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XXIII. Researches in Acoustics.—No. IX

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2. There is great difficulty in accounting for the reduction of the metal by purely chemical processes.

3. Arrhenius's hypothesis of ionic dissociation in flames is a chemically acceptable way of accounting for the liberation of sodium when its salts are heated in flames.

4. There is no direct evidence and no decisive indirect evidence that the sodium spectrum is the direct consequence of the chemical action in which the atoms are engaged.

Postscript.

In a recent number of Wiedemann's *Annalen* * there is a paper by Paschen in which he shows by bolometric measurements that the invisible spectra of hot gases exhibit distinct maxima of intensity—that, in short, gases do give discontinuous spectra on being heated, independently of chemical action. On these grounds and others, some of which are similar to those explained in the foregoing paper, Paschen does not consider that Pringsheim's conclusions can be accepted.

[To be continued.]

XXIII. *Researches in Acoustics.*—No. IX. *By* Alfred M. Mayer, *Ph.D.*†

Contents.

1. *On the Law connecting the Pitch of a Sound with the Duration of its Residual Sensation.*

In October 1874 I published in the American Journal of Science Paper No. 6 of Researches in Acoustics, which contained an account of my attempts to establish the law connecting the pitch of a sound with the duration of its residual sensation. The law given in that paper was the expression of the results of the first experiments, extending through several octaves, ever made on the duration of sonorous sensations.

Subsequently, in April 1875, I published in the American Journal of Science ‡ the results of similar experiments which

* Vol. 50. p. 409 (1893).

† Communicated by the Author.

‡ The papers cited above were published in the Philosophical Magazine of May 1875, in one paper, "Researches in Acoustics, No. VI."

Madame Ema Seiler had made at my request. She made a long series of experiments with the same apparatus I had used. Her determinations, though agreeing with mine in having approximately the same variation of the residual sensation with the pitch, yet differed considerably in the absolute quantities which she found for the durations of these sensations. That the two series of observations should differ was to be expected from the known variation of the sonorous sensations among different observers; but the principal cause of the difference is to be attributed to the apparatus (fig. 3) used in these experiments. This apparatus generated sounds in addition to the one to be specially observed, so that the determinations were difficult to make except by one whose hearing was peculiarly trained and naturally gifted in the power of excluding other sound-sensations from the one alone to be studied. In the ability to analyse composite sounds Madame Seiler was noted; and I had no doubt at the time of the publication of her results that they were more worthy than mine to form the basis of a physiological law. This I stated in my paper of 1875, and the experiments described in the present paper, made with improved methods, show that the opinion then entertained was correct.

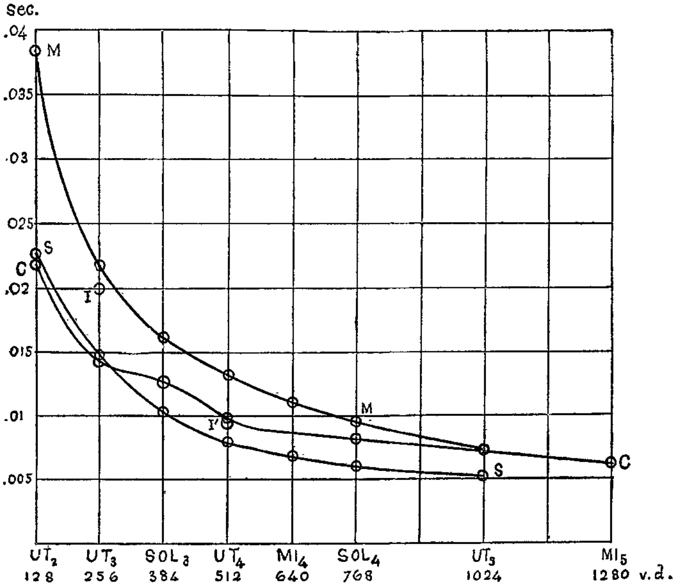
That there is a physiological law which gives the relation between the pitch of a sound and the duration of its residual sensation is shown by the numerous experiments contained in this paper. But those published in 1874 and 1875 sufficed to establish that fact; yet those experiments have never been repeated by physiologists.

I have waited nineteen years in the hope that others would make similar experiments, so that the combination of the results of various experimenters would give an expression of the law which might be regarded as general and accepted as expressing the average residual sensations of sounds.

It is true that Professor C. R. Cross and H. M. Goodwin published a series of similar experiments in "Some considerations regarding Helmholtz's Theory of Consonance" (Proc. Amer. Acad., Boston, June 1891). They obtained the smallest consonant intervals of simple sounds by blowing sheets of air across the mouths of resonators. The reciprocals of the differences of the frequency of the vibrations forming the intervals thus found are plotted in the curve C C of fig. 1. I and I′ give their determinations of the durations of the residual sensations of UT_3 and of UT_4, deduced from their observations of the coalescence of these sounds when interrupted by a perforated disk rotating between a resonator and its corresponding fork.

The curve S shows Madame Seiler's determination of the residual sonorous sensations; M shows mine. It is evident that the meandering, undecided curve, C, cannot be the expression of a law, and that the data I and I′ cannot be combined with those contained in the curve S, or in the

Fig. 1.



curve M. In a general way the curve C shows that the smallest consonant interval of two tones contracts as the pitch of the tones, forming the interval, rises.

The physicists Mr. Alexander J. Ellis and Professor J. A. Zahm have discussed the bearing of the law (as given by the experiments of 1874 and 1875) on the elucidation of many facts in consonance and dissonance, to which I referred in my paper of 1874*.

On the Duration of the Residual Sonorous Sensation.

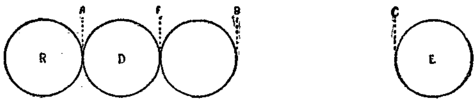
The duration of a residual sonorous sensation is really the duration of the entire period in which a sensation of sound is perceived after the vibrations outside of the ear, giving rise to that sensation, have ceased to exist. While the total

* See Ellis's translation, of 1875, of Helmholtz's *Die Lehre von den Tonempfindungen*, pp. 173, 701, 795, and Ellis's "Illustrations of Just and Tempered Intonation," Proc. Musical Assoc. of London, June 7, 1875; Zahm, 'Sound and Music,' Chicago, 1892.

duration of the after-sensation produced by the stimulus of light can be measured, as in the case of an electric flash, the determination of the total duration of the after-sensation of a sound appears, in the light of our present knowledge and with the means of experiment at our command, to be a problem very difficult to solve.

The object of this research was not to determine the total duration of the after-sensation of a sound, but to measure that duration in which the after-sensation of a sound does not perceptibly diminish in intensity.

Fig. 2.

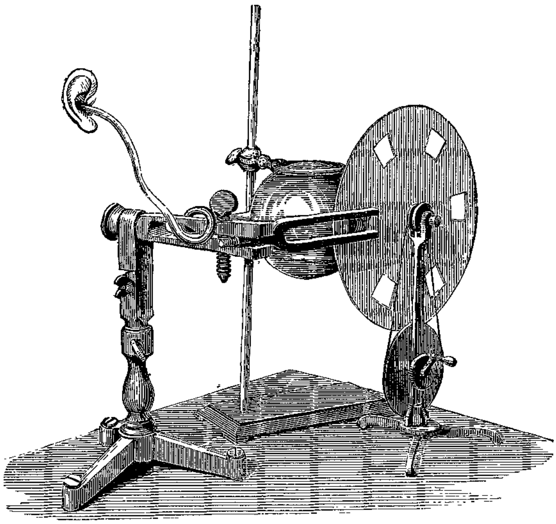


In fig. 2, D and E represent openings in a screen impervious to sound. The distance between the openings equals thrice the diameter of an opening. A tube, R, having the same interior diameter as the openings, is supposed to convey sound-vibrations against the screen, while the tube itself moves from left to right with its mouth sliding along the surface of the screen. In the position A, the sound is just about to traverse the opening D to the ear on the other side of the screen. As R progresses over the opening D, the sound traverses the opening till R has reached the position F B. Then, in the path of the tube from B to C, no sound traverses the screen. When the edge B of the tube has reached the position C, the sound is again just on the eve of traversing the screen through the opening E. As the distance A to B equals B to C, the periods during which the sound traverses the screen equal those in which it does not do so. If these alternations of sound and silence should succeed one another so rapidly that the sensation of the sound is uniform in its intensity, it may at first sight appear that during the time that the tube takes to go from B to C the after-sensation of the sound has not diminished in intensity. But is B to C to be taken as the measure of the duration of uniform sensation? As the tube R moves over D a sound with a varying intensity traverses the opening in the screen. We cannot suppose that the residual sensation caused by the stimulus of the sound traversing a minute opening in the screen equals that caused by the sound which traverses the screen when the circles R and D coincide. In such experiments, however, we are driven to take as the duration of the undiminished residual sensation the time that the centre of the tube, R, takes to go from the centre of D to the centre of E.

In this illustration I have, for simplicity and conciseness, supposed the tube R to move over the openings D and E. In the actual experiments D and E are two of several holes in a disk, arranged in a circle, and the disk rotates while the tube R is fixed. Another tube placed in the prolongation of the tube R, on the other side of the disk, conveys the interrupted sound to the ear.

Evidently the manner in which the tube conveying the sound to the disk is open and closed by the revolving disk has to be considered in researches made with this apparatus. I give two cases whose discussion has led me to modify,

Fig. 3.



with marked efficiency, the apparatus, shown in fig. 3, which was used in the researches published in this Journal in 1875. In that apparatus the interruptions of sound were made by a perforated disk revolving in front of the mouth of a resonator, while the interrupted sound was conveyed to the ear by a tube attached to the small opening in the nipple of the resonator.

This mode of obtaining the interruptions of the sound is objectionable because the resonator is not in tune with the fork except when the former is fully opened, and also because the perforated disk rotating across the mouth of the resonator gives rise to two secondary sounds and a resultant sound, fully described in my paper of 1875 *. These sounds, from their intensity in this form of experiment, mask the proper sound of the fork, making the determination of the durations of the sonorous sensations both difficult and uncertain. Also, in these experiments the action of the interrupted sound on the ear is distressing, even injurious; for the hearing of one of my ears was permanently impaired by the experiments I made with this apparatus nineteen years ago.

In the apparatus presently to be described, the fork vibrates in front of the mouth of the resonator, and the interruptions in the flow of sound are caused by the perforated disk revolving in front of the small opening in the nipple of the resonator, as shown in fig. 7.

Discussion of the Effects of the Relative Sizes of the Openings in the Revolving Disk and of the Opening in the Tube conveying the Sound to the Disk.

First Case. Suppose that the opening of the nipple of the resonator and the openings in the disk have the same diameter. In the actual experiments these openings were 1 centim. in diameter. The nipple of the resonator had a tube of that diameter adapted to it.

Fig. 4.

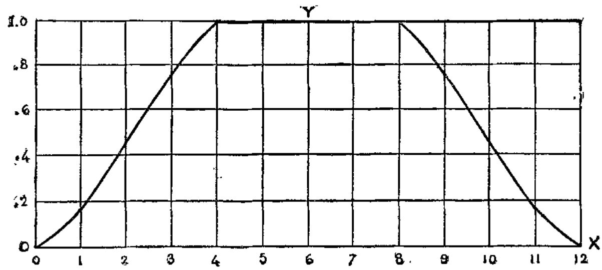
Fig. 4 is the graphic representation of the results of computing the varying areas of the opening of the tube of the resonator, as an opening in the disk passes in front of it. This diagram is to be studied in connexion with fig. 2. The entire length on the axis of abscissas OX gives the space A to B of fig. 2, divided into sixteen parts. The ordinates of the curve give the relative areas of opening for corresponding positions on the axis of abscissas. The ordinate

* See a paper by Lord Rayleigh, "Acoustical Observations, III. Intermittent Sounds," Phil. Mag., April 1880, in which the author gives an explanation, in mode and in measure, of the secondary sounds and of the resultant sound, observed by me in these experiments.

marked 8 Y shows the full area of opening when the circles of the tube of the resonator and of the disk coincide. It will be observed that the tube is opened and closed slowly, and is only instantaneously fully opened at 8.

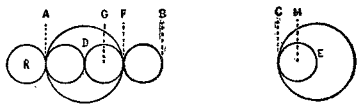
Second Case. The openings in the disk remain 1 centim. in diameter, but the opening in the nipple of the resonator is $\frac{1}{2}$ centim. in diameter. Fig. 5 shows the relations between the areas of opening of the nipple of the resonator and the

Fig. 5.



path A to B (fig. 6) of this opening, R, as it is supposed to move across the opening D in the disk. Fig. 5 is to be studied in connexion with fig. 6. Fig. 5 shows that the opening and closing of the nipple of the resonator take place rapidly, and that the nipple remains fully opened from 4 to 8, that is, during one-third the time that the opening in the disk takes to traverse the opening in the resonator. The advantages gained by this mode of experimenting are considerable. The periods of sound and of silence are sharply marked, and, as we shall now show, the fact that the hole in the resonator has half the diameter of the hole in the disk gives us the means of approaching nearer to the measure of the veritable time during which we have an after-sensation of uniform intensity.

Fig. 6.



In fig. 6 R represents the opening in the nipple of the resonator, supposed to pass over the opening D in the disk. In this case, as in fig. 2, the space A to B in which sound traverses the revolving disk is equal to the space B to C in which silence supervenes, for the distance separating two holes in the disk equals twice the diameter of a hole, or four times the diameter of the hole in the nipple of the resonator. But in this form of the experiment we are again in doubt as to what space should be taken as measuring the duration of the uniform residual sensation. The period from the full opening of the resonator in one position to the full opening in the succeeding one is equal to the time the disk takes to go from G to H, which is equal to the distance D to E between the centres of the openings in the disk minus the diameter of the opening in the resonator, or, DE minus $\frac{1}{6}$ DE. This distance, G to H, evidently measures the periods between maximum and maximum intensities of succeeding sound-pulses; and we have taken this distance, in terms of velocity of rotation, as the measure of the period of uniform residual sound-sensation, because we have no certain knowledge of the relative durations of the residual sensations corresponding to vibrations which pass the disk with increasing intensity, from 0 to 4, fig. 5, and with decreasing intensity, from 8 to 12.

In our experiments we measured the number of flashes of sound entering the ear by knowing the number of revolutions of the disk per second, and the number of holes in the disk. From this knowledge we compute the time it took the disk to go over D to E in fig. 6, the distance between centres of two neighbouring holes; then we reduced this time by one sixth, which is the ratio of the diameter of the opening in the nipple of the resonator to the distance D to E, and took this reduced time as the duration of the uniform residual sensation. The duration of the sonorous sensation determined in this manner is evidently nearer the truth than that obtained with apparatus in which the hole in the tube conveying the sound to the disk and the holes in the disk have the same diameter.

The Apparatus and Methods used to Measure the Durations of Residual Sonorous Sensations.

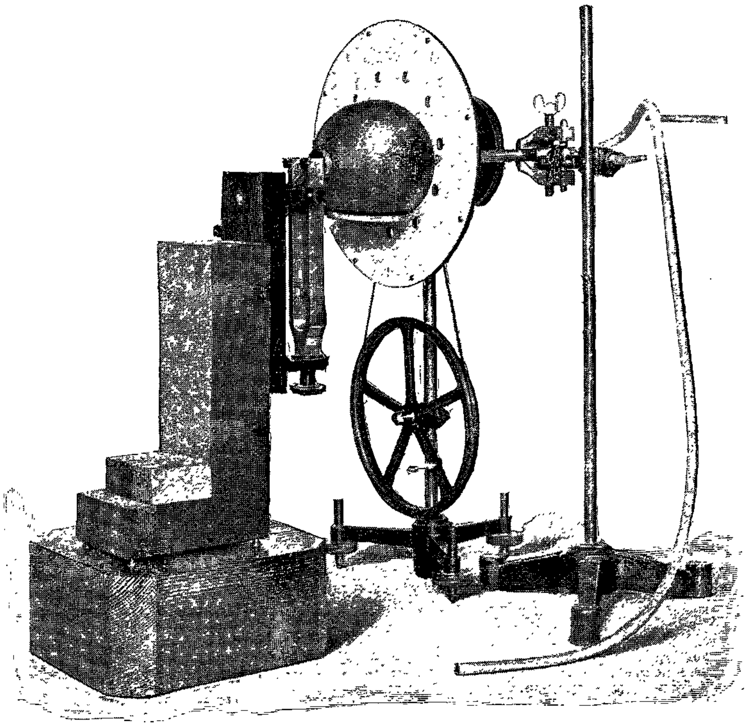
[A.] I shall first describe the apparatus which I found the most efficient for these measures, and then describe three other forms of apparatus used.

In fig. 7 is seen a perforated disk mounted on the axle of a rotator. In front of the disk is the resonator with its nipple close to the surface of the disk. A tuning-fork is opposite the mouth of the resonator. On the other side of the disk, with its axis in line with the axis of the nipple of the resonator, is the tube which conveys the interrupted sounds to the ear. The opening in this tube has the same diameter as the openings in the revolving disk. Behind the line of this tube is seen, on the end of the axle, a fly-wheel of copper weighing a kilogramme.

The bevelled face of the driving-wheel of the rotator has three grooves cut in it, in steps. Corresponding grooves are cut in the pulley of the axle.

To run a rotator of this description smoothly, without jars or vibrations, it is necessary that the cord which passes over the driving-wheel and pulley should be very flexible and have a circular section of uniform area throughout its length.

Fig. 7.



I obtained such a cord by soaking a cord of porpoise leather in neat's-foot oil, and then drawing it many times through wire-drawing plates. The ends of the cord are connected by a short hook-and-eye coupling. The driving-wheel can be slipped up or down the standard of the rotator to adjust the tension of the cord.

The driving-wheel is turned by a handle of aluminium of the form shown in the figure. It is necessary to have a handle of small diameter in order to turn the wheel with a uniform velocity. The fingers which clasped the handle were coated with plumbago dust.

In all the experiments the driving-wheel of the rotator was revolved either once in a second or twice in a second. This is accomplished, after some practice, in the following way:—The rotator, on which is mounted the disk and fly-wheel, is placed near a clock giving loud beats of seconds, and the driving is revolved by the guidance of the hand and ear. The results of the experiments showed that the velocities thus given to the disk were sufficiently uniform, and the measures of the durations of sonorous sensations sufficiently concordant and precise to obtain the data of the physiological law.

I adopted this method of rotation in preference to mechanical means for controlling and measuring the revolutions of the disks. To determine when the interrupted sounds have blended requires, so to say, a flexible apparatus whose velocity is under the immediate control of the hand and ear. This is important in making the final judgement between sounds, one of which appears to have too few interruptions, the other a few more interruptions than are necessary to give a continuous uniform sensation. It is evident that when we can at once slightly increase or diminish the velocity of rotation of the disk, we have the means of making comparisons rapidly succeeding one another. A rotator, driven as described, forms more a part of the observer than one driven and regulated by mechanism.

As there were three grooves in the driving-wheel and three in the pulley on the axle, and as the driving-wheel was revolved either once or twice in a second, eighteen different velocities could be given to the rotating disk.

Disks were made having numbers of holes from 5 to 19, so that, with 18 velocities and the various numbers of holes in the disks, it was easy to select a disk, driven with a known velocity, which gave the exact number of interrupted sounds per second to blend.

The 18 ratios of velocities of the driving-wheel and of the pulley on the axle of the rotator were obtained as follows:—A circle of cardboard, divided into one hundred parts, was clamped on the rotator in front of a disk. The driving-wheel was rotated either once or twice in a second, so that the conditions were the same as in the experiments. From 10 to 100 revolutions of the driving-wheel were made before the ratio was determined. The division to which a fixed index pointed on the divided circle gave the fraction of a revolution. The whole number of revolutions was given by a simple counter which moved with very little friction.

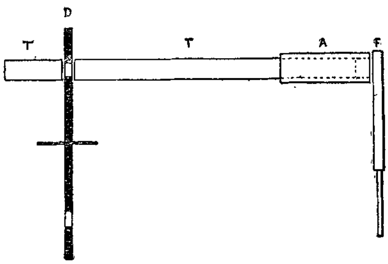
The rotating disks were made of mahogany, 5 millim. thick, with disks of cardboard about 2 millim thick screwed to the wooden disks. The circumferences of the holes in the wooden and cardboard disks exactly coincided. The circle of holes in the disks was placed 5 centim. from its border. The rotator and disks were so carefully made that the nipple of the resonator was only $\frac{3}{10}$ millim. from the surface of the revolving disk. The mouth of the tube conveying the interrupted sounds to the ear was about the same distance from the surface of the other side of the disk. The disks were clamped on the axis of the rotator between smaller flat disks of brass, not shown in the figure.

The diameter of the holes in the disk, and of the interior of the tube conveying the sounds to the ear, was 1 centim. The diameter of the opening in the nipples of the resonators was $\frac{1}{2}$ centim.

The disks were made of mahogany which had been in my possession for thirty years. It was well seasoned and had nearly the thickness required for the disks. This wood was used because it holds the form given it better than any wood I have had experience with.

Sound passes through mahogany and other woods even when a centimetre in thickness. Sound also passes through cardboard, but not so readily as through wood. I found that by placing cardboard on wood I formed a screen of heterogeneous materials which presented an effective obstruction to the passage of sound.

Fig. 8.



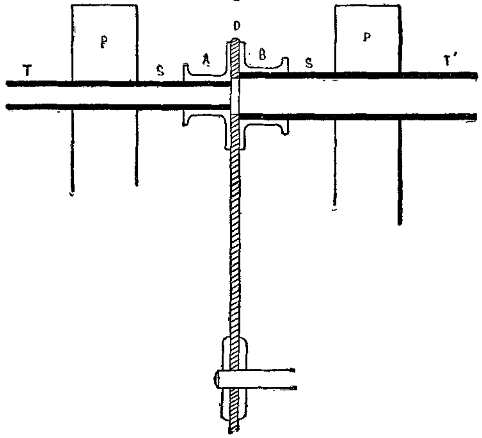
[B.] In the second form of apparatus I replaced the resonator by resonant tubes as shown in fig. 8, where F is the fork, T the tube, with a tube of larger diameter, A, sliding on T, so that the air in the tube could be adjusted to vibrate with the fork. On the other side of the disk D is the tube T′, to which is attached the tube of caoutchouc which leads to the ear. This arrangement is like that used by Dr. R. Kœnig, and described on page 140 of his *Quelques Expériences d'Acoustique*, Paris, 1882, and used by him for the observation of the sounds produced by interruptions of continuous sounds.

Experiments made with this apparatus gave the same results as those made with apparatus [A], but the sounds given are so feeble compared with those coming from the resonator (fig. 7), that the periods of sound and silence (or, rather, of sound and much diminished sound) are not sharply separated. It followed that the judgment of a continuous sensation on the ear could not be so neatly made with the use of the resonant tubes as when the resonators were employed.

[C.] To obtain sharper demarcation of sound and silence by having no aperture for the lateral escape of sound between the rotating disk and the nipple of the resonator and between the disk and the tube conveying the sound to the ear, I made the following apparatus (fig. 9). I turned disks of brass flat and of uniform thickness. These disks were revolved on a rotator driven by gear wheels made of "fiberoid," so that the movement should be noiseless. The number of teeth on the wheels and holes in the disks were such that I was enabled to make three determinations corresponding in the number of interruptions of sound to those already obtained with apparatus [A].

Two brass tubes, T and T′, fig. 9, one having an interior diameter of ½ centim., the other an interior diameter of 1 centim., slid accurately and with little friction in two tubes,

Fig. 9.

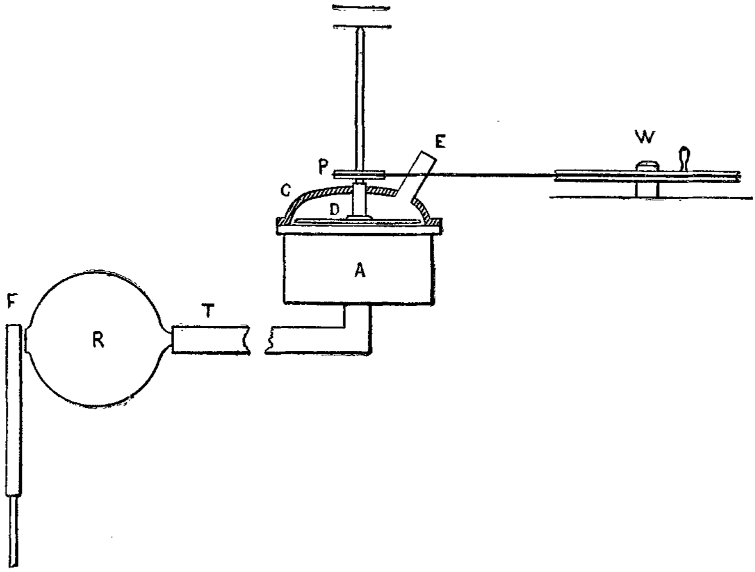


A and B, with flanges on their ends. These flanges were pressed against the surfaces of the disk D by two delicate helical springs fitting over the tubes at S and S′, between the flanges of A and B and the standards P and P′. The tubes T and T′ were as close as possible to the disk while it rotated. The flanges A and B were of such diameter that no sound could issue between them and the surfaces of the rotating disk, because the flanges entirely covered the hole in the disk while this hole traversed the tubes T and T′. A tube of caoutchouc led from the tube T′ to the ear.

Although a film of oil was between the flanges and the disk, and the disks were so accurately made that the greatest departure from uniformity of thickness amounted to only $\frac{1}{25}$ millim., yet the sounds produced by the sliding of the disk between the flanges caused much distraction in the perception of the sound from the resonator, which was adapted to the tube T, of $\frac{1}{2}$ centim. in diameter. The results obtained with this apparatus agreed with those given by apparatus [A].

[D.] I had formed great hopes of having the best apparatus for the determination of the duration of a residual sensation in

Fig. 10.



the one shown in fig. 10. A is the lower drum of a Helmholtz double siren; D, the perforated disk of the siren, which was rotated by the driving-wheel W. The disk D was enclosed in a cover, C, of the form shown in fig. 10, clamped to the drum of the siren. The sound issued from the box thus formed, and was conveyed to the ear by the tube E, to which was attached a tube of caoutchouc. The sound to be experimented on was conveyed from the fork, F, and resonator, R, through a long tube, T, to the drum of the siren. By placing pulleys, P, of various diameters on the axle of the disk of the siren, and by opening one or another of the various circles of holes in the drum A, I had the means of obtaining a considerable range in the number of interruptions of sounds per second.

The results given by this apparatus were the same as those obtained with [A], but the objection to its action is the production of sounds by the apparatus itself, caused by the rotation of the perforated disk. These sounds distracted the attention from the phenomenon of the continuous sensation produced by the interruptions of the sound from the fork, and so masked it that I consider this apparatus the least efficient of those I have described.

These experiments on the blending of interrupted sounds are not pleasant to make. The ear soon becomes fatigued, and the perception of sound is dulled. After an experiment the ear has to be rested during a considerable time before the experiment can be repeated satisfactorily. Thus much time is consumed, and these experiments cannot be made in a few days, but weeks are required to arrive at satisfactory measures; also, considerable time is consumed in gaining mastery over the apparatus. To make these experiments less tedious, fatigue of the ear is to be avoided. This is done by not allowing the interrupted sounds to enter the ear longer than during two or three seconds; then a rest of five to six seconds is taken, while the fork is kept in action and the disk revolved with the same velocity; then another three-second period of sound is given the ear. This is best done by placing the rubber tube in the meatus of the ear and pinching the tube between the fingers. By relieving the pressure more or less we can regulate the intensity of the sound which enters the ear, or we can shut the sound off. The other ear is tightly closed with beeswax softened with a little turpentine.

Within the limits of the intensities of sound used in these experiments, I found no change in the duration of the sensation with change in the intensity of the sound. It seems probable that such connected changes exist. If they do exist, then it would appear, from the smoothness of the curve I have obtained from the experiments, that the relative mean intensities of the sounds used did not vary sufficiently to make apparent any change in the duration of the after-sensation with change in the intensity of the stimulus.

Table of Results of Experiments.—The Empirical Formula which gives the Relation of the Pitch of a Sound to the Duration of its Residual Sensation.

The results of the experiments made with the various forms of apparatus just described are given in the following Table I.

TABLE I.

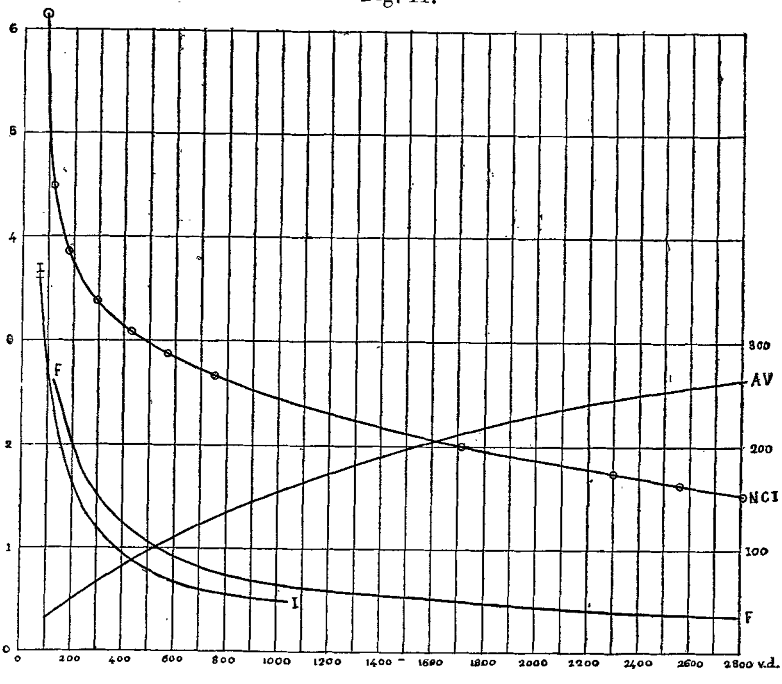
A.	B.	C.	D.	E.	F.	G	H.	I.	K.	L.
UT_1	64	23·1	$\frac{1}{26·9}$	·0361	·0369	+·0008	......	......	......	1·38
UT_2	128	36	43·2	·0231	·0228	−·0003	·0280	·0324	+·0044	1·77
SOL_2..............	192	......	......	......	......	......	·0232	·0237	+·0005	
UT_3	256	62	$\frac{1}{74·4}$	·0134	·0133	−·0001	·0190	·0189	−·0001	2·06
MI_3	320	73	$\frac{1}{87·5}$	·0114	·0112	−·0002	·0160	·0158	−·0002	2·12
SOL_3........	384	88	$\frac{1}{105·6}$	·0094	·0097	+·0003	·0137	·0137	·0000	2·18
UT_4	512	108	$\frac{1}{130}$	·0077	·0078	+·0001	·0110	·0109	−·0001	2·37
MI_4	640	126	$\frac{1}{151·2}$	·0066	·0067	+·0001	·0092	·0092	·0000	2·53
SOL_4'..............	768	143	$\frac{1}{171·6}$	·0058	·0059	+·0001	·0080	·0081	+·0001	2·68
UT_5	1024	170	$\frac{1}{204}$	·0049	·0049	·0000	·0066	·0066	·0000	3·01
MI_5	1280	......	......	......	......	......	·0057	·0057	·0000	
SOL_5..............	1536	.....	......	......	......	......	·0052	·0052	·0000	
UT_6	2048	......	......	......	......	......	·0045	·0044	−·0001	
MI_6	2560	......	......	......	......	......	·0039	·0039	·0000	
SOL_6..............	3072	......	......	......	......	......	·0036	·0036	·0000	

Column A gives the names of the sounds, and B the numbers of their vibrations (v.d.) per second ; C, the number of beats of interrupted sound received by the ear in a second to blend into a continuous uniform sensation. In column D are the durations of the residual sonorous sensations, expressed in vulgar fractions. The numbers in column D are obtained, as already explained, by diminishing the reciprocals of the numbers in column C by $\frac{1}{6}$. Under E are the fractions in column D in decimals. Under F are the durations of the residual sensations, computed by the formula

$$D = \left[\frac{33,000}{N+30} + 18\right] \cdot 0001,$$

in which D equals the duration of the residual sensation given by a sound of N number of vibrations per second. Under G are the differences between the computed and observed values of the residual sensations. These differences are so small, except in the case of SOL_3, that we may adopt the empirical

Fig. 11.



formula as expressing the law which connects the pitch of a sound with the duration of its residual sonorous sensation.

This formula, however, only refers to my own auditory sensation. It is a physiological law; and I imagine that the durations of these residual sonorous sensations will vary more with different observers than do analogous visual sensations. It is therefore to be wished that others will repeat these experiments and obtain determinations which, when combined, will give a law which may be accepted as the expression of the average durations of the residual sonorous sensations.

The results in column E are given graphically in the curve I of fig. 11. The unit on the axis of abscissas is 100 v.d. The unit on the axis of ordinates is ·01 second.

The determination of the duration of the residual sensation of UT_1 [64 v.d.] was very carefully made, and made many times. The experiments gave ·0361 second for the duration of this sound. The formula gives ·0369, which is $\frac{1}{45}$ greater than the observed duration. The greatest difference existing between the observed and computed duration of the remaining sounds of the table is in the case of SOL_3 [384 v.d.], where the computed is $\frac{1}{31\cdot 3}$ greater than the observed duration.

In column L of Table I. are given the number of wave-lengths of the sounds which pass into the ear through a hole in the rotating disks when these interrupted sounds blend into uniform sensations. The average number of wave-lengths which pass a hole is about $2\frac{1}{4}$. As the sound rises in pitch, more wave-lengths pass the hole; thus, while only 1·38 wave-length passes in the case of UT_1, three wave-lengths pass in the case of UT_5.

An examination of fig. 6 shows that sound passes to the ear while a hole in the disk passes over three diameters of the hole in the nipple of the resonator, while the distance between the centres of neighbouring holes in the disk equals six diameters of the hole in the resonator. Hence, to ascertain the number of wave-lengths which enter the ear during the passage of a hole in the disk across the hole in the resonator, we must divide the number of vibrations per second of the sound, given in column B, by twice the corresponding number in column C.

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2. *On the Smallest Consonant Intervals among Simple Tones.*

When two simple tones which differ slightly in pitch are sounded simultaneously, beats are produced, which become more frequent as the difference in pitch increases, and with this increase in the interval between the tones the dissonance becomes harsher, reaching a maximum dissonance when the number of beats are about $\frac{4}{10}$ of the number required to blend; then becoming less dissonant as the interval increases till, at last, the two tones blend into a consonance. These are the phenomena observed from SOL_1 [96 v.d.] to the highest tones used in music.

Having the law which gives the number of beats (produced by the interrupted sounds of tones of various pitch), which blend, one might naturally infer that the smallest consonant interval could be computed by that law. Given the pitch of a tone, we compute by the law the number of interruptional beats of this tone which blend, and adding this number to the frequency of the given tone we should apparently have the pitch of the upper tone which makes with the lower the smallest consonant interval *. This, however, is not so. Take, for example, UT_3 [256 v.d.]. The number of interruptional beats of this sound which blend is 62, and $256+62=318$, which, according to the law, should make a consonant interval with UT_3. But experiment shows that a tone of $256+58=314$ v.d. forms the nearest consonant interval with UT_3.

To render less tedious the comprehension of the results of many experiments on the smallest consonant intervals among simple tones, I shall at once give a table (Table II.) of the results of these experiments, and then give the account of the experiments that furnished the data of the table.

In column A are given the lowest tones of the consonant intervals which were experimented on. In column B are the number of vibrations to be added to the tones of column A to form the higher note of the smallest consonant interval, as deduced from the experiments on the duration of the residual sensations of interrupted sounds. In column C the numbers of vibrations by which the tones in column A have really to be increased to form the higher notes of the smallest consonant intervals. In column D are the numbers of vibrations to be added to those of column A to form the smallest consonant intervals as computed by the formula

$$N : N + \frac{1}{\left(\frac{42500}{N+23} + 23\right) \cdot 0001},$$

* Rigorously, we should take in the computation the number of beats which blend corresponding to a sound of a pitch which is the mean of the pitch of the lower and upper sounds of the interval.

TABLE II.

Number of Experiment.	A. Tones.	B. Number of additional v.d. to form the smallest consonant interval as deduced from the experiments on the residual sensations of interrupted sounds.	C. Number of additional v.d. really required to form the smallest consonant interval.	D. Number of additional v.d. required to form the smallest consonant interval according to the formula $N:N+\frac{1}{\left(\frac{42500}{N+23}+23\right)\cdot 0001}$.	E. Difference between D and C.	F. Smallest consonant intervals in semitones of equal-tempered scale.
[1]	SOL_{-1} [48]	19·0	No consonant interval.	16·0	......	None.
[2]	UT_1 [64]	23·1	No consonant interval.	19·5	......	None.
[3]	SOL_1 [96]	30·7	41	26·3	−14·7	$6\frac{15}{100}$ semitones.
[4]	UT_2 [128]	36·0	38	32·8	− 5·2	$4\frac{1}{2}$ "
[5]	SOL_2 [192]	49·7	48+	47·4	− 0·6	$3\frac{86}{100}$ "
[6]	UT_3 [256]	62·0	58	57·0	− 1·0	$3\frac{53}{100}$ "
[7]	MI_3 −[316]	72·0	68	67·4	− 0·6	$3\frac{34}{100}$ "
[8]	LA_3 −[432]	91·2	85·4	85·9	+ 0·5	$3\frac{12}{100}$ "
[9]	RE_4 −[575]	113·3	107·4	106·3	− 1·1	$2\frac{95}{100}$ "
[10]	SOL_4−[766·2]	137·8	129·8	130·1	+ 0·3	$2\frac{70}{100}$ "
[11]	LA_5 [1706·6]	219·3	210·4	212·0	+ 1·6	2 "
[12]	RE_6 [2304]	251·4	245	242·3	− 2·7	$1\frac{76}{100}$ "
[13]	MI_6 [2560]	262·8	256	253·5	− 2·5	$1\frac{64}{100}$ "
[14]	Fork [2806]	272·0	266	263·0	− 3·0	$1\frac{54}{100}$ "

in which N equals the number of vibrations of the lower tone of the interval, and

$$N+\frac{1}{\left(\frac{42500}{N+23}+23\right)\cdot 0001}$$

=the number of vibrations of the higher tone of the interval. In column E are given the differences between the computed values [D] and the observed values [C]. The formula gives quite closely the true values from SOL_2 [192 v.d.] to MI_6 [2560 v.d.]. In column F are given the smallest consonant intervals as determined experimentally from SOL_1 [96 v.d.] to the tone of 2806 v.d., expressed in semitones of the equal-tempered scale.

In fig. 11 these intervals, computed from the numbers in column C, are expressed graphically by the curve N.C.I. The units of ordinates (on the left of figure) are semitones, and the units of abscissas are 100 vibrations. This curve shows in a striking manner the contraction of the smallest consonant interval as we ascend the musical scale; while SOL_1 [96 v.d.] requires a sound separated from it by $6\frac{15}{100}$ semitones to give the smallest consonant interval, SOL_6 of 3072 v.d. forms a consonant interval with a higher sound separated from it by only $1\frac{1}{2}$ semitones.

The curve AV, of fig. 11, which has the same units of abscissas as curve N.C.I., and the units of whose ordinates, in number of vibrations, are on the right of the diagram, shows graphically the number of vibrations to be added to the tones, on the axis of abscissas, to obtain the smallest consonant intervals.

The experiments which form the basis of the statements given in Table II., I shall now describe.

Having but few forks below UT_3 in pitch, and those not numerous enough to determine with accuracy the consonant intervals, I requested my friend Dr. Rudolph Kœnig, of Paris, to determine for me the smallest consonant intervals among sounds below UT_3 in pitch. The numbers in the table referring to tones SOL_{-1}, UT_1, SOL_1, UT_2, and SOL_2, experiments nos. 1, 2, 3, 4, and 5 of Table II., were furnished by Dr. Kœnig's experiments. The account of these experiments, which Dr. Kœnig* so obligingly made for me, I give in his own words:—

Dr. Kœnig's Experiments.

"Paris, le 21 Mars 1893.

"... Je veux maintenant répondre à vos questions concernant la consonance des intervalles formés de notes graves et pas trop fortes, et faire d'abord cette remarque générale, que la perceptibilité des roulements et raucités produits par des battements, qui dépend avant tout de l'intensité relative des deux notes primaires, paraît être presque entièrement indépendant de leur intensité absolue, quand elles sont graves, tandis que le rôle de l'intensité absolue augmente avec leur hauteur. En effet, les résultats des observations faites avec des diapasons relativement faibles, et sans résonateurs, sont pour les octaves graves presque absolument identiques avec les résultats donnés par les gros diapasons et résonateurs du grand tonomètre, et publiés dans le tableau, page 113, des *Quelques Expériences d'Acoustique,* tandis que pour les intervalles avec les notes fondamentales UT_2 et SOL_2, les différences des résultats devient déjà sensible. Je pourrais aussi résumer ces faits en disant que l'influence de l'intensité absolue des deux sons primaires sur la perceptibilité de battements, qui est absolument nulle pour les battements lents, augment pour les roulements et raucités avec les nombres des battements qui les produit, car, pour example, les 8 battements de $UT_1 : RE_1$ sont aussi distinctement entendus avec des notes UT_1 et RE_1 très faibles, qu'avec des notes les plus fortes, tandis que le roulement des 32 battements de $UT_1 : SOL_1$, se fait beaucoup plus sentir quand les deux notes sont fortes, que quand elles n'ont qu'une faible intensité.

"Voici maintenant la liste de mes observations :—

"[1] *Intervalles avec la note fondamentale* $SOL_{-1}=48$ v.d.

"Les diapasons employés, du modèle de l'interrupteur universel, Cat. No. 253, ont des branches d'environ 9 millim. d'épaisseur sur 15 millim. de largeur.

"On ne trouve dans toute l'octave pas un seul intervalle sans battements fortement perceptibles, soit séparément soit comme roulement. Le roulement paraît le plus faible vers la sixte, $SOL_{-1} : MI_1$.

"[2] *Intervalles avec la note fondamentale* $UT_1=64$ v.d.

"Les diapasons employés, du modèle, Cat. No. 234, ont des branches d'environ 10 millim. sur 17 millim.

"Tout se passe dans l'octave entière comme avec les gros diapasons avec résonateurs, seulement le roulement confus près de la quinte est faible et disparaît presque devant les battements secondaires, mais il reparaît au-dessus de SOL_1+ 4 v.d. et devient plus fort en passant au roulement simple des battements supérieurs. A la quinte exacte, UT_1 : SOL_1, où les battements secondaires manquent, on entend très bien le roulement des 32 battements, mais plus faible qu'avec des gros diapasons et résonateurs.

"Les mêmes expériences répétées avec des diapasons à branches de 19 millim. sur 15 millim. donnent le même résultat.

"En somme, ici aussi il n'y a pas d'intervalle consonant dans toute l'octave.

"J'ai encore particulièrement examiné l'intervalle $UT_1 : MI_1$, mais on y entend le roulement des 16 battements même encore quand les sons primaires sont devenus déjà si faibles qu'on ne les perçoit presque plus.

"[3] *Intervalles avec la note fondamentale* $SOL_1 = 96$ v.d.

"Diapasons employés à branches de 10 millim. sur 17 millim.

"$SOL_1 : LA_1$, 10·6 battements séparément entendus.
" $: SI_1$, 24 " roulement simple fort.
" : 124 v.d., 28 " " plus faible.
" $: UT_2$, 32 " " très faible.
" : 134 v.d., ... faible raucité.

"SOL_1 : 136 }
" : 138 } Peut-être regardé comme consonant, mais à

"SOL_1 : 140 v.d. les battements secondaires deviennent déjà sensibles et augmentent vite en intensité. Quand ils disparaissent à la quinte exacte,

"$SOL_1 : RE_2$, celle-ci fait entendre une raucité à peine perceptible. Au-dessus de RE_2 les battements secondaires reparaissent pour disparaître vers 148 v.d.

"SOL_1 : 150 v.d. Il y a encore un peu de raucité mais on pouvait à la rigueur regarder les intervalles d'ici à

"SOL_1 : 166 v.d. comme consonants. Au-dessus de 166 v.d. le roulement des battements supérieurs commence, et augmente en intensité, et à partir de 180 v.d. les battements supérieurs sont séparément perceptibles.

"En résumé, il y a entre la quarte et la quinte une petite étendue, et au-dessous et au-dessus de la sixte une plus grande, où se trouvent des intervalles consonants.

"[4] *Intervalles avec la note fondamentale* $UT_2 = 128$ v.d.

"Diapasons employés pour UT_2 jusqu'à UT_3, avec branches de 10 millim. sur 17 millim., pour la note fondamentale UT_2 avec branches de $7\frac{1}{2}$ millim. sur 14 millim.

"Tout se passe à peu près comme avec les gros diapasons et résonateurs (Cat. No. 197), seulement le roulement de $UT_2 . MI_2$ n'est plus qu'un simple raucité, qui diminue vite pour disparaître à 166 v.d.

" UT_2 : FA_2 est consonant, et la consonance persiste jusqu'à 188 v.d. où les battements secondaires commencent à la troubler jusqu'à la quinte, UT_2 : SOL_2, qui est consonant. Quand ils ont troublé la consonance des intervalles au-dessus de la quinte jusque vers UT_2 : 197 v.d. les intervalles sont de nouveau consonants et le restent jusque vers UT_2 : 222 v.d. où alors le roulement des battements supérieurs commence.

" En résumé, les intervalles d'un peu au-dessous de la quarte jusqu'à près de la quinte, la quinte, et les intervalles d'un peu au-dessus de la quinte jusque au-dessus de la sixte sont consonants.

" Avec des diapasons du modèle de Cat. No. 38 *a*, aux branches de 15 millim. sur 20 millim., montés sur caisses, le roulement des 32 battements de UT_2 : MI_2 est fort ; UT_2 : SOL_2 est consonant, mais comme MI_2 : SOL_2 donne le même nombre, 32, de battements que UT_2 : MI_2, en faisant sonner ensemble UT_2, MI_2, SOL_2, les 32 battements de UT_2 : MI_2 et de MI_2 : SOL_2 se renforcent et produisent un roulement formidable.

" [5] *Intervalle avec la note fondamentale* SOL_2=192 v.d.

" Diapasons employés avec branches de 10 millim. sur 17 millim.

" SOL_2 : LA_2. Roulement simple assez fort.
,, : 244 v.d. Raucité prononcé.
,, : SI_2. Raucité faible.

" La consonance commence encore avant SOL_2 : UT_3.

" Il résulté de l'ensemble de ces observations, que pour les intervalles des notes très graves les faits ne s'accordent pas du tout avec votre loi, mais aussi que pour les intervalles des notes de plus en plus aigues, l'accord entre les faits et la loi devient toujour meilleur et finit par être presque parfait quand la note fondamentale atteint SOL_2."

I experimented on the hearing of twelve persons with all the forks at my command among which I could obtain consonant intervals. It is true that the number of intervals thus furnished, by 80 forks, did not amount to many. Many such intervals can only be obtained in the laboratory of Dr. Kœnig, where is his " grand tonomètre " giving the frequency of all sounds from 16 to 21845 complete vibrations per second. However, I secured, between UT_3 and SOL_6, enough intervals among the forks to establish the law and the facts given in Table II.

All the persons experimented on, except myself and one other observer, were accomplished musicians, several of them violinists of more than exceptional ability. Three were graduates of the Conservatory of Music of Leipzig.

These experiments were all made in the same manner, viz. by taking the fork giving the lower tone and sounding it successively with others which gave more and more beats per second, till these beats blended into a continuous smooth sensation, forming the smallest consonant interval. As musicians rather avoid than dwell on dissonant intervals, I educated each one in the special subject of the roughness of the sensation given by beats by making the beats more and more frequent till near consonance, then giving an interval which is admitted by every one to be consonant. In this way their hearing was trained in what I wished them to discern, viz. that separation in the pitch of two forks which just gives a consonant interval.

The variation among the decisions of these different observers never equalled two vibrations; generally they agreed exactly. The agreement among observers in the judgment of a consonant interval is remarkable. I give the mean of these experiments. The number of the paragraphs refer to the number of the experiment given in Table II.:—

[6] $UT_3 : MI_3 = 256 : 320$ gave a consonant interval. I narrowed the interval by lowering the pitch of MI_3 from 320 to 315, 314, 313·5, and 313.

UT_3 : 315 consonant.
„ : 314 just consonant.
„ : 313·5 slightly rough.
„ : 313 decidedly rough.

These experiments show that $256 : 256 + 58$ = smallest consonant interval.

[7] $MI_3 : SOL_3 = 320 : 384$ is decidedly rough. Separated the interval by lowering the MI_3 fork from 320 to 316; then

$MI_3 : SOL_3 = 316 : 384 = 316 : 316 + 68$ = smallest consonant interval.

[8] $LA_3 : UT_4 = 439 : 517·4$ decidedly harsh. Separated interval by lowering LA_3 of 439 v.d. to 432 v.d.; then

$LA_3 : UT_4 = 432 : 517·4 = 432 : 432 + 85·4$ = smallest consonant interval.

[9] $RE_4 : FA_4 = 576 : 682·65$ slightly rough. Increased the interval by lowering RE_4 of 576 v.d. to 575 v.d.; then

$RE_4 : FA_4 = 575 : 682·65 = 575 : 575 + 107·65$ = smallest consonant interval.

[10] SOL_4 : fork of 896 v.d. = 768 : 896 slightly rough. Increased the interval by lowering SOL_4 of 768 v.d. to 766·2; then

SOL_4 : 896=766·2 : 896=766·2 : 766·2+129·8=smallest consonant interval.

[11] LA_5 : SI_5=1706·6 : 1920 consonant. Narrowed the interval by lowering SI_5 of 1920 to 1917 ; then

LA_5 : SI_5=1706·6 : 1917=1706·6 : 1706·6+210·4=smallest consonant interval.

[12] RE_6 : MI_6=2304 : 2560 consonant. Narrowed the interval by lowering MI_6 of 2560 v.d. to 2549 ; then

RE_6 : MI_6=2304 : 2549=2304 : 2304+245 = smallest consonant interval.

[13] MI_6 : Fork No. 11_6=2560 : 2816=2560 : 2560+256 is just perceptibly rough.

[14] Fork 11 : SOL_6=2816 : 3072 slightly dissonant. Increased interval by lowering Fork 11 of 2816 v.d. to 2806; then

Fork 11 : SOL_6=2806 : 3072=2806 : 2806+266=smallest consonant interval.

In the experiments just described the intervals of the tones that gave consonance were made by simple tones of small intensity and without the slightest trace of the upper partial tones of the forks, and the two forks were vibrated so that they gave, as near as I could judge, the same intensity of sound. The results given only refer to intervals so formed. To obtain them the forks were gently vibrated by strokes of rubber hammers that varied in hardness with the pitch of the forks. The lower the pitch of the fork, the softer should be the hammer. A hammer of hard rubber striking low-pitched forks will develop the upper partial tones of the forks, and so vitiate the experiments that a really consonant interval might be judged as dissonant.

The results of all the experiments may be summed up as follows :—From SOL_2 of 192 v.d. to MI_6 of 2560 v.d. the smallest consonant intervals are closely given by the formula

$$N : N+\frac{1}{\left(\frac{42500}{N+23}+23\right)\cdot 0001}.$$

For sounds below SOL_2 the interval as computed by the formula is too small to agree with the true interval. For sounds above MI_6 (2560 v.d.) the intervals computed by the formula, like those below SOL_2, are too small. That the experimental determination of the smallest consonant intervals throughout four octaves, upward from SOL_2, or throughout the tones given by the violin, should agree so closely with the formula indicates the existence of a law connecting the magnitude of the smallest consonant interval with its position in the musical scale.

Dr. Kœnig has shown that a consonant interval does not exist among simple sounds of pitch below SOL_1 [96 v.d.], yet I have found that the sound of UT_1 [64 v.d.], when interrupted by a rotating perforated disk, blends perfectly, to my ear, when these interruptions occur 23·1 times in a second. It may appear strange that although 23·1 interruptions per second of the sound UT_1 blend, yet a consonant interval does not exist throughout the octave of UT_1 till the interval of $UT_1 : UT_2$ is reached; but the beats produced by the rotating perforated disks are produced by the interruptions of one tone, whereas when two simple tones are conjoined two sets of beats are produced, inferior and superior: thus, when UT_1 forms an interval with $UT_1 + 23$ v.d., the inferior beats are 23 per second and the superior beats are 41 per second, and the interaction of these inferior and superior beats produces secondary beats, which give to the interval a confused rumbling sound *. Of this interval $UT_1 : UT_1 + 23{\cdot}1$, Dr. Kœnig wrote to me as follows:—"Your 23·1 interruptions of UT_1 correspond, in number, to the inferior beats of the interval of the simple tones $UT_1 : UT_1 + 23{\cdot}1$, but it is just at this magnitude of interval that the superior beats begin to assert themselves, to produce with what remains perceptible of the inferior beats the confused rumbling, which evidently would be but a slight roughness (disappearing entirely at a further increase of the interval), if the superior beats, whose intensity from this point increases with the interval, did not exist."

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3. *The Durations of the Residual Sonorous Sensations as deduced from the Smallest Consonant Intervals among Simple Tones.*

If we assume that two simple tones form the smallest consonant interval because the beats produced by these conjoined sounds have blended into a smooth continuous sensation, then we may deduce the durations of the residual sonorous sensations from the observed smallest consonant intervals in the following manner:—The reciprocals of the numbers in column C of Table II. are taken as expressing the durations of the sonorous sensations given by tones whose numbers of vibrations are the mean of those of the lower and higher tones of the corresponding consonant intervals, for,

* See *Quelques Expériences d'Acoustique,* par Rudolph Kœnig (Paris, 1882), pp. 89, 107, 113.

when two sounds of different pitch blend, there is no reason why the duration of their residual sensation, as given by the reciprocal in column C, should refer to the lower sound more than to the higher. Therefore we have taken these reciprocals from column C as expressing the durations of sounds having the mean pitch of the two associated sounds forming the interval. The residual sensations thus found were projected in a curve, drawn to a large scale. From this curve were obtained the durations of the residual sonorous sensations of the tones of the musical scale. These durations are given in column H of Table I.

In column I of Table I. are given these durations as computed by the formula

$$D=\left(\frac{48{,}000}{N+30}+21\right)\cdot 0001.$$

In column K are given the differences between the durations computed by the formula and the durations given in column H. The differences show that the formula expresses closely the durations of the residual sensations thus deduced from the determinations of the smallest consonant intervals, except in the case of UT_2; for which tone the computed number of vibrations to be added to it to form the higher tone of its smallest consonant interval, as shown in Table II., is 5·2 vibrations less than the number really required.

In fig. 11 these durations, as determined from the smallest consonant intervals, are plotted in the curve F, so that the comparison of the durations of the residual sonorous sensations thus determined may be readily compared with those given (by the curve I) of the residual sensations as determined by the blending of sounds interrupted by rotating perforated disks.

The ordinates of the curves I and F of fig. 11 are obtained in fractions of a second by changing the numbers 1, 2, and 3 on the left of fig. 11 into ·01, ·02, and ·03.

These two curves of fig. 11 present the same general character of a rapid upward flexure at the points corresponding to the pitch of about 600 v.d.

The durations of the sound-sensations as deduced from the smallest consonant intervals of the forks average about $\frac{1}{3}$ greater than those given by the beats of interrupted sounds. It may be supposed that the durations of the sonorous sensations deduced from the smallest consonant intervals of simple tones are greater than those determined by sounds interrupted by the perforated disks, because in the resultant actions of the vibrations of the tones, forming the smallest consonant intervals, the periods of silence, or the periods of great diminution of sound, are a fraction of the periods of sound, or of the periods of maximum intensity of sound. To test this opinion I combined the sinusoids corresponding to the two tones of various smallest consonant intervals. On taking as the residual duration of the sound, not the time from maximum to maximum of vibration (as in the deduction of the durations from the smallest consonant intervals), but the interval of time during which much diminished intensity of sound exists, as shown in the combined curves, I found that the durations of the sonorous sensations were thus reduced, on the average, about $\frac{1}{2}$, whereas the reduction in time should be only $\frac{1}{3}$ to make these durations agree with those determined by the rotating perforated disks. The explanation suggested is therefore not tenable.

For the period of much diminished intensity of sound I took that length (in time) of the resultant curve which is bounded, at each end, by an amplitude of vibration $\frac{1}{2}$ of the maxima amplitudes of the curve. We here are in doubt as to the relative intensities of the sensations given by two sound-vibrations whose amplitudes are 2 : 1, and whose energies are as 4 : 1. We at once face an obstacle which, from our want of knowledge, is insurmountable: for, assuming that either the law of Weber, or the formula of Fechner deduced from it, correctly gives the relations existing between the intensity of a stimulus and its corresponding sensation, we cannot apply either of these laws, because we do not know the *absolute energies* of the sound-vibrations whose sensations are to be compared. Thus, if we adopt the law of Weber, with the least perceptible difference in the sensation of two sounds equal to $\frac{1}{3}$ of their energy, as given by the experiments of Volkmann *, we find that if 1 and 4

* In the investigations on this subject of which I have knowledge, the experimenters have used either noises, or sounds of complex composition mingled with noise, and the ways in which they have determined the relative energies of sounds, or noise-producing vibrations, are open to criticism. I do not know of similar experiments made with simple sounds or tones. I would suggest that the problem of determining the difference in the energies of two simple sounds to give a perceptible difference in the sensations they cause may be solved as follows:—A fork or rod is vibrated with a constant amplitude, and this amplitude is accurately measured with a micrometer-microscope. A second fork, or rod, placed alongside of the first fork or rod, has a much smaller amplitude of vibration, which can be varied, and is also measured with a microscope. The second fork differs from the first slightly in pitch, so that, say, three beats per second are given. The amplitude of the second, or of the first fork, is varied till the perception of beats just vanishes, or just appears, while the ear is kept at a fixed distance from the forks.

are the absolute energies of the sound-vibrations, we get for the ratio of their corresponding intensities of sensations 1 : 2·6; but if the absolute energies of the sounds are 10 and 40 (and their ratio is also 1 : 4), we get for their relative sensations 1 : 1·48. Or, what is the same, on the curve expressing the law of Weber, or of Fechner, the ratio of the sensations of two sounds as given by their corresponding ordinates depends on the number of units in the abscissas forming the ratio of the energies of these sounds.

Professors Cattell and Fullerton, from extended experiments "On the Perception of Small Differences" *, very carefully made and skilfully reduced, have formed the opinion that neither the law of Weber nor Fechner's formula is correct, and *à priori* considerations lead them to the opinion that it is probable that the sensation is directly as the stimulus. If the sensation increases directly as the stimulus, then we can obtain the relative sensations of two sounds whose relative energies are known. Adopting this relation, we have 1 : 4 as the ratio of the maximum sensation in the periods taken

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If we take for the relative intensities of the sound-giving vibrations the ratio of the squares of the amplitudes of the forks, the least perceptible difference in sensation corresponding to the differences in the energies of the sounds may be computed. As example, suppose the second fork has $\frac{1}{20}$ of the amplitude of vibration of the first. Then the energy of the maximum sounds of the beats will be $\overline{20+1}^2=441$, and the energy of the minimum sound of the beating will be $\overline{20-1}^2=361$, and $\frac{441}{361}$ = the ratio of the stimuli giving the least perceptible difference in sensation. Sound-vibrations of different amplitudes and of different pitch will have to be experimented with, and the fork giving the greater amplitude of vibration should, in successive experiments, be lower in pitch, and then higher in pitch, than the fork giving the lesser amplitude of vibration, for reasons set forth in my research (1) "On the Obliteration of the Sensation of one Sound by the simultaneous action on the ear of another more intense and lower sound; (2) On the Discovery of the Fact that a Sound even when intense cannot obliterate the sensation of another Sound lower than it in pitch" (Phil. Mag. Dec. 1876; 'Nature,' Aug. 10, 1876). Such a research will be difficult and tedious, and will require many precautions in arranging the experiments.

Any one may readily observe the phenomena described by sounding a fork with long amplitude of vibration, and, gradually bringing up to the ear a second fork with a small amplitude of vibration, giving with the first three beats per second. As the latter fork gradually approaches the ear the beats become stronger, reaching a maximum of intensity, and then diminishing till, at a certain distance of the fork from the ear, they vanish in the more intense sensation of the more intense sound, to reappear when the faintly vibrating fork has been brought closer to the ear.

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for those of much diminished sound to the maximum sensation in the periods of much increased sound, as given by the measurements of the amplitudes of the resultant curves of the smallest consonant intervals.

In explanation of the facts and laws given in this paper I have no hypothesis to offer. It appears to me that the present condition of our knowledge of audition demands that we should ascertain more facts relating to it before we frame hypotheses on the mechanism and action of the apparatus of hearing.

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XXIV. *A Study of the Polarization upon a Thin Metal Partition in a Voltameter.*—Part II. *By* JOHN DANIEL.*

IN this paper two questions will be discussed: first, the passage of ions through a gold-leaf partition in a voltameter; second, the minimum current-strength at which the ions are deposited visibly upon the partition for various electrolytes. This will be called the "critical current." This paper is a continuation of the work done last spring in Berlin in the quantitative measurement of the polarization upon metal partitions ranging in thickness from ·0001 millim. to ·02 millim. for various current-strengths in a 30-per-cent. H_2SO_4 voltameter. In those experiments there was no development of gas nor polarization on a gold-leaf partition (·0001 millim. thick) for the highest current used, which was four tenths to five tenths ampere.

The present apparatus consists *essentially* of a glass voltameter vessel with platinum electrodes separated by the metal partition under investigation, so that there is no path for the current except through this partition; an accurate current-measurer, and a strong, steady battery. The voltameter consists of an outer glass jar 8 centim. high, 8 centim. wide, and 8 centim. long; and a glass jar 8 centim. high, 5 centim. wide, and 5 centim. long, placed inside the first jar. A platinum kathode suspended by a platinum wire is placed inside the inner jar, and a similar electrode serves as anode in the larger jar, though a copper anode was sometimes used when the electrolyte was $CuSO_4$. A hole 2 centim. in diameter was bored in one side of the smaller jar. Glass

* Communicated by Prof. O. J. Lodge, D.Sc., F.R.S.