

attention. Needless to say, no one knows the facts in question better than our distinguished author, and it is a pity that he did not take time to express himself more precisely. Later the author drops the term "arbitrary coefficients" and speaks of "coefficients represented symbolically" (page 11).

The foundation of the Galois theory covers in this tract the first twenty-nine pages; then eleven pages are given to cyclic equations, four pages to abelian equations and ten pages to metacyclic equations. The last chapter is upon "Solution by standard forms" and contains, on six pages, in outline, the treatment of the quintic by means of the icosahedral irrationality. This outline can be used with interest in conjunction with Klein's *Ikosaeder*.

FLORIAN CAJORI.

Introduzione alla Geometria Proiettiva degli Iperspazi. By E. BERTINI. Pisa, Spoerri, 1907. 425 pp.

THE subject of geometry of n dimensions has undergone a noteworthy advancement during the last few decades. In this advancement the Italian geometers have taken a very prominent part. It is therefore fitting that this able work on projective geometry of n dimensions should be the work of an Italian.

The author avoids the difficulties of spacial intuition by starting from a purely arithmetical definition of a point and of a space. There exist, in ordinary geometry, numerous examples of systems of entities which can be put into one to one algebraic correspondence, without exceptional elements, with the values of the ratios of a system of parameters. The properties of such systems are of two kinds, those depending on the particular nature of the entities and those depending only upon the fact of the correspondence with the group of values of the parameters. The latter properties the author considers to form the subject matter of projective geometry of n dimensions. A point is therefore defined as a set of ratios of $n + 1$ parameters x_0, x_1, \dots, x_n , and a space of n dimensions as "the totality formed by all the sets of values (real and complex) of these ratios."

The first two chapters are devoted to a discussion on this basis of the ideas of subordinate spaces, section, projection, etc. The possibility of a satisfactory synthetic treatment is also pointed out, however, and a number of references to such treatments are given.

The next three chapters, of about eighty-four pages, are de-

voted to the discussion of projective and dualistic correspondences. Chapters six and seven, containing each about twenty pages, treat of the properties of quadrics and systems of quadrics. Chapter eight, of twenty-five pages, is devoted to the discussion of hypersurfaces and chapter nine, of thirty pages, to the discussion of algebraic spreads in general. In chapters ten and eleven, containing about fifty pages, an able and interesting discussion of linear systems of hypersurfaces is given. Chapter twelve, of fifteen pages, treat of rational curves, with especial attention to rational normal curves. Chapter thirteen, containing twenty-three pages, treats of rational ruled surfaces. In chapter fourteen, of about twenty pages, the author discusses the relations of linear systems of plane curves to two dimensional spreads in space of n dimensions. Chapter fifteen, of twenty-five pages, is devoted to a thorough discussion of the surface of Veronese and its three dimensional projection, the Steiner surface.

Of especial importance is the appendix of about fifty pages. This is divided into three chapters. The first deals with branches of algebraic curves, the second with quadratic transformations and the third with correspondences on one dimensional spreads.

It will be seen that the book covers only a portion of the field of geometry of n dimensions ; but the subjects treated are discussed with a great deal of thoroughness. The treatment is in fact, in some cases, rather profuse. It would, perhaps, have been better to omit some of the minor discussions except for a reference to where the treatment may be found.

A regrettable feature of the book, from the standpoint of the general reader, is the scarcity of references. This lack is recognized by the author himself, but he points out that the book is intended primarily for undergraduates in the Italian universities, for whom an extensive bibliography did not seem so necessary.

The reader will find in no other place so complete and comprehensive an exposition of our present knowledge of projective geometry of n dimensions as is given in this book. It is clearly and logically written and unusually free from errors. It will doubtless stimulate a much wider interest in this field of research.

C. H. SISAM.