

These conclusions are confirmed by numerous experiments; and no experiment has been found to contradict them.

The decision in favour of this theory was supplied by the fact that glowing and cold wires separated only by heated air, with the exclusion of combustion-gases, showed an electrical difference. Here also the latter is dependent on the nature, and the quality of the surface, of the electrodes employed, and on their state of incandescence. A too strong heating of the wires, and, therewith, also of the separating air stratum, proved unfavourable to the development of free electrical tension—a circumstance probably due to the augmentation of the conducting-power of that separating stratum. In accordance with this, wires introduced into the flame, so long as they are both immersed in the combustion-gases (which are relatively good conductors), never give the maximum of potential-difference; rather this enters only when one of the wires comes into contact with only the outer air stratum of the flame (which is endowed with a very high resistance).

The occurrence of a thermoelectric counterforce within the galvanic flame-arc is also naturally explained by the above theory.

The questions proposed at the commencement are therefore to be answered thus:—Hankel's theory is not in accordance with experiment; and the two kinds of excitation assumed by Buff and Matteucci must be regarded as *simultaneously* causing the apparent electricity of flame.

Wolfenbüttel, February 1882.

XX. *On the Equilibrium of Liquid Conducting Masses charged with Electricity.* By LORD RAYLEIGH, F.R.S.\*

IN consequence of electrical repulsion, a charged spherical mass of liquid, unacted upon by other forces, is in a condition of unstable equilibrium. If  $a_0$  be the radius of the sphere,  $Q$  the charge of electricity, the original potential is given by

$$V = \frac{Q}{a_0}.$$

If, however, the mass be slightly deformed, so that the polar equation of its surface, expressed by Laplace's series, becomes

$$r = a(1 + F_1 + F_2 + \dots + F^n + \dots),$$

\* Communicated by the Author.

then

$$V = \frac{Q}{a_0} \left\{ 1 - \Sigma(n-1) \iint \frac{F_n^2 d\sigma}{4\pi} \right\};$$

and the potential energy of the system reckoned from the equilibrium position is

$$P' = - \frac{Q^2}{8\pi a_0} \Sigma(n-1) \iint F_n^2 d\sigma.$$

In actual liquids this instability, indicated by the negative value of  $P'$ , is opposed by stability due to the capillary force. If  $T$  be the cohesive tension, the potential energy of cohesion is given by

$$P = \frac{1}{2} a_0^2 T \Sigma(n-1)(n+2) \iint F_n^2 d\sigma^*.$$

If  $F_n \propto \cos(pt + \epsilon)$ , we have for the motion under the operation of both set of forces,

$$p^2 = \frac{n(n-1)}{\rho a_0^3} \left\{ (n+2)T - \frac{1}{4\pi} \frac{Q^2}{a_0^3} \right\}.$$

If  $T > \frac{Q^2}{16\pi a_0^3}$ , the spherical form is stable for all displacements. When  $Q$  is great, the spherical form is unstable for all values of  $n$  below a certain limit, the maximum instability corresponding to a great, but still finite, value of  $n$ . Under these circumstances the liquid is thrown out in fine jets, whose fineness, however, has a limit.

The case of a cylinder, subject to displacement in two dimensions only, may be treated in like manner.

The equation of the contour being in Fourier's series

$$r = a(1 + F_1 + \dots + F_n + \dots),$$

we find as the expression for the potential energy of unit length

$$P' = - \frac{Q^2}{l^2} \Sigma(n-1) \int \frac{F_n^2 d\theta}{2\pi},$$

$Q$  being the quantity of electricity resident on length  $l$ .

The potential energy due to capillarity is

$$P = \frac{1}{2} \pi a T \Sigma(n^2-1) \int \frac{F_n^2 d\theta}{2\pi},$$

and for the vibration of type  $n$  under the operation of both

\* See Proc. Roy. Soc. May 15, 1879.

sets of forces,

$$\rho^2 = \frac{n^2 - n}{\rho a^3} \left\{ (n + 1) T - \frac{2Q^2}{\pi l^2 a} \right\}.$$

The influence of electrical charge in diminishing the stability of a cylinder for transverse disturbances may be readily illustrated by causing a jet of water from an elliptical aperture to pass along the axis of an insulated inductor-tube, which is placed in connexion with an electrical machine. The jet is marked with a recurrent pattern, fixed in space, whose wave-length represents the distance travelled by the water in the time of one vibration of type  $n=2$ . When the machine is worked, the pattern is thrust outwards along the jet, indicating a prolongation of the time of transverse vibration. The inductor should be placed no further from the nozzle than is necessary to prevent the passage of sparks, and must be short enough to allow the issue of the jet before its resolution into drops.

The value of  $T$  being known (81 C.G.S.), we may calculate what electrification is necessary to render a small rain-drop of, say, 1 millimetre diameter unstable. The potential, expressed in electrostatic measure, is given by

$$V = \frac{Q}{a_0} = \sqrt{(16\pi a_0 T)} = 20.$$

The electromotive force of a Daniell cell is about '004; so that an electrification of about 5000 cells would cause the division of the drop in question.

XXI. *On an Instrument capable of Measuring the Intensity of Aerial Vibrations.* By LORD RAYLEIGH, F.R.S.\*

**T**HIS instrument arose out of an experiment described in the 'Proceedings of the Cambridge Philosophical Society' †, Nov. 1880, from which it appeared that a light disk, capable of rotation about a vertical diameter, tends with some decision to set itself at right angles to the direction of alternating aerial currents. In fig. 1, A is a brass tube closed at one end with a glass plate B, behind which is a slit C backed by a lamp. D is a light mirror with attached magnets, such as are used for reflecting-galvanometers, and is suspended by a silk fibre. The light from the slit is incident

\* Communicated by the Author.

† See also Proc. Roy. Soc. May 5, 1881, p. 110.