THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[FIFTH SERIES.]

MAY 1877.

XLIII. *On Rotation of the Plane of Polarization by Reflection from the Pole of a Magnet.* By JOHN KERR, *LL.D., Mathematical Lecturer of the Free Church Training College, Glasgow*[*].

1. I WAS led some time ago to think it very likely, that if a beam of plane-polarized light were reflected under proper conditions from the surface of intensely magnetized iron, it would have its plane of polarization turned through a sensible angle in the process or fact of reflection. The known facts upon which this expectation was founded are indicated briefly under the five following heads.

(1) The effects discovered by Faraday in his famous polariscopic experiments in the magnetic field.

(2) Many instances in optics to this effect—that a reflected vibration may have its character determined wholly or partly by the refractive power of the reflector, or, more generally, by the specific properties of the reflecting body in relation to transmitted light. I may adduce Brewster's law of the polarizing angle, also Fresnel's theory of reflection from glass &c., a theory which is still accepted and applied in delicate photometric work as affording a good expression of facts, and which treats refraction and reflection as closely related parts of one dynamic whole. I may adduce also the laws of reflection from the surfaces of Iceland spar and other birefringent bodies. It is true that in this last case the catoptric effects are extremely faint in comparison with the dioptric—a fact which is clearly unfavourable to the proposed case of reflection from iron, as contrasted with the resolved cases of transmission through

---

[*] Communicated by the Author.

heavy glass &c. But I think that the following facts bear with equal or greater force the contrary way.

(3) The enormous differences (in relation to magnetic force) between iron and steel on the one side, and Faraday's transparent diamagnetics on the other.

(4) The effects obtained by Verdet in his application of Faraday's magneto-optic method to the salts of iron. The strongest instance is that of the perchloride. A dense solution of perchloride of iron in wood-spirit gives a rotation of light contrary to, and nearly twice as great as, that given by heavy glass under the same conditions.

(5) The known laws of metallic reflection, particularly the fact that silver, zinc, steel, and other metals are distinguished from each other in a perfectly definite manner as reflectors, each metal having specific relations to the principal component vibrations (perpendicular and parallel to the plane of incidence) with reference both to change of phase and change of amplitude.

The preceding facts were sufficient to suggest a plan of procedure, as well as to give me a strong expectation of success.

During the month of August last, in the course of some careful experiments in the direction thus indicated, I obtained several interesting results which appeared conclusive. Soon afterwards I gave a description of the experiments before the British Association. Since that time I have made one or two additional observations, and have got rid of a serious error into which I had fallen in my first view of the facts. In this paper I propose to give an account of all my principal experiments and views upon the subject. And first, for future reference, I shall lay down the sum of the results in one sentence.

The new Fact.

2. When plane-polarized light is reflected regularly from either pole of an electromagnet of iron, the plane of polarization is turned through a sensible angle in a direction contrary to the nominal direction of the magnetizing current; so that a true south pole of polished iron, acting as a reflector, turns the plane of polarization righthandedly.

Apparatus and Arrangements.

3. *The Magnet.*—This is an upright horseshoe electromagnet, and a very good instrument, I think, of its size. Only one limb of the horseshoe is used at a time, the current being sent through one of the coils, and the observations being made on the enclosed core. Each of the cores is a solid cylindrical bolt of soft iron, 10 inches long and 2 inches in diameter, which is therefore the diameter of each polar surface. Each

of the coils weighs 14 pounds; and the wire makes about 400 turns. The particular coil employed in any case is put into circuit (generally as a double wire of 200 turns) with a small Grove's battery of only six cells; and this is the highest power applied in my experiments. In the circuit is placed also a commutator, which is at my hand, so that, while I watch the polariscope, I have the magnetic state of the core under perfect control.

4. *Polar surfaces.*—These were originally well planed, and perpendicular to the axes of the cores. For the present purpose they had to be smoothed and brightened by polishing, a process which I found troublesome and excessively tedious, from the refractory nature of the material. The polishing was done with fine dry emery powder, applied by chamois leather to one of the surfaces, and by a rubber of fine silk stuff to the other. Each rubber was backed by a flat and smooth block of iron, which was worked carefully by hand over the end of the core. The last stage of the polishing was similar to the earlier stages, but without new additions of emery. When the process was finished, each polar surface (though not such a speculum as would satisfy an optician) acted as a pretty good plane metallic mirror, its plane perpendicular to the axis of the core. Placed in a room in ordinary daylight, each mirror gave good regular images of all surrounding objects that were in any degree illuminated; and in a darkened room, the image of a neighbouring candle-flame was generally very good, brilliant, distinct, and sharply and truly outlined, except towards the rim of the mirror, parts not used in the observations. The surface that had been finished by chamois leather was rather more brilliant than the other, but not so perfectly well planed.

I should say here that, from all that I have seen in these experiments and in some earlier trials, I consider the finest attainable polish very desirable. In my present apparatus, I would prefer a much finer polish to any increase whatever of magnetic power (3).

5. *Placing of the pieces.*—The electromagnet is placed on a solid table, near the edge, and is inclined with its polar surface towards the light by means of a small block placed under the stand. The source of light is a paraffin-flame, narrow and very brilliant, distant a foot or less from the polar surface. Close to the flame stands the first Nicol. The beam of plane-polarized light so rendered is incident horizontally (at an angle of 60° to 80° to the normal) on the polar surface, and is regularly reflected. On this side of the polar surface, a few inches distant, comes the second Nicol, which is supported on a lateral

stand, and so placed that, when I look fairly through it, I see the image of the flame in the iron mirror.

6. *Principal azimuths of first Nicol.*—As the polariscope is worked here in the usual way, by restoration from the best possible extinction, there are only two positions of the first Nicol which are suitable to start from. The plane of polarization of the light incident upon the iron mirror must be either parallel to the plane of incidence or perpendicular to it, because in every other case the reflected light is elliptically polarized and therefore inextinguishable by the analyzer. I generally make the plane of polarization coincide with the plane of incidence; and I manage this in the first place very approximately by trial. I lay the first Nicol with its principal section sensibly horizontal. Looking through the second Nicol, and watching the image of the flame in the polar mirror, I turn the second Nicol quickly through the position of minimum intensity backwards and forwards, while the first Nicol is turned slowly, also backwards and forwards, until I obtain a minimum-intensity zero.

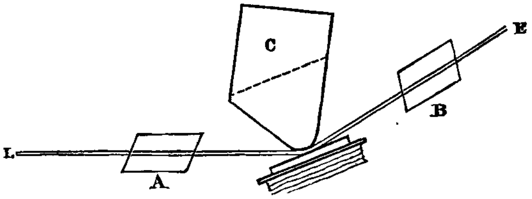
It is a matter of capital importance in the experiments to have the Nicols placed in this position of pure extinction; and the arrangement is not so easily made as might be supposed. Perhaps it is from imperfection of polish, and perhaps from the very nature and structure of the reflecting metal; but whatever be the reason, the mirror is never perfectly black in the polariscope; and though the intensity of the illumination is very faint when the Nicols are in exact position, it is still sufficient to embarrass the observer's judgment when he has to decide between pure extinction and impure. The difficulty can be overcome by a simple and regular process, as will be seen immediately. In the mean time I assume that we can obtain a pure initial extinction in the polariscope.

7. *Submagnet.*—I have now mentioned every thing that is of any importance in the arrangements, except one condition, without which I have never obtained any optical effect; and that is, an intense concentration of magnetic force upon the iron mirror. For this purpose I employ a block of soft iron, one of several polar pieces belonging to the magnet, 2 inches square and 3 inches long, which has been planed off at one end into a blunt wedge with well-rounded edge. Two splinters of hard wood, which have been thinned and toughened by hammering, are laid upon the sloping polar surface about an inch apart, and parallel to the plane of incidence. Holding the wedge in my left hand, I plant it edge downwards upon the splinters, with its rounded edge perpendicular to the plane of incidence, and right above the centre of the mirror. The

effect of this arrangement is, that when the circuit of the magnetizing current is closed, there is a very powerful concentration of magnetic force upon the mirror, and particularly on that part of it which is utilized optically in the experiments, on so much of it, namely, as the chink between wedge and core leaves exposed, on one side to the lamp, and on the other side to the observer's eye. The lines of magnetic force are sensibly perpendicular to the reflecting surface. The iron mirror is a true polar surface; and its intensely contrasted states, as north, south, neutral, are perfectly under control through the commutator.

The wedge intercepts a large part of the image of the flame. The pieces are generally so placed that the part left of the image is a strong middle segment, both top and bottom being cut off. The object now watched in the polariscope is a broad streak of light, crossing the chink at right angles from top to bottom, very sharply defined, and perfectly suitable as an object in delicate polariscopic work.

Splinters of different thicknesses are employed in different experiments, and in variations of one experiment. My only rule is, that the chink between block and core be as narrow as the requirements of the optical observation will allow. On an average, the width of the chink in the following experiments is about $\frac{1}{20}$ of an inch. The arrangements now described (3 .. 7) are shown simply in the adjacent diagram.



L is the source of light, E the observer's eye, A and B the first and second Nicols, C the wedge of soft iron.

8. *First experiment.*—The pieces arranged as in the diagram, the chink between block and mirror as narrow as possible, the plane of polarization of the light incident on the polar mirror parallel or perpendicular to the plane of incidence, and the second Nicol turned into the position of pure extinction. The observer now watches the chink through the second Nicol, and works the commutator. When the circuit is closed, the streak of light immediately reappears. The effect is very faint at the best; but it is very distinct and perfectly regular, unless the apparatus is in some way out of order, the mirror dimmed, or the battery working below its average power. Under ordinarily

good conditions, at the instant when the circuit is closed, the light shows itself faintly in proper form, size, and position across the formerly uniform chink, and so continues without sensible change as long as the current passes. Break, and the light immediately disappears. Reverse, and the light again appears and continues till the instant of break, when it disappears at once.

The beam reflected by the mirror of magnetized iron is certainly not plane-polarized, as is the incident beam (and the reflected beam also before magnetization); for when the light is restored by magnetic force from pure extinction as above, it cannot be extinguished by any rotation of the second Nicol in either direction; nor (as far as I can judge of these faint effects and with the present means) is the light sensibly weakened by any such rotation. The analyzer's position of extinction before magnetization is also (exactly or nearly) the position of minimum intensity after magnetization.

In many repetitions of this experiment, the angle of incidence varied from 60° to 80°, and was generally about 75°.

9. At this point I must ask the reader's attention to several terms, and to the sense in which I shall use them. By a rotation of the first Nicol to the right, I mean a rotation which is right-handed (like the motions of the hands of a watch which faces the observer) when viewed from the point of incidence on the iron mirror. By the north pole of a magnet, I mean "that which points, on the whole, from the north, and, in northern latitudes, upwards"*.

10. *Second experiment.*—Taking this experiment as a continuation of the first, and providing for the best effects, I suppose all the arrangements as before: I suppose also that make, break, and reverse of the commutator give bright, black, bright in the polariscope distinctly, however faintly.

(1) Leaving the circuit open and every thing else untouched, I simply turn the first Nicol ever so little to the right. The amount of the rotation is important. I have said it was ever so little; and this generally gives effects distinct enough. But when working for the best results, I determine the displacement of the first Nicol by this condition, that the intensity of the light restored in the polariscope by the displacement be

---

* Sir W. Thomson's papers on Electrostatics and Magnetism, § 445. It will be seen from the quotation that this is no innovation of mine. Having had this nomenclature brought to my attention recently by Sir William Thomson and very strongly recommended by him, I made it a matter of careful consideration and have determined to adopt it. Like poles of the great Earth-magnet and of our artificial magnets ought to be similarly named; and the northern pole of the Earth-magnet cannot with any propriety be called a south pole.

sensibly equal to that of the light restored formerly by magnetization in the first experiment. This being done, I watch the faint light in the polariscope, and work the commutator as formerly. But I must now specify the magnetic states of the mirror.

When the mirror becomes a north pole, the light flashes up at once to a sensibly higher intensity, which is sustained without change as long as the current passes. When the circuit is broken and the mirror demagnetized, the light falls at once from the higher intensity to the primitive faint intensity, and so continues as long as the circuit is open. When the mirror becomes a south pole, the light falls from the primitive faint intensity, down either to perfect extinction or extremely near it. In favourable cases of this kind (that is, in cases properly managed and in a well-darkened room) it is very striking to look at the chink through the analyzer, searching in vain for the faintest trace of the streak of light, and remembering the displacement of the first Nicol. When the circuit is finally broken, the light reappears at once as at first.

(2) Leaving the circuit open and every thing else untouched, I watch the faint light in the polariscope, and turn the first Nicol backwards to the left, into the position of extinction and a little beyond it, regulating the amount of rotation by the intensity of the restored light as in the first case. I now watch the light through the analyzer and work the commutator. It would be superfluous to describe the magnetic changes of the iron mirror, and the corresponding changes in the polariscope; the description would be word for word as before, with one essential alteration. It is the south pole that now strengthens the light, and the north pole that extinguishes or weakens it.

This experiment is much more easily managed than the first. Let a good sensible extinction of the streak across the chink be obtained by optical trial in the manner already described (6), the plane of polarization of the incident light being either parallel or perpendicular to the plane of incidence; and let the first Nicol be turned to the right, so far only as to render the extinction sensibly impure. When the three states of the mirror (north, neutral, south) are now made to succeed one another rapidly, the contrast of bright, faint, dark in the polariscope comes out in almost every case very distinctly.

Very often I have seen the second experiment give clear effects as now described, in cases where, through partial exhaustion of the battery, the first experiment gave no sure effect whatever.

11. I have given these two experiments as a simple and exhaustive summary of a large number of observations which

were at first very perplexing, so irregular and apparently inconsistent were the phenomena. The chief cause of my perplexity I found afterwards to be a very interesting thing; and that was what I may truly call the exquisite delicacy of the magnetic mirror as a test for fixing the position of the plane of polarization of the incident light. One or two simple notes of actual observations will illustrate this point more distinctly than any general statement could do.

Things often happened thus. Working as in the first experiment and with ordinary caution, I started from good extinction, and found the north pole restoring the light, and the south pole much the same as open circuit. Trying to obtain better initial conditions if possible, I threw the two Nicols well out of position, and worked them carefully back to good extinction; and now, without any other observable change in the conditions, I found things reversed, the south pole clearly restoring the light, and the north pole much the same as open circuit. Here the magnetic mirror simply detected the impurity of the initial extinction, and characterized it, by strong contrasts of intensity in the polariscope, as due to a slight misplacement of the first Nicol (otherwise barely or not at all detectable), to the right in the first case, and to the left in the second.

Working sometimes with one of the mirrors (that which had been polished by chamois leather, and which was not so well planed as the other), at a particular part of its surface, and at large angles of incidence, I found the upper end of the streak clearly restored by the north pole and the lower end not, while the lower end of the streak was clearly restored by the south pole and the upper end not. There can be no doubt that in this case the magnetic mirror detected a slight difference of slope at those parts of the mirror which reflected the upper and lower ends of the streak. Say that the one part sloped a little downwards to the left, and the other a little downwards to the right; then the planes of incidence at the two places would be out of coincidence with the plane of polarization of the incident light, to the left in the first case, and to the right in the second.

Similarly, I have sometimes seen the right side of the streak restored by the north pole and the left side not, while the left side was restored by the south pole and the right side not. Irregularities and inconsistencies of this kind were explained perfectly by the second experiment as soon as it was discovered.

Finally, I observe here that the arrangements for the first experiment are best obtained through those for the second; and this is a point of some practical importance. Arranging the apparatus as for the first experiment and with the greatest care, I find the effects of the two magnetizations unequal in almost every case. Say that the north pole restores distinctly, and the south pole weakly or not at all. Leaving the circuit open, I turn the first Nicol to the left as little as possible, and then bring the second Nicol into the position of extinction, and test by working the commutator and watching the light in the polariscope. Several careful operations of this kind are sometimes requisite.

Summary and Interpretation of the facts.

12. In these experiments light is reflected from an iron mirror at an incidence of 60° to 80°, passing through a first Nicol before reflection and through a second Nicol after.

Initial conditions.—The iron mirror unmagnetized, the principal sections of the two Nicols perpendicular and parallel respectively to the plane of incidence.

Essential operations.—Starting thus from pure extinction in the polariscope, we apply any one or two of four operations. Two of these are merely mechanical, extremely small rotations of the first Nicol from its initial position, a right-handed rotation (R) and a left-handed (L). The other two are physical, intense magnetizations of the mirror, as a north pole (N) and as a south pole (S). These four operations will be named here and afterwards by suggestive and easily remembered letters as above; and they will be grouped in pairs invariably, R and N together, thus:

(R, N), (L, S).

13. When any one of the operations is applied singly, the light is restored from pure initial extinction in the polariscope.

When any two of the operations are applied simultaneously, and their relations determined by comparison of effects in the polariscope, they are found to be conspiring operations if they belong to the same pair, and contrary operations if they belong to different pairs. Considering, then, any one of the operations, we see that there are two ways of strengthening its effect in the polariscope, and two ways of weakening it. To strengthen the effect of R, apply either operation of the pair (R, N); turn the first Nicol a little more to the right, or magnetize the mirror as a north pole. To neutralize or weaken the effect of R, apply either operation of the pair (L, S); turn the first Nicol a little to the left, or magnetize the mirror as a south pole.

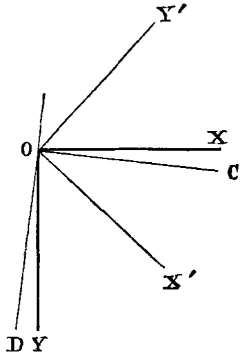
To obtain a complete interpretation of the facts, we have only to assume that the immediate optical effects of the four operations (R, N), (L, S) are similar in kind for all, and similarly directed for those of either pair, but oppositely directed for different pairs. R and L turn the plane of polarization; so therefore, according to this view, do N and S. R and N turn the plane of polarization in one direction; L and S turn it in the contrary direction. But even from an optical point of view there is still an important difference between the mechanical operations and the physical; for in the one case (R or L) the full effect of the operation is impressed upon the light before incidence, while in the other case (N or S) the effect is impressed somewhere and somehow in the very process of reflection.

To get a more definite statement of this interpretation, consider the pair of conspiring operations (R, N). In the case of operation N, and to an eye which looks into the polar mirror, the nominal direction of the magnetizing current round the core is right-handed (9). In the case of operation R and to the same eye, the direction of rotation of the plane of polarization, or the direction of rotation of the trace of that plane upon the reflecting surface, is evidently left-handed (9). We infer that a right-handed current gives a left-handed rotation of the plane of polarization. And this completes the first experimental proof of the general statement made in art. 2.

14. To test the truth of this view of the facts, I thought of three methods which appeared accessible:—first, to apply each of the four operations (R, N), (L, S), and to characterize them separately by definite compensating actions in the polariscope; secondly, to apply the operations N and S in combination with small permanent rotations of the second Nicol; thirdly, to return to the case of perpendicular incidence, which I had already tried roughly without success. I shall prepare the way for an account of the first of these methods by a short mathematical discussion.

Compensation of effects of operation R.

15. Let the angle of incidence be about 75°; and suppose that the initial conditions are as in the first and second experiments (12), and particularly that the direction OX of the vibration is perpendicular to the plane of incidence. The operation R being now applied, and the incident vibration being turned thus through a small angle $XOC=\alpha$, it is required to find the character of the reflected light, particularly with a view to compensation. The two rectangular components (one in OX) of the incident vibration (c in OC) are



$$c \cos \alpha \cos 2\pi \frac{t}{\tau},$$

$$c \sin \alpha \cos 2\pi \frac{t}{\tau},$$

or, more briefly,

$$a \cos \theta \text{ and } a' \cos \theta,$$

where θ is proportional to t. Let O Y be perpendicular to O X and to the reflected ray; then, to obtain the components x and y of the reflected vibration in the directions O X and O Y, we must apply to the preceding components the known laws of metallic reflection. We find thus

$$\left.\begin{array}{l} x = ha \cos \theta, \\ y = ka' \cos (\theta - \phi), \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where h and k are constants characteristic of the reflecting metal. As the angle of incidence is about 75°, and therefore very near the principal incidence, we may put

$$\phi = \tfrac{3}{2}\pi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Substituting in (1), and representing the amplitudes by b and b', we find

$$\left.\begin{array}{l} x = \quad b \cos \theta, \\ y = -b' \sin \theta. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From these equations or otherwise we see that the reflected vibration is elliptic, and that its principal rectangular components are perpendicular and parallel respectively to the plane of incidence. We see also that the elliptic polarization is left-handed in the case of operation R, and right-handed in the case of L.

Hence a simple method of compensating the effect of the operation R, or of the rotation α of the incident vibration. Introduce a difference of phase $\frac{\pi}{2}$ between the components x and y by means of a quarter-wave plate, and then turn the second Nicol in the proper direction through a small angle which is definitely related to α. This method I have not had an opportunity of trying.

To find another method. Let the elliptic vibration (3) be represented by its rectangular components x' and y', in directions O X′ and O Y′ inclined at 45° to O X; and let

$$x' = m\cos(\theta - \beta),$$
$$y' = m\cos(\theta - \gamma).$$

Identifying the second members of these equations with the proper sums of resolved parts of x and y, we find easily

$$m^2 = \tfrac{1}{2}(b^2 + b'^2),$$
$$\tan\beta = -\frac{b'}{b}, \quad \tan\gamma = \frac{b'}{b}.$$

And therefore, if δ be determined by the equation

$$\tan\delta = \frac{b'}{b} = \frac{ka'}{ha} = \frac{k}{h}\tan\alpha, \quad . \quad . \quad . \quad . \quad (4)$$

we see that, finally,

$$\left.\begin{aligned} x' &= m\cos(\theta + \delta), \\ y' &= m\cos(\theta - \delta). \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

By any adequate action upon the reflected ray at any point between the iron mirror and the analyzer, let the component x' be retarded relatively to y', so as to undergo a relative change of phase equal to 2δ. As the components x' and y' have already equal amplitudes, and are equally inclined to O X, it is evident that by this change of phase of x' the elliptic vibration (5) is transformed into a rectilinear in the primitive direction O X. And thus the compensation of effect of the operation R is fully effected, without displacement of the second Nicol.

If we assign to $\frac{k}{h}$ the value $\frac{1}{2}$, which is probably near the truth, as its value in the case of steel, measured both by Jamin and by Senarmont, lies between ·5 and ·6, and if we give effect to the condition that α is a very small angle, we find from equation (4), approximately,

$$2\delta = 2\tan^{-1}.\frac{k}{h}\tan\alpha = 2\frac{k}{h}\alpha = \alpha.$$

However, it is sufficient for our present purpose to observe that the compensating change of phase 2δ is a small quantity determined by α, and of the same order as α, and also of the same sign.

16. *The Compensator.*—This is a slip of plate glass held in the hands, and strained either by flexure round its thickness, or by simple tension or compression from the two ends. In the present experiments the slips used were of the best plate,

$\frac{1}{8}$ inch thick, $\frac{5}{8}$ wide, and $7\frac{1}{2}$ long, chosen carefully so as to be quite inactive in the polariscope while unstrained. Suppose one of these slips placed between the mirror and the second Nicol, its surface perpendicular to the reflected ray, and its length parallel to O X′; and let the glass be stretched in the direction of its length. Stretched glass acts upon transmitted light as a positive uniaxal with its axis parallel to the line of extension. In this case, therefore, the extraordinary component x' is retarded relatively to the ordinary y'; and the method found in the last article gives us this simple rule:—

To compensate the effect of a small operation R or L, the incident vibration being initially directed along O X, at right angles to the plane of incidence, and the reflected vibration being initially cut off by the second Nicol. Leaving the second Nicol in its initial position, and placing the compensating slip between the mirror and the second Nicol, its plate faces perpendicular to the ray, and its length parallel to O X′; stretch the slip along its length in the case of R, and compress it along its length in the case of L.

The direction O X′ will be taken as the standard direction of strain: it is at 45° to the plane of reflection, right hand down.

17. *Third experiment.*—All the arrangements are as in the first experiment, the angle of incidence about 75°, and the extinction in the polariscope perfect. As the experiment is a purely optical one, the circuit is kept open. To ensure uniformity of optical conditions, the block C is kept in position as in the diagram of (7), and the light is viewed through the chink as formerly.

(1) The first Nicol is turned righthandedly through a very small angle, and the light is distinctly restored. The compensating slip is introduced between the mirror and the second Nicol in the manner which has just been fully described. When the slip is stretched along its length, say between closely gripping finger and thumb at each end, with a force which increases continuously from zero up to a certain small value, the light restored by displacement of the first Nicol fades away to pure extinction, reappearing and brightening as the tension increases. When the slip is submitted to a longitudinal compression which increases continuously from zero, the light increases continuously and very distinctly from beginning to end of the increase of compression.

(2) The first Nicol is now turned to the left, through the position of extinction, and the light distinctly restored; and the compensator, kept always in the standard position, is stretched and compressed as formerly. Things are precisely as in the first case, except that the effects of tension and com-

pression are reversed, and therefore interchanged. It is now compression that extinguishes the light; tension strengthens it from first to last.

When the angle of rotation of the first Nicol is too large, which it may be while still very small, the neutralization by tension or compression is incomplete, the light fading to a very sensible minimum and then increasing; but the extinction is still perfect when the initial intensities have reached much greater values than those obtained by magnetization in the first experiment.

I found the present experiment a very interesting one, from the simplicity of the means, the purity of effects, and the beautiful distinctness of the contrasts. However, I do not give the experiment here for its own sake. The only use of it is to characterize the effects of R and L in the polariscope; and this work it does perfectly.

18. *Fourth experiment.*—The angle of incidence about 75°, and all the arrangements and procedure as in the first experiment, with addition of the compensator. As the intensity in the polariscope is very faint at the best, all proper means are adopted for increasing it—the room well darkened, the battery in good order, the surface of the mirror fresh, the chink between wedge and core merely wide enough to give a good object, and the initial extinction sensibly perfect.

When the light is restored from pure extinction by the operation N, and the compensator is placed and strained as in the third experiment, the light is weakened by tension and strengthened by compression, and the weakening by tension proceeds to pure extinction. The effect of the operation S is, on the contrary, weakened to extinction by compression, and strengthened from first to last by tension.

This is the general result; but some precautions had to be taken in the actual experiment. Sometimes heat from the hand, possibly also from the breath, gave rise in the compensating slip to strains which had large effects in the polariscope, effects larger indeed than that to be compensated. In such a case the slip was laid aside and a fresh one employed. It was found necessary also to keep the plate faces of the slip accurately perpendicular to the reflected beam, as a very small displacement from this direction gave a noxious effect. Observing these and other precautions, and working with proper care, I found after some practice that the phenomena were perfectly under control. Sitting down in front of the polariscope, and getting an assistant to hold the submagnet and work the commutator, I bring the compensator suddenly into the standard position, and find the extinction still pure. The circuit is now closed, and the light reappears through the compensator. I now compress the slip along its length with a force increasing slowly from zero, and find that the light increases continuously and very distinctly as the compression increases. I therefore pronounce the mirror a north pole, which the assistant finds right. To verify by the polariscope, I stretch the slip with a force increasing slowly from zero, and find that the light fades to pure extinction and then brightens. The effects are very faint, but quite unmistakable. In the last-mentioned case, for instance, I put the light out of sight by careful increase of the tension up to a certain small value, and keep it out of sight as long as I please by sustaining the force, straining my eye all the time to catch the faintest glimpse, till the instant when the slip is relieved of strain without change of position; and then the light reappears as at first. Working similarly in another case, I find these optical effects of tension and compression interchanged, compression extinguishing the light and tension strengthening it; and the mirror is found accordingly to be a south pole.

19. *Fifth experiment.*—This is a repetition of the second experiment with addition of the compensator; it is more easily managed than the fourth; and the results are equally convincing. In the first half of the second experiment, as already described (10), the three sets of operations applied successively were

$$(R, N), \quad R, \quad (R, S);$$

and the intensities in the polariscope in the three cases respectively were bright, faint, dark.

When the effects in the first and second cases are tested by the compensator, exactly as in the third and fourth experiments, they are both compensated to pure extinction by tension, and both strengthened from first to last by compression. And similarly in the second half of the second experiment, the single effect of L and the joint effect of L and S are both strengthened by tension, and both weakened down to sensible extinction by compression.

20. Summary of the results obtained in the last three experiments.

The effects of the operations R and L in the polariscope are compensated respectively by tension and by compression of glass in the standard direction: the effect of N is compensated precisely as that of R, and the effect of S precisely as that of L; the joint effect of R and N is compensated precisely as the separate effects of R and N, and the joint effect of L and S precisely as the separate effects of L and S: and in all these cases the compensation proceeds to sensible extinction.

The four operations (R, N), (L, S) were found in the second experiment to be related to one another, two and two, as conspiring or contrary; they are now seen to be related to one another more generally, and in the same combinations, as like or unlike. With reference to effects in the polariscope, the operations R and S are as clearly unlike as are the operations R and L, or the operations N and S; and, on the other hand, and with reference always to effects in the polariscope, R and N are as clearly like as are any two operations R, or any two operations N. It was assumed, in explanation of the facts brought out in the second experiment, that the optical effects of the four operations (R, N), (L, S) are the same in kind for all, and similarly directed for those of either pair, but oppositely directed for those of different pairs. All the new facts agree with this hypothesis and confirm it.

It has been observed already that the effects of the operations R and L are fully impressed upon the light before incidence, while the effects of N and S are impressed in the process of reflection; but, as far as we can judge from the present experiments (17, 18, 19), and as far as changes of phase of the principal components are concerned, this difference between the mechanical operations and the magnetic has little influence upon the final effect in the polariscope. We may therefore assume provisionally that, as far as changes of phase by metallic reflection are concerned, the rotation due to magnetic force is impressed effectively before incidence. We come now to the second method proposed in 14.

21. *Sixth experiment.*—Angle of incidence about 75°, initial arrangements as in the first experiment, plane of polarization of the incident light sometimes parallel and sometimes perpendicular to the plane of incidence, initial extinction as pure as possible.

(1) Leaving the first Nicol untouched, I turn the second Nicol righthandedly through a very small angle; and watching the faint light thus restored, I work the commutator as formerly. The operation N strengthens the light; and this effect is distinct and regular. The operation S has sometimes no effect, and sometimes weakens the light, always less distinctly than N strengthens it, and generally less and less distinctly as the rotation of the second Nicol is diminished.

(2) The second Nicol is turned to the left from its initial position through a very small angle. N and S now interchange effects; but otherwise the phenomena are as in the first case.

The effects obtained in repeated and careful trials were, with few exceptions, as I have now described them; but they were neither so strong nor so pure as those obtained in the second experiment. The strengthening actions of N in (1) and of S in (2) are evidently what was to be expected; for in (1) the second Nicol leaves the plane of polarization a little to the left, and N turns that plane a little more to the left. But the whole subject deserves a more particular discussion.

22. To find the intensity of the light which reaches the observer's eye in the sixth experiment.

Suppose the incident vibration directed along O X (figure of art. 15), at right angles to the plane of incidence. When the second Nicol is turned (righthandedly) through a very small (positive) angle $\mathrm{Y\,O\,D}=\epsilon$, the resolved part of the reflected vibration (of amplitude 1) in the direction O D has an amplitude $-\sin\epsilon$ or $-\epsilon$, and the intensity of the light transmitted to the eye is ϵ^2.

The effect of an additional operation S is to turn the primitive vibration out of the direction O X through a very small (positive) angle ρ, or to add to the primitive vibration in O X a very small vibration, of amplitude $\sin\rho$ or ρ, in a direction perpendicular to O X. There are therefore two vibrations presented now to the second Nicol—one in O X and sensibly of amplitude 1 as before, the other in O Y and of amplitude $k'\rho$ or ρ', where k' is a positive number less than 1, an unknown function of the angle of incidence. According to the hypothesis advanced in the end of art. 20, the difference of phases of these components has the same value ϕ as if the component ρ' in O Y were due to an operation R or L. The resolved parts of these components in the direction O D of transmission have amplitudes $-\sin\epsilon$ and $\rho'\cos\epsilon$, or $-\epsilon$ and ρ'; the intensity of the transmitted light is therefore equal to

$$\epsilon^2+\rho'^2-2\epsilon\rho'\cos\phi.$$

23. Before discussing this formula, I proceed to apply similar considerations very briefly to the second experiment. Suppose the direction O X of the primitive vibration still perpendicular to the plane of incidence, and that positive angles are still those due to righthanded rotations. If two operations, L and S, be applied simultaneously, the vibration is turned through a small angle α before incidence, and through a small angle ρ in the process of reflection. The amplitudes of the small reflected vibrations thus generated in the direction O Y of transmission may be represented by α' and ρ', where α' is the $k\alpha'$ of equations (1) of art. 15, and ρ' is the same as in art. 22. According to the hypothesis stated in art. 20, these vibrations are reflected in the same phase, and the intensity of the transmitted light is therefore equal to $(\alpha'+\rho')^2$

To apply this result to the first half of the second experiment. By trial we give to α such a value that sensibly $\alpha'=\rho'$, and then apply successively the three sets of operations

$$(\mathrm{R, N}),\quad (\mathrm{R}),\quad (\mathrm{R, S}).$$

The corresponding intensities in the polariscope are

$$(-\alpha'-\alpha')^2,\quad (-\alpha')^2,\quad (-\alpha'+\alpha')^2,$$

which are as the numbers 4, 1, 0. The actual results, as already specified, were bright, faint, black (10).

24. Returning to the sixth experiment. In discussing the expression

$$\epsilon^2+\rho'^2-2\epsilon\rho'\cos\phi,$$

found in art. 22, I shall suppose the rotation of the second Nicol always righthanded, or the angle ϵ always positive. The amplitude ρ' is positive for S, negative for N. The angle ϕ varies continuously with the angle of incidence, from zero at normal incidence, through $\frac{\pi}{2}$ at principal incidence (75° or 76°), up to π at grazing incidence. It will be observed that the $\frac{\pi}{2}$ at principal incidence in the present case is the $\frac{3}{2}\pi$ of equation (2) of art. 15, diminished by the π of reversal due to reflection.

(1) When the value of the angle of incidence is considerably less than 75°, $\cos\phi$ has some positive value c, and the additions made to the primitive intensity ϵ^2 by the operations N and S are

$$\rho'^2+2\epsilon\rho'c \text{ and } \rho'^2-2\epsilon\rho'c.$$

In this case, therefore, the effect of N in the polariscope is always an increase, and always more pronounced than the effect of S.

Let ϵ' be the value of ϵ which is determined by the equation

$$\rho'-2\epsilon c=0.$$

When $\epsilon=\epsilon'$, the effect of S in the polariscope is reduced to zero; when $\epsilon<\epsilon'$, the effect of S is a small increase; when $\epsilon>\epsilon'$, the effect of S is a decrease.

(2) When the value of the angle of incidence is considerably greater than 75°, $\cos\phi$ has some negative value $-e$, and the additions made to the primitive intensity ϵ^2 by N and S are

$$\rho'^2-2\epsilon\rho'e \text{ and } \rho'^2+2\epsilon\rho'e.$$

Here, therefore, contrary to what holds in the first case, the effect of S is always an increase, and always more pronounced than the effect of N. Here also, as ϵ increases from zero, the addition made to ϵ^2 by the weaker magnetic operation passes from positive, through zero, to negative.

(3) In the case of principal incidence, $\cos\phi=0$, and the additions made by N and S to the primitive intensity ϵ^2 in the polariscope are equal and always positive.

25. *Seventh experiment*, a repetition of the sixth, to test the preceding inferences.

(1) Angle of incidence about 70°. All the effects recovered as predicted, and as already obtained roughly in the sixth experiment. Recovered also perfectly at various incidences from 60° to 75°.

(2) Angle of incidence very large, about 85°. No sensible effect obtained in any case by application of the operations N and S, with the arrangements of the sixth experiment, or with those of the second. The reason very probably is that, as the angle of incidence approximates to 90°, the ratio of the amplitudes ρ' and ϵ becomes excessively small, by diminution of the rotation ρ towards zero.

Angle of incidence about 80°. The effects very faint, but clearly contrary to what was predicted: N strengthens the light as in the first case; S either weakens it or has no effect.

(3) Equal positive effects of N and S in the polariscope were never observed at 75° or any other incidence. The hypothesis advanced in 20 is therefore inexact: the rotation due to magnetic force is not impressed effectively before incidence. Neither is it impressed effectively after reflection (10...19). The difference of phases of the two reflected vibrations, ρ' in O Y, and 1 in O X, has therefore some value $\lambda\phi$ intermediate between ϕ and 0; and the intensity in the sixth experiment is equal to

$$\epsilon^2+\rho'^2-2\epsilon\rho'\cos\lambda\phi.$$

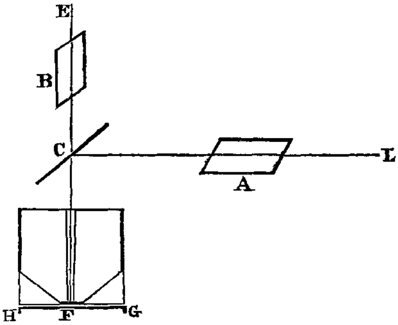
Judging from the earlier experiments, second to fifth, I think we are bound to assume that λ is very nearly equal to 1: but certainty upon the subject can be reached only through exact measurements. I come now to the third method mentioned in art. 14.

Case of Perpendicular Incidence.

26. *Submagnet.*—The old wedge C of art. 7 is now inadequate. The piece which I substitute for it is a block of soft iron, 2 inches square and 3 inches long, rounded at one end into a frustum of a very obtuse cone, of which the small base is hardly $\frac{1}{2}$ inch in diameter. A small boring is drilled through the block, and along the axis of the cone, narrowing regularly from $\frac{1}{4}$ inch at the flat end of the block to $\frac{1}{12}$ inch

at the conical end. The surface of the boring is well dimmed with a coating of lampblack. To ensure perfect stability of position when the piece rests upon its conical end, the original rectangular volume of the block was restored, the part added being a hard stone of plaster of Paris, which was easily moulded to the block of iron in the usual way. This is the first submagnet that gave me good and constant effects in the case of normal incidence; and it appears to be much the best that I have yet tried. Without a submagnet of some kind, I have never obtained a suspicion of an effect.

27. *Placing of the pieces.*—The old magnet (3) is placed on a solid table near the edge, with its polar surfaces horizontal; and the submagnet just described is laid upon one of the polar surfaces, its conical end downwards, the axis of core and boring coincident, and the block and core separated by a wide ring of writing-paper, or very thin card. Above the block, as in Norremberg's polariscope, stands a mirror of unsilvered glass, which receives a horizontal beam from the first Nicol, and reflects it downwards through the boring, perpendicularly to the surface of the magnetic mirror. In the diagram, H F G is the polar surface, L the source of light, which is the same paraffin-flame as formerly, E the observer's eye, A and B the first and second Nicols, C the transparent mirror. The course of the light from L to E is L A C F C B E. All the pieces are placed very stably, and the room is well darkened.



28. *Eighth experiment.*—All the pieces are placed as in the diagram, and so that the observer sees at F, through B, a bright and steady image of part of the flame L: the first Nicol is so laid that the plane of polarization of the light incident at C coincides with the plane of incidence; and the second Nicol is turned into the position of pure extinction.

(1) The second Nicol is turned righthandedly through a small angle, giving a distinct but faint restoration. The operation N strengthens the light thus restored; and the operation S weakens and sometimes extinguishes it.

(2) The second Nicol is turned lefthandedly through a small angle beyond pure extinction. The results are as in the first case, with reversal of actions of N and S. It is now

S that strengthens the light, and N that weakens or extinguishes it.

The phenomena now mentioned are very faint, a good deal fainter than those obtained in the second experiment; but they are certain, distinct, and perfectly regular. I need hardly say that this experiment is decisive, and that the effects are certainly due to rotations virtual and actual of the plane of polarization of the light which is presented to the analyzer, the virtual rotations being produced by displacements of the second Nicol, and the actual by the operations N and S. N conspires with a righthanded rotation of the second Nicol; and therefore N turns the plane of polarization to the left: S conspires with a lefthanded rotation of the second Nicol; and therefore S turns the plane of polarization to the right.

29. *Ninth experiment.*—No change in the arrangements, the initial extinction perfect.

(1) The first Nicol is turned righthandedly (from C as point of view) through a small angle, giving a faint but distinct restoration. S strengthens the light thus restored, and N weakens and sometimes extinguishes it.

(2) The first Nicol is turned lefthandedly, through pure extinction, to faint restoration. N strengthens the restored light, and S weakens or extinguishes it.

The phenomena are precisely as in the eighth experiment, and equally distinct and regular, but with reversed relations of S and N to movements of the Nicol: and this was to be certainly expected, because the first Nicol simply carries the plane of polarization with it, while the second Nicol simply leaves that plane behind it. As an illustration of this statement, and of the consistency of the results obtained in this article and the preceding, I offer the following optical experiment, though it will be to many of my readers unnecessary.

30. *Tenth experiment.*—The arrangements unchanged, the initial extinction perfect, the circuit kept open.

(1) First Nicol to the right, giving a faint restoration. The restored light is weakened to extinction by rotation of the second Nicol to the right; strengthened clearly, *ab initio,* by rotation of the second Nicol to the left.

(2) First Nicol to the left, giving a faint restoration. The light is weakened to extinction by rotation of the second Nicol to the left; strengthened clearly, *ab initio,* by rotation of the second Nicol to the right.

These effects are certain and regular; but sensibly perfect extinctions are obtained only in careful work, and with very small displacements of the first Nicol.

31. *Eleventh experiment.*—Starting with the same arrange-

ments as in the last three experiments, and working under the most favourable conditions attainable, I have often left the two Nicols in position at pure extinction, and tried the effects of the simple operations N and S. I have certainly got distinct effects many times in such circumstances, and assured myself that they were due to magnetizations of the iron mirror by getting them to appear and disappear at the instants of make and break of the circuit; but the effects were so excessively faint that I could not once characterize them as due to rotation of the plane of polarization. I have no doubt whatever that, with a stronger magnet and a finer mirror, and a more intense light, this experiment would be as satisfactory as any of the preceding.

32. *Twelfth experiment: influence of the Submagnet.*—The old wedge C of art. 7 has a slit sawn into it at right angles to the edge, as if to divide the block into two equal wedges. The slit is about $\frac{1}{12}$ inch wide, and terminates at the dotted line drawn across the block C in the diagram of 7. Returning to the diagram of art. 27, the bored block is removed, and the slit block put in its place, its largest plane face on the polar surface, and the slit perpendicular to the plane L C F. Block and core are separated successively by six sheets of increasing thickness, tissue-paper, thin writing-paper, drawing-paper, pasteboard, thick card-board, and a quarter-inch plank, each of the sheets being perforated properly at F, so as to expose the polar surface through the slit. All the other arrangements and the procedure are as in the eighth and ninth experiments. The old effects are obtained under these new conditions, but more faintly at the best. They are certainly strongest with the sheets of pasteboard and card-board, $\frac{1}{30}$ inch to $\frac{1}{12}$ inch thick. With the quarter-inch plank they are barely if at all perceptible. With the first and second sheets, the tissue-paper and thin writing-paper, I could catch no trace of the effects.

Summary of Experimental Results.

33. When plane-polarized light is reflected perpendicularly from the polar surface of an iron electromagnet, the plane of polarization is turned through a small angle in a direction contrary to the nominal direction of the magnetizing current.

When the light is reflected obliquely, the effect in the polariscope is mixed, partly due to magnetic force, and partly due to metallic reflection ; but in this case, as evidently as in the case of normal incidence, the action of the magnetic force is purely or chiefly photogyric, and the plane of polarization is turned always in a direction contrary to that of the magnetizing current.

The precise character of the mixed optical effect in the case of oblique incidence can be determined only by exact measurements. This much, however, appears to be clearly proved by the preceding experiments, that the rotation due to magnetization of the mirror is impressed upon the light neither effectively before incidence, nor effectively after reflection.

No effect was obtained in any case without the presence of a submagnet. I think it certain that the only use of this piece is to concentrate or intensify the magnetic force upon the iron mirror by inductive action.

The powers applied were barely adequate to produce all the effects. Some of the phenomena were quite imperceptible when the battery began to work, and afterwards, when it had worked at intervals for three or four hours. Much better effects may certainly be expected with higher electromagnetic powers and finer optical appliances.

Glasgow, 26th March, 1877.

XLIV. *Notes on the Theory of Sound. By* R. H. M. BOSANQUET, *Fellow of St. John's College, Oxford.*

[Continued from p. 278.]

2. *On the Energy per second of a Pendulum-vibration in Air.*

THE flow of energy per second along a column of air transmitting a pendulum-vibration is

$$2\rho v^3\left(\frac{\pi A}{\lambda}\right)^2, \text{ or } 2\times 1.4\Pi v\left(\frac{\pi A}{\lambda}\right)^2,$$

where ρ is the density of the air,
Π the atmospheric pressure,
v the velocity of sound,
A amplitude of vibration,
λ wave-length.

This was proved in a paper in the Philosophical Magazine, March 1873. The kinetic and potential energy were estimated separately; each is equal to half the above quantity.

This result may be obtained more conveniently by supposing a disk to oscillate in an infinite cylinder. The changes of pressure on the two sides of the disk are always equal and opposite; and the work done in any small movement is the product of the displacement by the difference of the pressures. The total work done by the disk is the energy supplied to keep up the