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XVIII. On the elasticity of brass

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mical combustion of the iron and troilite contained in the meteorite. The last point has been already considered; on the other I have only to observe that, were the hollowing-out of a meteorite due to such little whirlwinds on its surface, we should find traces of the rotatory action in a whorl-like distribution or marking of the crust. We do not find this, whereas we do find very often distinctly marked traces of the orientation of the meteorite at a given period in its course, indicated by the backward flow of the then molten crust, which, like some old lava stream *in petto*, still presents lines following the course of that flow on the surface, and a denser incrustation in the rear than on the front, from which the backward-streaming lines take their course.

The subject here discussed, however, is bound up with questions as to the original form in which meteorites enter our atmosphere, questions that have recently been discussed from two very opposite points of view by F. Mohr in Liebig's *Annalen* (vol. clxxix. pp. 257–282), and by Professor Tschermak, in the June Supplementary Number of the Philosophical Magazine for this year. I will take a nearly opportunity of asking you to record my views on both of them.

I am, yours &c.,

N. Story Maskelyne.

112 Gloucester Terrace, W.,
July 17, 1876.

XVIII. *On the Elasticity of Brass.*

By Lieut.-Col. A. R. Clarke, *C.B., R.E., F.R.S., &c.**

In the course of a recent measurement of the distance between the knife-edges of Kater's Pendulum, it became necessary to consider the question of the change of length which takes place when the position of the pendulum is changed from vertical to horizontal, and also the alteration in length caused by interchanging the knife-edges in swinging. The values of the modulus of elasticity which are found in the ordinary books of reference are given, so far as I have seen, without any statement of the authority on which they rest; and it does not appear that any precise experiments have been made in this country on the elasticity of brass, though those of iron and steel are well determined. Some circumstances having led me to doubt the accuracy of the value taken from the Table, a simple experiment on the flexure of a brass scale was made, which gave a very different value.

In order to get some precise results, I obtained four speci-

* Communicated by the Author.

mens of brass, of which the dimensions and weights are shown in the following Table:—

Rod.	Length.	Breadth of opposite faces I. and III.	Breadth of opposite faces II. and IV.	Weight.
	in.	in.	in.	lb.
No. 1.	39·85	0·2370	0·2400	0·72161
„ 2.	39·85	0·2392	0·2385	0·73018
„ 3.	39·85	0·2875	0·2127	0·76143
„ 4.	39·85	Circumference =1·0610		1·09018

Nos. 1 and 2 were cut from the same sheet of brass, but the latter was well hammered; No. 3 is cut from a different sheet. These three specimens were very carefully filed, and their dimensions are very uniform throughout. The quantities in the Table result each from ten measures at equal intervals; and the probable error of each quantity may be $\pm 0{\cdot}0003$ in. The round rod No. 4 is very fairly circular in section; it is probably drawn, not rolled. The probable error of its circumference may be taken at $\pm 0{\cdot}001$ in.

In order to ascertain their elasticity, the rods were provided with three supports, one at each extremity, and a third at the centre capable of movement by a micrometer-screw in a vertical direction. Thus, when the rod is straight, or when the centre support is in line with the others, the bar rests at the left for one fortieth of an inch of its length, and over its entire breadth, upon the brass rectangular support; the central movable support is a knife-edge; and at the right end the contact is on a surface of one fortieth of an inch square, being in the mid breadth of the rod. Now if w, h, k, l are the weight, breadth, depth, and length, and E the modulus of elasticity of the rod, $\frac{1}{2}ln$ the breath of each support, so that $l(1-n)$ is the distance between the edges of the supports, then

$$e = \frac{wl^3}{4\mathrm{E}hk^3}\left(1-\tfrac{5}{2}n\right),$$

where e is the space (measured by the micrometer-screw) moved over by the centre of the rod between its position of resting wholly on the supports at the ends, and the position of being carried entirely by the centre support and having just contact only with the extreme supports. The three supports were fixed in a window-sill, the greatest rigidity being requisite in order to obtain trustworthy results. The experiments were all made at nearly the same temperature (65° Fahrenheit); and great care was taken to avoid change of temperature during the measures. The faces of the rods were numbered I., II., III., IV.; and the flexure was measured with each face up; so that each bar gives four values of E: the round rod was also observed in four positions. In the case of this rod,

$$E=\frac{wl^3}{12\pi e k^4}(1-\tfrac{5}{2}n),$$

where k is its radius.

To combine the four values of E in each case, we should proceed as follows:—if dw, dl, ... be the corrections necessary to the observed values of these quantities, we must express the consequent corrections to the computed values of E. Let the first two (that is, the values of E resulting from the opposite faces I. and III.) be multiplied each by a factor $\frac{1}{4}(1+\theta)$, and the other two by $\frac{1}{4}(1-\theta)$; then it is easy to express the probable error of the resulting value of E in terms of the probable errors of the observations; and the value of θ to be used is that which renders this expression a minimum. In the present case, however, the first two rods are very nearly square in section; and the third rod giving identical values (or nearly) for E, the multipliers are immaterial. We therefore take the mean of the four computed values with a probable error

$$\pm E\left\{\frac{\partial w^2}{w^2}+9\frac{\partial l^2}{l^2}+4\left(\frac{1}{h^2}+\frac{1}{k^2}\right)\partial h^2+\frac{25}{4}\partial n^2+\frac{1}{f^2}\partial e^2\right\}^{\frac{1}{2}}$$

for the square rods, and

$$\pm E\left\{\frac{\partial w^2}{w^2}+9\frac{\partial l^2}{l^2}+16\frac{\partial k^2}{k^2}+\frac{25}{4}\partial n^2+\frac{1}{f^2}\partial e^2\right\}^{\frac{1}{2}}$$

for the round rod, where

$$\frac{1}{f^2}=\frac{1}{16}\left(\frac{1}{e_1^2}+\frac{1}{e_2^2}+\frac{1}{e_3^2}+\frac{1}{e_4^2}\right),$$

the symbol ∂ meaning probable error; ∂h is taken to be the probable error of either h or k; and the four e's are assumed to be determined with the same accuracy.

This last supposition is not, however, quite exact. In the following Table the values of e are given for each face of each rod, the probable error being that resulting from the agreement of the eight determinations made with each face up.

Rod.	Face I. up. Face III. up.	Face II. up. Face IV. up.
	in.	in.
No. 1.	0·20670 ± 3 0·20716 ± 4	0·21328 ± 2 0·21342 ± 2
" 2.	0·20943 ± 1 0·20885 ± 4	0·20831 ± 2 0·20803 ± 2
" 3.	0·26910 ± 3 0·26910 ± 1	0·14719 ± 1 0·14715 ± 2
" 4.	0·14875 ± 2 0·14853 ± 5	0·14882 ± 2 0·14912 ± 1

The probable errors here refer to the last figure of the preceding decimal. An examination of this Table shows that there are, notwithstanding the extreme care with which the observations were made, some unknown constant errors in existence outweighing the apparent probable errors of the different measurements, and therefore interfering with the application of the theory given above, at least for rods Nos. 1 and 2. I hope by some future observations to discover the source of the constant errors in question. The following Table gives the values of E for each rod expressed in millions of pounds.

Rod.	Face I.	Face III.	Face II.	Face IV.	Mean.
No. 1.	16·806	16·769	16·737	16·726	16·759
„ 2.	16·945	16·992	16·937	16·960	16·958
„ 3.	16·181	16·181	16·192	16·196	16·188
„ 4.	15·083	15·105	15·076	15·045	15·077

While making these experiments I met with a very elaborate work published at St. Petersburg, entitled *Recherches Expérimentales sur l'élasticité des Métaux faites à l'observatoire physique central de Russie*, par A. T. Kupffer. Of brass, nine specimens were experimented on—three of cast brass, three of plate brass, and three of hammered. The experiments were made :—first, by causing the rods to bend under given weights, and measuring with vertical circles the inclinations of the ends of the rods so bent; secondly, by causing the rods to vibrate transversely when fixed by one end in a vertical position, the other end being weighted, and counting the vibrations. The results obtained by the two methods agreed very satisfactorily. The following Table gives the moduli of elasticity expressed in millions of pounds for the nine specimens.

Cast.	Rolled.	Hammered.
12·262	15·338	16·012
11·537	16·244	16·523
14·540	15·884	15·753

These quantities result from a very large number of observations.