Philosophical Magazine Series 5

ISSN: 1941-5982 (Print) 1941-5990 (Online) Journal homepage: http://www.tandfonline.com/loi/tphm16

LIII. Theoretical essay on the distribution of energy in the spectra of solids

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To cite this article: M. Wladimir Michelson (1888) LIII. Theoretical essay on the distribution of energy in the spectra of solids , Philosophical Magazine Series 5, 25:156, 425-435, DOI: 10.1080/14786448808628207

To link to this article: http://dx.doi.org/10.1080/14786448808628207

Published online: 29 Apr 2009.

may be conceived of quite generally as the ratio of cause to effect.

That the objections to the employment of the term magnetic resistance have been founded on the assumed requirement of identity instead of analogy between the affections of different subjects, and cannot be sustained.

And that the extension of the employment of the term Resistance in the above manner leads to some remarkable analogies which go far to justify independently the employment of the term Magnetic Resistance. And it has been pointed out that the ordinary mode of statement involves inconsistent and impossible ideas as to the relation of cause and effect in the phenomena, whereas the application of the term Magnetic Resistance compels us to precisely ascertain these, and put them in their right places.

Note.—An objection has been recently raised by M. Hospitalier* to my definition of Magnetic Resistance, which comes to this, that in defining the effect I take the induction through unit surface instead of the total induction. This objection is again based on the supposed necessity for conforming identically to Ohm's law in Electricity. The foregoing remarks will have made it clear that I consider this unnecessary, the question being, What is the most convenient measure of the effect? Since the magnetic potential attainable is practically limited, though it increases with the dimension, and the induction through unit of area practically attainable is also limited (by saturation), the ratio of these two quantities lies always within certain limits, though it increases with the dimensions. In fact the magnetic resistance thus estimated is of linear dimension, and may be regarded as of linear scale in plans. If, therefore, this quantity be selected to be tabulated, the discussion of questions of design from a practical point of view is greatly facilitated, and the relations of the quantities involved are more easily followed and more simply expressed.

LIII. *Theoretical Essay on the Distribution of Energy in the Spectra of Solids.* *By* M. Wladimir Michelson†.

THE recent remarkable publications of Prof. Langley upon the invisible spectrum‡, and especially the promise of a

* 'Electrician,' xx. p. 164, note.

† Translated from the *Journal de Physique*, t. vi. Oct. 1887. Abstract of a paper recently published in the *Journal de la Société Physico-chimique russe*, vol. xix. No. 4, p. 79 (1887). Communicated by the Author.

‡ S. P. Langley, "Sur les Spectres invisibles" (*Ann. de Chim. et de Phys.* Dec. 1886, pp. 433–506). See also Amer. Journ. Sci. vol. xxxii. Aug. 1886; and Phil. Mag. vol. xxi. pp. 394–409, and vol. xxii. pp. 149–173.

special memoir upon the radiations of solids at different temperatures, induce me to publish at once, at least in abstract, some theoretical considerations upon this subject. I hope to give a more complete discussion of the question when the new data of Prof. Langley's spectro-bolometric researches shall enable me to confront my theory with experiment in a more detailed manner.

1. *Hypothesis and General Law.*—The absolute continuity of the spectra emitted by solids can only be explained by the complete irregularity of the vibrations of their atoms. The discussion of the distribution of radiant energy amongst the simple vibrations of different period is, then, to be undertaken by the calculus of probabilities.

Let us consider a homogeneous isotropic solid of which all the atoms are in identical circumstances, so that, for example, they are not grouped into separate molecules. Each atom has a definite position of equilibrium towards which it is continually driven back by the surrounding atoms, and about which it describes infinitely small oscillations. I express this fact by supposing that each atom moves freely in the interior of a spherical elastic shell of infinitely small radius ρ, which has the position of equilibrium as its centre. The atom rebounds from the interior surface of this sphere according to the law of impact for perfectly elastic bodies, preserves its absolute velocity during several free paths, and then changes its velocity in consequence of the unsymmetrical action of the surrounding atoms.

Let us endeavour to find upon this hypothesis what will be the most probable trajectories of the atom in the interior of the sphere of displacement. Let us admit that, for the initial position of the atom, all possible distances from the position of equilibrium are equally probable; then the probability that this distance shall lie between the limits r and $r+dr$ will be expressed by

$$\frac{dr}{\rho}. \quad \text{. . . . .} \quad (1)$$

Let us take the radius O N (fig. 1) of the sphere passing through the initial position, M, of the atom for polar axis. Let us denote the angle N M P, which the direction of motion of the atom makes with this axis, by ϕ. Let us admit, as in the theory of gases, that all directions are equally probable. Then the probability that ϕ will lie between the limits ϕ and $\phi+d\phi$ will be

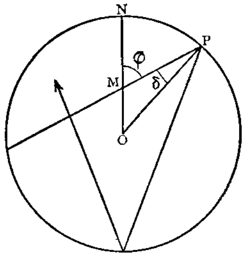
$$\tfrac{1}{2}\sin\phi\, d\phi.$$

Let us call the angle of incidence MPO, δ. This angle is connected with ϕ by the relationship

$$r\sin\phi=\rho\sin\delta.$$

Since the same value δ corresponds to the values ϕ and $\pi-\phi$ of the angle ϕ, the probability that an atom whose initial dis-

Fig. 1.



tance from the centre of the sphere lies between r and $r+dr$ shall strike the spherical surface at an angle lying between δ and $\delta+d\delta$, is given by the expression

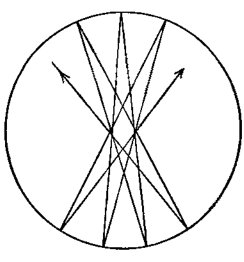
$$\sin\phi\, d\phi = \frac{\rho^2}{r^2}\,\frac{\sin\delta\cos\delta\, d\delta}{\sqrt{1-\frac{\rho^2}{r^2}\sin^2\delta}}. \quad . \quad . \quad . \quad . \qquad (2)$$

We obtain the same probability for any atom whatever by multiplying this expression by $\frac{dr}{\rho}$, and extending the total to all possible values of r for a given δ; that is to say, by integrating with reference to r between the limits $\rho\sin\delta$ and ρ,

$$w=\int_{\rho\sin\delta}^{\rho}\frac{\rho}{r^2}\,\frac{\sin\delta\cos\delta\, d\delta}{\sqrt{1-\frac{\rho^2}{r^2}\sin^2\delta}}\,dr=\left(\frac{\pi}{2}-\delta\right)\cos\delta\, d\delta. \quad . \qquad (3)$$

It is evident that this probability has a maximum value when $\delta=0$; that is to say, that motions along diameters of the spheres of displacement are the most probable. It will happen most frequently that, between two neighbouring disturbances, each atom will describe trajectories differing but little from the stellar form (fig. 2).

Fig. 2.



If we suppose that each periodic change in the motion of the atom produces a wave of the same period in the surrounding æther, and if we consider that the waves produced by the radial motion indicated above will not only be the most frequent, but also the most intense, we must admit, as a first approximation, that it is only these motions which determine the composition of the radiations emitted by our solid.

But the period of vibration τ corresponding to this radial motion is evidently connected with the instantaneous velocity v of the atom by the relationship

$$\tau=\frac{4\rho}{v}. \quad \ldots \ldots \ldots \quad (4)$$

This relationship may be employed for the transformation of the known formula of Maxwell, which expresses the most probable distribution of energy amongst a large number of any material points whatever continually exchanging their velocities by means of forces having a potential*. If we denote by N the total number of atoms, by m the mass of each of them, by e the base of Napierian logarithms, by $k=\frac{3}{2m\overline{v^2}}$ a constant inversely proportional to the mean *vis viva* $\frac{m\overline{v^2}}{2}$ of an atom, Maxwell's law gives the number of atoms whose velocities lie between the limits v and $v+dv$; this number is

$$v_v=\frac{4N}{\sqrt{\pi}}(km)^{\frac{3}{2}}e^{-kmv^2}v^2dv. \quad \ldots \quad (5)$$

Replacing v by $\frac{4\rho}{\tau}$, according to (4), we obtain

$$v_\tau=\frac{256N}{\sqrt{\pi}}\rho^3(km)^{\frac{3}{2}}e^{-\frac{16km\rho^2}{\tau^2}}\tau^{-4}d\tau. \quad \ldots \quad (6)$$

This formula gives the number of atoms of which the principal periods of vibration lie between the limits τ and $\tau+d\tau$. It is the probable law of distribution of the periods of vibration between the atoms of our solid.

In consequence of the law of superposition of vibrations, we may admit that the intensity of a simple radiation of period τ ought to be :—

(1) Proportional to the number v_τ of atoms of the source of radiation vibrating in the same period.

* See upon this subject L. Boltzmann, "Ueber die Beziehung zwischen dem zweiten Hauptsatze der mechanischen Wärmetheorie und der Wahrscheinlichkeitsrechnung &c." (*Sitzungsber. der Wiener Akad. der Wiss.* Bd. lxxvi. pp. 373–435, Oct. 1887); and also Watson, 'A Treatise on the Kinetic Theory of Gases' (Oxford, 1876), p. 12, &c.

(2) Proportional to a function of the energy of these same atoms. In consequence of the relationship (4) and of the high value of $\frac{1}{\tau}$ for all the radiations which we shall have to consider, this function may be reduced to a power of $\frac{1}{\tau^2}$.

(3) Finally, in the direct ratio of an unknown function of the absolute temperature of the body. This function constitutes a factor which ought to represent the mean reinforcement or weakening produced in each primary wave by the whole of the resultant vibrations* and by absorption in the radiating body itself. We will denote this function by $f(\theta)$.

Thus, then, denoting positive constants by A and p, we put the intensity of the simple ætherial undulation of the period τ,

$$I_\tau = A v_\tau \left(\frac{1}{\tau^2}\right)^p f(\theta). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Considering, as usual, the absolute temperature θ as proportional to the mean *vis viva* of an atom, we may replace the constant k in formula (6) by $\frac{M}{\theta}$, where M is independent of θ. Let us then introduce this expression (6) in equation (7). Then putting, for the sake of brevity,

$$A \frac{256N}{\sqrt{\pi}} \rho^3 (Mm)^{\frac{3}{2}} = B, \quad 16\rho^2 Mm = c: \quad . \quad . \quad . \quad (8)$$

we have

$$I_\tau = B\theta^{\;\frac{3}{2}} f(\theta) e^{-\frac{c}{\theta\tau^2}} \tau^{-(2p+4)} d\tau. \quad . \quad . \quad . \quad (9)$$

Replacing here the variable τ by the variable $\lambda = V\tau$, where λ is the length of the æthereal wave, and V the velocity of propagation of light. Thus denoting constant coefficients suitably modified by B and c, we shall have

$$I_\lambda d\lambda = B\theta^{-\frac{3}{2}} f(\theta) e^{-\frac{c}{\theta\lambda^2}} \lambda^{-(2p+4)} d\lambda. \quad . \quad . \quad . \quad . \quad (10)$$

This formula gives the intensity of a simple radiation of wave-length λ as a function of this length and of the absolute temperature of the source.

* According to the analysis of Helmholtz, resultant waves must be produced in all cases where the square of the elongation is not to be neglected. They explain, according to this author, the phenomena of sound of Sorges and Tartini (combination tones). In the case which we are considering they ought to form especially upon the surface of the radiating body, the forces which retain the atoms in their positions of equilibrium not being symmetrical in all directions.

It is towards a law of this sort that the distribution ot radiant energy in each spectrum must tend as its continuity becomes more and more perfect, that is to say, as the elective absorptions along the path of the rays diminish. To obtain a more complete expression of the law it would still be necessary to determine the value of p and the form of $f(\theta)$; but our formula is capable of giving some interesting results without our even having to make such a specialization.

It is evident that, under this general form, our law embraces as particular cases all the empirical laws of emission proposed hitherto, such as those of Newton, of Dulong and Petit, and of Stefan.

If we were to attribute to θ a constant value, and if we were to take λ as abscissa, and I_λ as ordinate, equation (10) would be the equation of the curve of energy in the normal continuous spectrum of a solid source at the temperature θ. These are the curves which M. Crova has called "isothermic curves."* In order to study the general properties of these curves, we will suppose θ constant and take the derivative of the expression I_λ.

2. *Limits of the Spectrum.*—It is easy to see, from formula (10) and its derived formula, that if $f(\theta)$ and p have definite values, we shall have for $\lambda=0$ and $\lambda=\infty$,

$$I_\lambda=0 \quad \text{and} \quad \frac{dI_\lambda}{d\lambda}=0.$$

This signifies that all the curves of energy represented by equation (10) are tangents to the axis of λ at the origin of coordinates, and that they have this axis as an asymptote. At the two extremities of the spectrum the radiant energy diminishes to zero; but whilst towards the violet it disappears almost suddenly because of the rapid diminution of the factor $e^{-\frac{c}{\theta\lambda^2}}$, the less refrangible energy extends indefinitely towards the side of increasing λ. This fact has been recently observed by Prof. Langley†.

3. *Maximum Intensity.*—Each of the curves represented by equation (10) presents only a single maximum defined by the condition

$$\lambda_{\max}=\sqrt{\frac{c}{p+2}}\,\frac{1}{\sqrt{\theta}}. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

We see that the position of maximum intensity in our spectra

* A. Crova, "Etude des Radiations," &c. (*Ann. de Chim. et de Phys.* [5] t. xix. p. 472).

† S. P. Langley, "Sur les Spectres invisibles" (*Ann. de Chim. et de Phys.* Dec. 1886).

depends upon temperature, and formula (11) gives the law of this dependence :—*Whatever the law of emission may be, the wave-length corresponding to the maximum energy is inversely proportional to the square root of the absolute temperature of the source.*

The only experimental measurements, so far as I know, which have been published hitherto upon this subject are contained in a recent diagram by Prof. Langley*. In order to compare these data with the relationship $\theta\lambda^2_{max} =$ const. equivalent to (11), I have deduced from this diagram, by a graphical process, the following values of λ_{max}:—

Temperatures.			
c°.	θ°.	λ_{max}.	$\theta\lambda^2_{max}$.
178	451	4·90	10828
330	603	4·05	9891
525	798	3·63	10515
815	1088	3·27	11634

Although the constancy of $\theta\lambda^2_{max}$ is far from being perfect, yet in view of the manner in which these four values have been obtained, and the uncertainty attaching to the determination of elevated temperatures, we may say that these observations appear to confirm our theory.

4. *Measurement of Temperatures.*—If we suppose that this law were confirmed by more numerous observations, it would then furnish us with a very simple method of measuring the high temperatures of radiating bodies†. This method would be superior to the spectro-photometric method elaborated by M. Crova, inasmuch as it is not dependent upon the choice of the tints compared, and would give, not "optical degrees," but absolute temperatures. Knowing the constant of equation (11), it would suffice, in order to measure the temperature of a radiating body, to determine the wave-length corresponding to the maximum intensity of its normal spectrum.

If we take account, as much as possible, of the absorptions suffered by the solar rays, and if we admit that the great density and enormous mass of the sun ought to give to its primitive spectrum all the qualities of continuous spectra, we may apply this method to calculate the temperature of the sun. According to the observations of Prof. Langley‡, the maximum

* *Loc. cit.* p. 462.

† The idea of a similar method, but without any definite law, has been propounded by M. Crova, *loc. cit.* p. 479.

‡ S. P. Langley, 'Researches on Solar Heat,' a report of the Mount Whitney Expedition, p. 144, pl. xv. curve iii. (Washington, 1884).

intensity of the solar spectrum observed beyond our atmosphere would be situated about $\lambda = 0^{\mu}\cdot 5$. Hence if, according to the preceding table, we take for $\theta\lambda^2_{max}$ the round number 10,000, we obtain about 40,000° for the absolute temperature of the sun. It should be remarked that this method only supposes an analogy in the distribution of energy in the primitive spectrum of the sun and in that of lampblack ; but the total emissive power may be very different for these two bodies.

5. *Total Energy of Radiation.*—By calculating the area of each curve represented by the equation (10), I obtain the total energy of radiation of the spectrum

$$E = \int_0^\infty I_\lambda d\lambda = \tfrac{1}{2} B c^{-(p+\frac{3}{2})} \Gamma(p+\tfrac{3}{2}) f(\theta) \theta^p, \quad . \quad . \quad (12)$$

where Γ denotes the Eulerian integral of the second order.

Comparing this formula with that obtained by eliminating λ between the two equations (10) and (11), that is to say, with the maximum intensity of the spectrum,

$$I_{max} = B \left(\frac{p+2}{ec}\right)^{p+2} f(\theta) \theta^{p+\frac{1}{2}}. \quad . \quad . \quad . \quad . \quad (13)$$

Making abstraction of constant coefficients, expressions (12) and (13) only differ because the second contains another factor $\sqrt{\theta}$. Hence the maximum intensity increases with the temperature more rapidly than the total energy of radiation, and their ratio

$$\frac{I_{max}}{E} = \frac{2c^{-\frac{1}{2}}}{\Gamma(p+\frac{3}{2})} \left(\frac{p+2}{e}\right)^{p+2} \sqrt{\theta}, \quad . \quad . \quad . \quad (14)$$

increases in the direct ratio of the square root of the absolute temperature of the source.

Multiplying together equations (11) and (14), we obtain

$$\frac{I_{max}\lambda_{max}}{E} = \frac{2e^{-(p+2)}}{\Gamma(p+\frac{3}{2})} (p+2)^{p+\frac{3}{2}} = \text{const.} \quad . \quad . \quad (15)$$

This interesting relationship shows that *the total radiant energy emitted by a solid bears a constant ratio to the product of the maximum energy of the normal spectrum by the corresponding wave-length*, or, differently expressed, that the area of each curve of energy bears a direct ratio to the area of the rectangle having for sides the coordinates of the summits of this curve (see further on, fig. 3). This constant ratio depends only upon the value of p, which probably is the same for all bodies ; for in our equations it is this value which characterizes the mode of action between the ponderable atoms and the surrounding æther.

This relation, which does not require a knowledge of the absolute temperature, is better fitted for experimental verification than the preceding relations. For the moment we do not possess direct data for such a verification. It is only in a very imperfect manner that I have been able, by measuring the ordinates and areas of the curves given by Prof. Langley, to obtain the following values of I_{max} and E :—

θ.	λ_{max}. μ	I_{max}.	E.	$\frac{I_{max}\lambda_{max}}{E}$.
451 . . .	4·90	5·2	182	0·140
603 . . .	4·05	21·5	532	0·163
798 . . .	3·63	39·7	1074	0·134
1088 . . .	3·27	64·9	1730	0·123

Bearing in mind the imperfection of the method of verification, the numbers of the last column, which according to our theory should be equal, do not seem unfavourable to it.

6. *Tracing the Curves.*—To be able to trace the curve of energy it is necessary to give special values to p and $f(\theta)$. If, for example, we admit with M. Stefan* that the radiant energy emitted by a body bears a direct ratio to the fourth power of its absolute temperature, we shall be able to put

$$p=1,\quad f(\theta)=R\theta^3;$$

from which equation (10) will take the form

$$I_\lambda=B_1\theta^{\frac{3}{2}}e^{-\frac{c}{\theta\lambda^2}}\lambda^{-6}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Having determined the two constants B_1 and c of this equation from a diagram of the normal invisible spectrum of lampblack at 178° C., given by Prof. Langley, I have calculated and traced the theoretical curves for this temperature and for 327° C. = 600° abs. (fig. 3). We see that these curves have all the qualities that Prof. Langley attributes to the curves determined experimentally by him ; that is to say :—

(1) The radiant energy represented by the ordinates diminishes on both sides, yet extends indefinitely towards the side of the less refrangible rays.

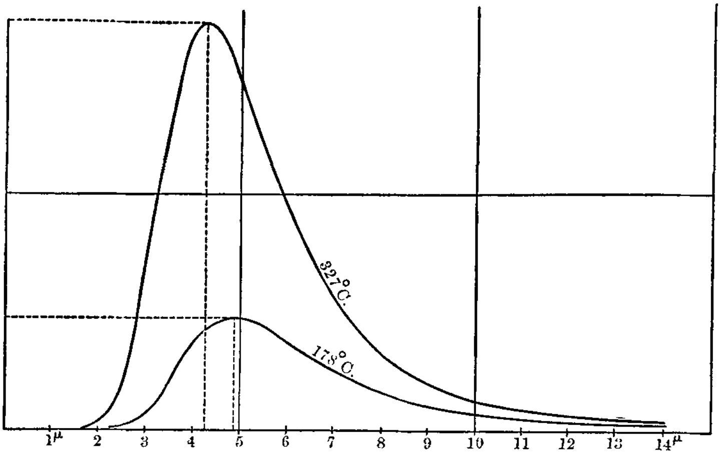
(2) When the temperature increases all the ordinates increase, but in unequal proportion. Those which represent the energy of the more refrangible waves increase always more rapidly than those which correspond to the longer undulations.

(3) Hence the fact, independently observed, that the maxi-

* Stefan, *Sitzber. Wien. Akad.* vol. lxxix. p. 391 (1879).

mum ordinate is progressively displaced toward the more refrangible region. We have indicated above the probable law of this displacement.

Fig. 3.



(4) The theoretical curves of the normal spectrum, like Prof. Langley's prismatic curves, are not symmetrical; the largest area is situated on the right-hand side of the maximum ordinate, that is, on the side of the less refrangible rays.

(5) In accordance with this, the fall of each curve is more rapid on the side of the shorter undulations. For the two cases of which we have given diagrams, all sensible heat disappears before the curves reach the limit of the visible spectrum: there is only dark heat.

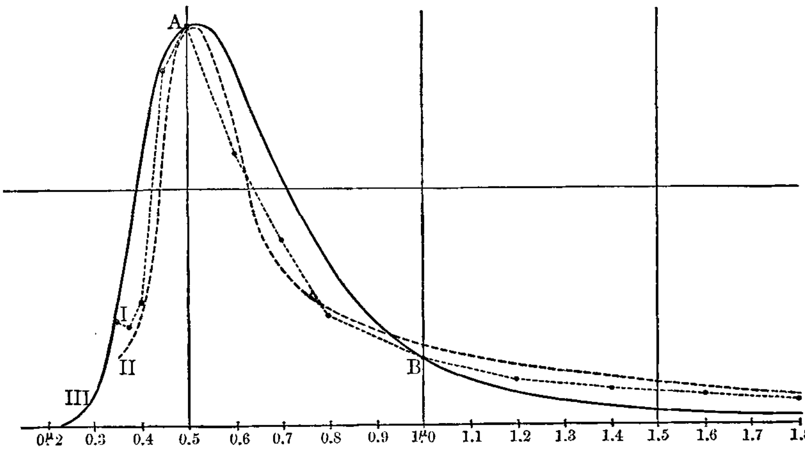
7. *Solar Curve.*—I have also endeavoured to compare my theoretical curves, as to general form, with the curve of solar energy beyond the atmosphere. In fig. 4 the separate points correspond to the numbers given by Prof. Langley in No. 9 of the table 120 of his work, 'Researches on Solar Heat, &c.' The dotted curve has been traced from curve 3 of plate 15 of the same work, and the continuous curve gives the theoretical curve represented by the equation

$$I_\lambda = Ae^{-\frac{M}{\lambda^2}}\lambda^{-6}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and passing through the observed points A and B.

To take account of the difference between theory and observation (a greater difference in this case than in the preceding one), we must remember that not only equation (17), but also the more general formula (10), can only be considered as a first approximation ; that these equations have been deduced for solid bodies, and not for such bodies as the sun ; and, lastly, that in our atmosphere, as well as in the colder regions of the gas surrounding the sun there must occur the pheno-

Fig. 4.



menon of degradation of the radiant energy. In consequence of this effect, which has not been eliminated by Prof. Langley, a portion of the more refrangible radiations would be transformed into longer undulations, which would tend to modify the spectral diagram exactly in the sense indicated by observation.

Recognizing the provisional character of the ideas suggested in this paper, I hope that it will be possible to give them a greater development and precision when once we are in possession of more complete experimental data to guide us in this study. We should then be able to express by one sufficiently exact formula the law of distribution of radiant energy as a function of the wave-length and the temperature of the source. Such a formula would be of great service in all questions arising in the study of spectra.