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XXVIII. *On Magnetomotive Force.*

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To the Editors of the Philosophical Magazine and Journal.

Gentlemen,

THE following paper is an attempt to develop the analogy between magnetism and the voltaic circuit, which was enunciated by Faraday. The assumptions of this theory are generally admitted to be true; but they have not, so far as I know, been consistently pushed to their consequences.

It appears to me, further, that this point of view is the only one for which there exists any experimental evidence. The fundamental assumptions of Poisson's theory are admittedly false; and it is only by the introduction of fictitious quantities that the existing mathematical theory has been rendered in any degree capable of representing the facts.

Faraday compared a magnet to a voltaic battery immersed in water*; and he established by experiment the principal analogies on which this comparison is founded.

The first principle of the voltaic circuit is, that the current produced by a given electromotive force in a circuit depends on the resistance of the circuit as a whole.

I shall use the expression "magnetomotive force" to indicate the analogue of electromotive force. It is a difference of magnetic potential, just as electromotive force is a difference of electric potential.

Now the fundamental hypothesis at the base of the ordinary mathematical theory of magnetism is, that there are magnetizing forces $\mathfrak{H}$ which are of the dimensions of the magnetic induction $\mathfrak{B}$ which they produce, and that the magnetizing force permeates every medium, and produces in magnetic media magnetic induction proportional to the force and to a coefficient of permeability μ, quite independently of the existence of any magnetic circuit.

This is the simplest way of putting it. I will state the case presently in terms of the quantity known as "magnetization," which is the quotient of moment by volume.

There are two objections to this. First, in relation to complete circuits.

Consider a sphere of iron, or a disk magnetized normally to its plane. Then (Maxwell, ii. p. 66) the magnetic induction through it is small. Now let it form part of a bar, and let the bar be bent round into a ring, so as to establish a magnetic circuit. We know that the magnetic induction through the same piece under the same force will become enormous. Can

* Experimental Researches, iii. p. 424, par. 3276.

it be said that it is a natural expression of such facts to assume that the magnetism depends on the force and on the conductivity of the material of disk or sphere, leaving the existence of the circuit out of the question? It would be equally sensible to take a piece of copper out of a voltaic circuit, and found our theory of current electricity on the hypothesis that an electromotive force acting on the copper produced a current through it proportional to its conductivity and to the force, irrespective of the completion of any circuit.

There is in magnetism this circumstance, which gives somewhat more colour to the hypothesis than there would be in the above case—namely, that space conducts magnetism much better than it does electricity; and in consequence *some* magnetic induction can always be set up in iron by a magnetomotive force—just as, if we lived in the sea, and had voltaic batteries and dynamo machines there, the action of an electromotive force would always produce *some* current, the circuit being completed by the conducting-power of the sea-water.

The second objection is to the substitution of the so-called "magnetizing force" for a magnetomotive force. This is just as if, living in the sea, we associated electromotive forces with the currents in the sea-water which would inseparably accompany them, took these currents for the measures of the forces, and called them the electrizing forces.

In carrying out the ordinary theory on this basis, we have to suppose that the magnetizing force $\mathfrak{H}$ within a magnetic body has the power of remaining separate and distinct from the magnetic induction as a whole, though the two are quantities of the same nature. This has always seemed to me to present insuperable difficulties as a physical conception.

So soon as we replace the "magnetizing force" by a difference of potential or magnetomotive force, we can assimilate the whole conception to that of the origin of an electric current under an electromotive force. The quantity $\mathfrak{H}$ becomes merely the magnetic induction in vacant space, and $\mathfrak{B}$ that in magnetic matter. $\mathfrak{B}$ replaces $\mathfrak{H}$, and is not supposed to include it as before. According to the ordinary theory,

$$\mathfrak{B}=\mathfrak{H}+4\pi\mathfrak{J} \text{ or } \mu=1+4\pi\kappa;$$

where

$$\mu=\frac{\mathfrak{B}}{\mathfrak{H}},\quad \kappa=\frac{\mathfrak{J}}{\mathfrak{H}},\quad \mathfrak{J}=\frac{\text{moment}}{\text{volume}}.$$

The change of conception and the real meaning of the formula can be shown as follows:—

Let $\mu=1+\lambda$. Suppose an infinite* bar acted on by a mag-

* The bar must be infinite, not merely long. Rowland found the influence of the ends still sensible in the longest bars he tried.

netizing force $\mathfrak{H}$. The old theory says there are all together $\mu A\mathfrak{H}$ lines of force (A = section of bar), and

$$\mu A\mathfrak{H} = A\mathfrak{H}(1+\lambda), \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $A\mathfrak{H}$ are the lines of force of the magnetizing force itself, $A\mathfrak{H}\lambda$ those added by the induction.

These last form the poles. And, since there are 4π lines of force round a unit pole, strength of pole $= \frac{\lambda A\mathfrak{H}}{4\pi}$.

Again,

$$\text{Moment} = \text{pole} \times \text{distance of foci},$$
$$\text{(ultimately)} = \text{pole} \times \text{length of bar},$$
$$= \frac{\lambda\mathfrak{H}}{4\pi} \times \text{volume};$$

and

$$\mathfrak{I} = \frac{\text{moment}}{\text{volume}} = \frac{\lambda\mathfrak{H}}{4\pi}.$$

Substituting for $\lambda\mathfrak{H}$ in (1),

$$\mu\mathfrak{H} = \mathfrak{H} + 4\pi\mathfrak{I};$$

whence come the equations of the ordinary theory first above written.

From our point of view $\mu = \lambda$ in the above, the action of the magnetic matter replacing that of space instead of being added to it; and our fundamental equation becomes

$$\mu = 4\pi\kappa, \text{ or } \mathfrak{B} = 4\pi\mathfrak{I}.$$

I believe that there is no evidence whatever for the view that represents $\mathfrak{H}$ as subsisting independently throughout the magnetic body.

If we are really to carry out Faraday's theory of magnetism, we must take into account the entire resistances of the circuits formed by iron and air, and then determine the magnetic induction through the circuit as the quotient of the "magnetomotive force" by the total resistance.

We may define the unit of "magnetomotive force" as that which, acting through a unit of magnetic resistance, produces a unit of field-intensity or magnetic induction.

Consider a solenoid having its ends joined. Then, if the resistance unit be that of 1 centim. of the length in air, x is the resistance of length x of the solenoid. Similarly, if the solenoid be filled with an iron ring of permeability μ, x/μ is the resistance of length x of the ring. And if x_0 be the whole length of the ring, M the whole magnetomotive force,

$$\frac{M}{\frac{x_0}{\mu}} = \mathfrak{B},$$

or
$$M = \frac{x_0}{\mu}\mathfrak{B} = x_0\mathfrak{H},$$

where M is the whole difference of magnetic potential which acts on the induction as it traverses the circuit once.

If C be the current in the coils, n the number of coils,
$$M = 4\pi Cn,$$
since the point considered has gone once round each spire of the coil.

Put $M = 1 = 4\pi Cn$. Then, if we put $n=1$,
$$C = \frac{1}{4\pi};$$
and the C.G.S. unit of current is 10 ampères;
$$\therefore\ C = \frac{10}{4\pi} \text{ ampères, } = \cdot 8 \text{ ampère nearly.}$$

Hence the unit of magnetomotive force is that which acts on a circuit singly linked with one spire of a current of $10/(4\pi)$ ampères. Thus a soft-iron horseshoe with ends nearly meeting, round which a wire carrying such a current is wound once, would exhibit nearly the unit of magnetomotive force between its poles. (See *post*, on broken circuits.)

Example of a ring solenoid.—Let the length of the solenoid round the axis be 100 centim.,

Current $= 10/(4\pi)$ ampères, number of coils $= 1000$;
$$\therefore\ M = 1000;$$
and
$$\frac{M}{x} = 10 = \mathfrak{H} = \text{intensity within the ring in air.}$$

The area of the section of the resistance comes in as a factor on both sides. Strictly the unit resistance would be that of 1 centim. length of an air-cylinder whose cross section has an area of 1 square centim. If we suppose the area of the section of the air-space enclosed in the solenoid to have this value, its radius would be $1/\sqrt{\pi}$ centim.

In general all the lines of force pass through some one section, generally the equatorial section of a bar, so that the total magnetic induction is the product of the magnetic induction through unit area and the area of this section. It is usually convenient to express the resistance in terms of the length of a cylinder having the same sectional area. This area appears ou both sides, and may be struck out.

Suppose the above ring-shaped solenoid to be wound about an iron ring whose permeability $=\mu$. Then

$$\frac{\frac{M}{100}}{\mu}=\mu \, . \, 10=\mathfrak{B}$$

gives the magnetic induction in the iron. It was by measuring this quantity in rings that Rowland determined the values of μ under different magnetic inductions (Phil. Mag. xlvi. p. 140).

Now it is possible from the above equation, supposing one of Rowland's tables to be correct, to tell what the magnetic induction and permeability for soft iron would be in the above case. The induction is ten times the permeability; we have therefore only to find the corresponding point in Rowland's table for soft iron, reduced to C.G.S. measure. I take the liberty of transcribing the two columns required from Phil. Mag. xlvi. p. 151. $\mathfrak{B}$ is Rowland's Q reduced to C.G.S. by dividing by 10.

$\mathfrak{B}$.	μ.	$\mathfrak{B}$.	μ.
71·5	390·7	7473	2367
600·5	868·7	8943	2208
966·7	1129	10080	1899
2460	1936	12270	1448
2923	2078	12970	1269
3082	2124	13630	1137
4959	2433	14540	824·1
5482	2470	15770	461·8
5782	2472	16270	353·8
6651	2448	16600	258·0
		17500	0

The pair of values most nearly corresponding to $\mathfrak{B}=10\mu$ is $\mathfrak{B}=12970$, $\mu=1269$.

The observation that μ as a property of the iron must be a function of its condition, and probably of $\mathfrak{B}$, was made by Rowland. We see that it cannot possibly depend on $\mathfrak{H}$ as is usually supposed, as this is simply the field-intensity produced in air by the given magnetomotive force, and has nothing to do with the iron.

In the present case (iron ring) $\mathfrak{H}$ is the numerical ratio of $\mathfrak{B}$ and μ, and so a function of the iron; but, in broken circuits into which air-resistances enter, μ, the permeability of the circuit, depends in general chiefly on the air-resistances. So that, for given values of $\mathfrak{H}$, the functions of the iron may have widely differing values, according to the value of the external resistance of the circuit.

As another example we may suppose the current in the last case reduced to $\frac{1}{10}$ of its amount—*i. e.* to ·08 ampère say. Then $M=100$, and $\mu=\mathfrak{B}$. This points to a value between the third and fourth steps of the table, where each of these functions would be about 1250.

Let us now suppose the ends of the solenoid separated, and apply the analogue of the law which regulates the E.M.F. between the terminals of a battery.

If R be the internal resistance of the battery,

r the external resistance,

E total E.M.F.,

e E.M.F. between terminals;

then

$$e=\frac{r}{R+r}E.$$

Similarly let X be the internal resistance of a magnetic solenoid,

x the external resistance,

M the total magnetomotive force of the circuit,

m the magnetomotive force between the ends of the solenoid;

then

$$m=\frac{x}{X+x}M.$$

Here we assume that the whole magnetomotive force acts within the solenoid. This is not strictly true; for every part of the circuit is subject to some portion of it; but it is nearly enough true for approximate purposes.

This is generally in accordance with fact (see Faraday, Exp. Res. iii. p. 428, par. 3283). The case of the soft-iron horseshoe surrounded by one or more coils of wire may now be considered. X will be small, and x, the air-resistance, great; $\therefore$ m nearly $=$ M, as was observed in speaking of the unit magnetomotive force. If, on the other hand, an armature be applied having a resistance x much less than X, the free magnetomotive force at the terminals is reduced, or m becomes a small fraction of M. Other cases can be discussed in the light of the analogy of the voltaic circuit. The solenoid without iron corresponds to a battery of high internal resistance; it may be regarded as joined up through the comparatively small resistances at the end, and presents but little free magnetomotive force.

Let us now consider the case of a body of great conductivity exposed to a uniform magnetic field, such as that of the earth's horizontal magnetism. It is clear that, in consequence of the conductivity, the potential at the ends of the conductor tends to be equalized; and if the conductivity were infinite it would be equalized throughout the body. The whole of the body must therefore be regarded as being at the potential which in its absence its centre of symmetry would have had. The fall of potential on approaching the body is greater than in the undisturbed state. This gives rise to stream-line problems, which are the same as those ordinarily dealt with.

We see that, in such a field, no new lines of force can be developed in any circuit; for the action of the uniform magnetomotive force on the opposite portions of the circuit is the same in amount and opposite in direction.

We may use the known solutions to obtain the permeability of a sphere, by which we mean the ratio of the number of lines of force through its equatorial section to the number through the same section in air. This is 3 for a sphere of infinite conductivity. This is deduced by Stefan in a recent number of Wiedemann's *Annalen*, xvii. p. 956. It can also be obtained from fig. 4, p. 489, of the Reprint of Sir William Thomson's Papers, by comparing the square of the ordinate of the outside line inflected so as just to meet the sphere with the square of the radius of the sphere. This gives

$$(1{\cdot}375 \times \sqrt[3]{2})^2 = 3{\cdot}001.$$

In both these cases the solution only refers to the case of a uniform field of infinite extent, which excludes circuits (as remarked above). I shall presently examine this excluded case.

In the meantime an important point may here be noticed. If we calculate the *magnetization* of a sphere of infinite conductivity by the usual formula (Maxwell, vol. ii. p. 65), we obtain the number $3/(4\pi)$. Now if we seek to deduce the permeability from this by the usual formula $\mu = (1 + 4\pi k)$, we find the number 4 instead of 3 as given by the above investigations. This obviously arises from the formula being based on the hypothesis that the "magnetizing force" penetrates unchanged through the body, and is to be added to the distribution of stream-lines which has been determined. It is very difficult to admit this. Our point of view, according to which the magnetizing agent is a magnetomotive force and not a field intensity, removes this difficulty; and the formula for μ reduces to $\mu = 4\pi k$, which gives 3, as before, in the present case.

We can obtain a more general approximate solution for the case of a sphere subjected to a magnetomotive force such that the sphere forms part of circuits through which the force acts, in a form suitable for experimental verification.

Let a coil be wound on a reel having a cylindrical opening within. Length of reel = diameter of sphere = diameter of cylindrical opening. Then magnetic circuits will be formed passing through the sphere and linked once with each turn of the coil. It will be near enough for the present purpose to assume that the lines of force radiate at right angles from the surface of the sphere in all directions. This is the case close to the surface; and by far the greater portion of the resistance of the divergence arises close to the surface. It is, then, easy to show that the resistance of the divergence from each hemisphere is equal to that of a cylindrical air-space having the equatorial section of the sphere for base, and height = half the radius. In fact, if $s=2\pi r^2$,

$$\text{resistance of hemispherical shell} \qquad = \frac{dr}{s},$$

$$\text{total resistance} \quad = \int_a^\infty \frac{dr}{2\pi r^2} = \frac{1}{2\pi}\left[-\frac{1}{r}\right]_a^\infty = \frac{1}{2\pi a},$$

$$= \frac{\frac{a}{2}}{\pi a^2}.$$

If the sphere be of infinite conductivity, the total resistance is twice this, *i. e.* a cylinder of altitude a.

Remove the sphere. Then the resistance is that of the cylinder, with divergence from the flat ends. If we take these divergences each to have resistance $\cdot 6\, a$ of the cylinder, as we know to be the case approximately in the analogous case of the divergence of sound from the end of a pipe*, we have for the whole resistance in this case that due to cylinder + 2 ends, measured by $a(2+2\times\cdot 6)=3\cdot 2\, a$. Comparing this with the resistance of the sphere, which was measured by a, we have 3·2 for the permeability of the sphere, which agrees very fairly with what went before.

There remains the important case of a disk. According to our view the disk will have finite air-resistances around it, and when its thickness becomes small the air-resistances will not be sensibly altered by its removal. The conductivity of the circuit through a thin disk is therefore unity. According to the ordinary theory (Maxwell, vol. ii. p. 65), the *magnetization* of a disk for which $\kappa=\infty$ is $1/(4\pi)$. Here we meet again the same difficulty as in the case of the sphere: if we use the ordinary formula $\mu=1+4\pi k$, and assume that the *magnetizing force* flows through the disk as well as the lines of force that result from the magnetization, then $\mu=2$. But from our point of view the force is a magnetomotive force; the induction in the substance takes the place of that in space, and is not additional to it, and $\mu=1$, or the state of things is unaffected by the disk.

Permanent Magnets.

The following quotations appear to embody the facts as they are supposed to be :—

* See Lord Rayleigh on Sound, ii. p. 169.

(1) Maxwell, vol. ii. p. 45 :—" If a magnet could be constructed so that the distribution of its magnetization is not altered by any magnetic force brought to act on it, it might be called a rigidly magnetized body." . . . . . .

(2) Gordon, 'Electricity and Magnetism,' i. p. 148 :— " Into however many pieces we cut a magnet, each will have two opposite poles, whose strength is equal to that of the poles of the original magnet." P. 151—" The moment of a magnet is not altered by cutting it in pieces." P. 155—" If from any magnetized substance we cut any piece whatever, its magnetic moment is simply proportional to its volume."

Now these two hypotheses may apply to a theoretical magnetism which can be imagined; but they are both far from representing the actual behaviour of permanent magnets.

With respect to (1), I shall develop a hypothesis which leads to an account of the properties of permanent magnets at all events nearer the truth than (1). (2) is very far from being true.

(1) After this passage Maxwell proceeds:—" The only known body which fulfils this condition " (rigid magnetization) " is a conducting circuit round which a constant current is made to flow. Such a circuit exhibits magnetic properties, and may therefore be called an electro-magnet; but these magnetic properties are not affected by the other magnetic forces in the field."

Now from the point of view of the preceding investigation, we should not call the current-circuit a rigid electromagnet. We should speak of it as possessing a definite magnetomotive force, and say that it magnetizes the space or other objects in its neighbourhood. And this magnetism is by no means rigid, but depends on the resistance (or on the permeability) of the magnetic circuits through which it flows; *i. e.* it is modified by the introduction of iron into the field, which rigid magnetism would not be.

If, then, the hypothesis of frictionless Ampèrian currents in magnets be at all correct, even as an analogy, the first supposition as to the nature of permanent magnets will be that they possess a constant magnetomotive force in their substance in virtue of the Ampèrian currents; or this may be simply assumed without further hypothesis. And this magnetomotive force maintains a magnetic induction in the magnet which depends on the total resistance of the magnetic circuit.

Now, if this be true, suppose a long steel bar-magnet to be cut up into short pieces. The resistance of the whole consists of two parts—that of the steel, and the air-resistances at the two ends; there are also parallel air-circuits which start off all along the sides of the rod. These latter are under smaller differences of potential than the divergences at the end, and may, in the first instance, with rods not very long, be neglected in comparison with the resistances at the ends.

Now suppose the rod cut up into n pieces: we have the same total magnetomotive forces, the same total steel resistance, and the resistance of $2n$ ends. The magnetic induction in each piece is therefore altered in a ratio which depends on the ratio of the steel-resistance of one of the little pieces to the air-resistance of its two ends.

It is not possible to cut up a hard steel bar without disturbing the magnetism; in fact it is hardly possible to cut it up at all. I have therefore preferred to cut a soft steel bar into short lengths, finish these as accurately as possible so that they may be put together to form one long bar; then harden, glass-hard; then grind the ends with emery and oil till the pieces will pick each other up when firmly pressed together with a trace of oil on the faces; then magnetize.

The compound bar is then suspended in a cradle by means of a bifilar suspension arranged with its equilibrium-plane at right angles to the magnetic meridian. If it were true that the moments of the separate portions were the same whether joined up or not, the deflection should be the same in both cases. But it is not so. The deflection when the bar is joined up and pressed together is many times as great as that obtained when the pieces are so dispersed about the tray which carries them as to be fairly removed from each other's influence.

A rough preliminary arrangement showed the existence of a difference, but did not lead to the detection of the smallness of the effect produced by the separated pieces.

A bifilar suspension was then constructed. It has a pulley for the wire to pass over; adjustable slides with metre- and inch-scales carrying the holes for the wire to pass through; and a circular seat, with a circle divided to degrees; this is fixed on a firm crossbeam at a good height. The wires enter, through a hole in the cover, a cylindrical case, whose sides are made of narrow pieces of flat glass. Within this case swings the cradle which carries the magnet. It has three V-shaped troughs. The pieces can either be wedged together in the middle trough, or be placed at considerable distances in all three. The cradle is 16 inches long. It is suspended over a circle divided to degrees and having nearly 16 inches diameter. Pointers are attached to the ends of the cradle, which play in front of the circle. There is not more than $\frac{1}{16}$ of an inch to spare between points and circle on each side, so that the centering has to be very true for the cradle to swing free. The circle is read through the glass sides of the case.

The magnet employed is made from a cylindrical bar of cast tool-steel. It consists of eighteen pieces, fitted and hardened as above described. They were then fastened together in two lengths of 9 pieces each with wooden splints, and placed between the terminals of an electro-magnetic magnetizer constructed for the purpose. When screwed up firmly between the poles of the magnetizer a current was transmitted through the coils. In this condition it was tapped with a hammer for some time. When removed, each compound bar retained a considerable permanent magnetism.

The chief difficulty was to observe with sufficient accuracy the small deflection produced by the separated pieces. Small differences in this small quantity produce large differences in the calculated resistance (or permeability) of the steel. The measures now given probably reach to an accuracy of about a tenth of a degree. Up to this point I have endeavoured to take count of all the errors of the apparatus. The final measures are:—

Deflection due to 18 pieces joined up $=13^{\circ}{\cdot}0$
" " separated $= 1^{\circ}{\cdot}05$*.

Dimensions of magnet:— centim.
Whole length $=28{\cdot}50$
Length of each piece $= 1{\cdot}58$
Diameter $= 1{\cdot}97=2\mathrm{R}$

Let r be the resistance of one of the steel pieces expressed in centimetres of a similar air-cylinder,

$\alpha\mathrm{R}$ the resistance of one end;

then $$\frac{18r+2\alpha\mathrm{R}}{18r+18\times 2\alpha\mathrm{R}}=\frac{1{\cdot}05}{13}={\cdot}08 \text{ nearly}$$

(assuming the forces proportional to the deflections).

Then $r= {\cdot}053\times\alpha$ centim.

and length of piece $=1{\cdot}58$ centim.

$$\therefore \text{ ratio of } \frac{\text{length of air}}{\text{steel of same resistance}}=\frac{{\cdot}053\times\alpha}{1{\cdot}58}.$$
$$={\cdot}034\times\alpha.$$

* After the pieces had remained separated for some days in the bifilar, I noticed that the reading had changed in the direction of increased moment. A set of readings gave $13^{\circ}{\cdot}0$ and $1^{\circ}{\cdot}9$. After standing for some days joined up again, $13^{\circ}{\cdot}2$ and $1^{\circ}{\cdot}8$. These latter values correspond to $\mu=15$ nearly if $\alpha={\cdot}6$, to $\mu=29$ nearly if $\alpha={\cdot}3$. There appears to have been a sort of spontaneous rearrangement of the magnetism of the little pieces in the direction of less resistance, probably with diminution of α. The spontaneous change was an increase of moment, not a diminution. The point requires further examination.

If we assume $\alpha = \cdot 6$ from analogy to sound and electricity,

ratio$= \cdot 020$.

α is not likely to be greater than this ; it may be less, as the case is like that of a tube with side resistances removed to a certain extent.

The conclusion is: The magnetic induction of a permanent magnet may be supposed to be produced by a magnetomotive force derived from permanent Ampèrian currents, acting through the resistance of the steel.

In the case of the steel examined, this resistance was $\frac{1}{50}$ of that of space, if $\alpha = \cdot 6$. If α be less the resistance will be less in proportion.

Meyer (Wiedemann, *Ann.* xviii. p. 233) has determined the magnetization-function k of hard steel (p. 245). He finds generally values varying from 2 to 3 for small magnetizing forces, but in some cases as much as 9 or even 12. These correspond, according to our formula $\mu = 4\pi k$, to the following values of μ :—

k ...	2	3	8	12
μ ..	25·1	37·7	100·6	150·8.

According to our result μ would be 50 for the above steel. The number is quite uncertain, as the application of the coefficient α is as yet hypothetical. But it shows that the hypothesis is not in contradiction with known values. Determinations of all the quantities involved require instruments of a more accurate kind than those I have hitherto employed. These are being constructed for other purposes ; and I hope to examine the matter further.

The assumption of the existence of magnetomotive force and resistance in permanent magnets appears to be the necessary consequence of Faraday's comparison of the permanent magnet with the voltaic battery immersed in water. It is the simplest assumption by means of which the facts can be represented.

On the old theory the assumption of rigid magnetism might be modified to suit the facts by assuming the magnetism to be elastic instead of rigid. Suppose, then, that when the magnet is cut up into spheres, or disks say, the demagnetizing forces of Maxwell (ii. 57) act on the elastic magnetization. There will be a temporary diminution.

It will be understood that I regard these demagnetizing forces as arising out of the fictitious quantities created by analysis for the purpose of compensating the errors of the original hypotheses of Poisson's theory. I therefore prefer the method here indicated. This has the advantage of retaining permanent elements in the permanent magnet ; and it appears that, so far as the numerical value of the resistance can be obtained, it is at all events not glaringly incompatible with the values of the resistance to external magnetomotive force which have been obtained in an entirely different manner.

XXIX. *On an Arrangement for dividing Inch- and Metre-Scales.* *By* R. H. M. Bosanquet, *St. John's College, Oxford.*

To the Editors of the Philosophical Magazine and Journal.

Gentlemen,

THE screw of the slide-rest of the Royal Society's lathe in my laboratory has a pitch of $\frac{1}{8}$ inch. It was therefore an obvious arrangement for dividing decimal-inch scales, to fit this screw with a micrometer-wheel of 25 holes.

20 holes then correspond to $\frac{1}{10}$ inch.
4 " " $\frac{1}{50}$ inch.
2 " " $\frac{1}{100}$ inch.

The holes are worked with pins, and a V falling over the pins in the same way as in the micrometer of the same lathe described in Phil. Mag. x. p. 220.

Before this was completed it occurred to me that a metre-scale could be divided by the same arrangement, if there was any moderate number which would serve as a factor.

The common equivalent is 1 inch = 2·5400 centim. According to Everett's book of units the error of this is about 2 units in the fifth place of decimals; and therefore it is negligible for practical purposes.

$$\text{Hence 1 centim.} = \frac{8}{2{\cdot}54} \text{ eighths of an inch.}$$

$$= \frac{400}{127} \quad " \quad "$$

It was only necessary therefore to set a division of 127 holes round the micrometer-wheel, and 40 of these holes give a millimetre. If it were desired to divide to tenths of millim., of course 4 of the holes would be used. This division of 127 holes and the one of 25 for the inch-scale are executed on the same wheel, by means of which therefore both metre- and inch-scales can be divided without any shifting whatever.

In dividing millimetres a little piece of brass is used, which reaches from the pin in use to the hole at distance 40, so as to avoid having to count every time.

The division of 127 was executed with the micrometer by means of the following approximation depending on the 67 wheel.