such information as we can get concerning the brightness, mass, &c. of the stars. In dealing with the brightness of stars the writer would suggest that Professor Kapteyn's conception of the "absolute magnitude" of a star should be generally used. By the absolute magnitude of a star Professor Kapteyn denotes the magnitude which it would appear to have at such a distance that its parallax was $0''\cdot1$. If m is the star's observed magnitude and π its parallax, we have then for the absolute magnitude m_0

$$m_0 = m + 5 - 5 \log \pi$$

3 K

In calculating this and similar quantities the relative parallax already found for our stars should be corrected by adding the probable mean parallax of the comparison stars.

We thus obtain for Lalande 21185 $\pi = +0''\cdot351$ which, with the magnitude 7·3 and proper motion 4·77, gives

Absolute magnitude 10·0

The Sun's absolute magnitude is given by Kapteyn as 5·5, so that the star is 4·5 magnitudes fainter than the Sun, and gives about $\frac{1}{60}$ as much light.

The velocity of the star at right angles to the line of sight is 65 kilometres per second, with a probable error (so far as the present determination of the parallax is concerned) of about 3 km.

For γ *Virginis* we find the absolute magnitude of the two stars taken together to be 2·4. The two components are equal in brightness, so that the absolute magnitude of each one of them is 3·2; that is, each of them gives about nine times as much light as the Sun. The velocity of the system at right angles to the line of sight is 34 km. per second, while from Belopolsky's observations the velocity in the line of sight is 21 km., and the star is approaching us. This would make the velocity of the system in space 40 km. per second in a direction inclined about 60° to the line of sight. These values are, however, somewhat uncertain.

Using See's elements for the binary system ($a=3''\cdot99$, P=194 years, $e=0\cdot90$) we find

Major axis of orbit	= 50	astronomical units
Distance of stars at periastron	5	„
„ at apastron	95	„
Mass of system	3·3	

Auwers and Lewis have found that the masses of the two components are nearly equal, and so each of them must be about 1·6 times as massive as the Sun, whereas they each give about nine times as much light.

These stars must therefore be either less dense than the Sun or have a greater surface brightness, which accords well with the fact that their spectra are of the first type.

§ 17. In conclusion I wish to express my hearty thanks to the Director and staff of the Cambridge Observatory for the use of its instruments and of all its privileges, and for their cordial interest in the work; and in particular to Mr. A. R. Hinks for much valuable comment and criticism, and especially for taking a large number of plates for me while I was disabled by a long illness.

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