Philosophical Magazine Series 3

ISSN: 1941-5966 (Print) 1941-5974 (Online) Journal homepage: http://www.tandfonline.com/loi/tphm14

XXXV. On the true amplitude of a tessarine; on the derivation of the word theodolite; and on light under the action of magnetism

James Cockle Esq. M.A.

To cite this article: James Cockle Esq. M.A. (1850) XXXV. On the true amplitude of a tessarine; on the derivation of the word theodolite; and on light under the action of magnetism , Philosophical Magazine Series 3, 36:243, 290-295, DOI: 10.1080/14786445008646476

To link to this article: http://dx.doi.org/10.1080/14786445008646476

Published online: 30 Apr 2009.

ent hours of the day and night, at different seasons of the year, and through different media. May not the velocity through the ground vary with the direction of the current,—whether east and west, or north and south?

I place great confidence in these results, as every care was taken to eliminate all possible sources of error. Every magnet in use was in the observatory, and all the connexions and adjustments were under my own eye.

The adjustment of the receiving magnet was unaltered during the experiment, and no change occurred for more than thirty minutes after the experiments were finished. Each of the pens recorded, in every instance, the armature time in the distance from the primary to the vibratory dot. One-half this record was shown to be the armature time, as follows:—

The variable pen could not begin to descend until the standard pen was down. Hence, considering the armature time of the receiving magnet as insensible (as it was), with a short circuit the interval of the two records would be equal to the armature time of the standard pen. This interval, being measured, was found to be exactly one-half the interval between the primary and vibratory dot of the standard pen.

The length of this article forbids me to go into further detail. In case further particulars are desired, it will give me pleasure to furnish them by correspondence, or by a further publication.

Cincinnati Observatory,
November 16, 1849.

XXXV. *On the True Amplitude of a Tessarine; on the Derivation of the word* Theodolite; *and on Light under the action of Magnetism. By* JAMES COCKLE, *Esq., M.A., of Trinity College, Cambridge, Barrister-at-law*.*

ANY tessarine $w+i'x+j'y+k'z$, or t, may be put under the form†

$$M\{q\cos p+i'r\sin p+j'(1-q)\cos p+k'(1-r)\sin p\},$$

* Communicated by the Author. Mr. Cockle takes an opportunity of stating that remarks on the Tessarine Theory, as well as on the impossible equations and quantities so intimately connected with it, will be found in his *Horæ Algebraicæ*, published in vols. xlvii. to l. of the Mechanics' Magazine; and he also begs to refer the reader to the following papers published in the same work, and in which he has adverted to the same subject; viz. "On Algebraic Symbols," Mechanics' Magazine, vol. l. pp. 292–294; "On the Symbols of Algebra, and on the Theory of Tesssarines," Ibid. p. 534; "On the Tessarine Algebra," Ibid. pp. 558, 559; "On certain Researches of Mr. Boole, and the Symbol of Infinity," Ibid. vol. li. pp. 124, 125; "On Systems of Quadruple Algebra," Ibid. pp. 197–199; and see some further remarks "On Quadruple Algebra," Ibid. vol. li. pp. 557, 558; and "On Tessarines," Ibid. p. 610.

† See Mechanics' Magazine, vol. li. p. 610.

provided that in the expression for the true modulus M (which may be seen at p. 435 of the preceding volume of this Journal) both the undetermined signs are taken as positive. Let M, p, q, r be called the ELEMENTS of the tessarine t; then, if M$'$, p', q', r' be the elements of t', and M$''$, p'', q'', r'' those of t''; and, further, if t, t', and t'' be connected by the relation $tt'=t''$, the following equations will hold between the elements of the product and of the factors; viz.

$$q''\cos p''=\{1-(q,q')\}\cos p\cos p'-\{1-(r,r')\}\sin p\sin p',$$

$$r''\sin p''=\{1-(q,r')\}\cos p\sin p'+\{1-(q'r)\}\cos p'\sin p,$$

$$(1-q'')\cos p''=(q,q')\cos p\cos p'-(r,r')\sin p\sin p',$$

$$(1-r'')\sin p''=(q,r')\cos p\sin p'+(q',r)\cos p'\sin p,$$

where (a, b) denotes $a+b-2ab$. The above four equations will readily be verified by a comparison with those which I gave (at p. 437 of vol. xxxiii. of the Phil. Mag. S. 3) as connecting the *constituents* of the product with those of the factors; add the first and the third of them together, and we have

$$\cos p''=\cos p\cos p'-\sin p\sin p'=\cos(p+p'), \quad . \quad \text{(a.)}$$

whence, calling p the *true amplitude* of t, we infer that the true amplitude of the product of two tessarines is the sum of the true amplitudes of the factors. And we might have inferred the same thing by adding the second and fourth of the above equations, which would have given us

$$\sin p''=\sin(p+p'). \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(b.)}$$

The combination of (a.) and (b.) shows us, however, that, rejecting circumferences, $p''=p+p'$ is the only relation between p, p', and p''. It will be remembered, in the above investigation, that M$''$=MM$'$.

I may add that these relations, both between the moduli and the amplitudes of systems of tessarines, may be readily arrived at by combining the reasoning of paragraphs 10 to 12 of the Rev. Professor Charles Graves's paper on Triple Algebra at pp. 119–126 of vol. xxxiv. of this Journal*, with that used by

* In the same Number (Phil. Mag. for February 1849) I have given (see pp. 132–135) a "Solution of Two Geometrical Problems," by means of a Uniaxal Geometry. The rationale of the *virtual solution* is clear and logical. Let it be required to find a point situate at a distance a from a given point (the origin, for instance), and fulfilling certain (specified) conditions. In Uniaxal Geometry the solution proceeds by supposing the required point to be situate in the primary axis or *axe*—as to which latter term see Phil. Mag. S. 3. vol. xxxiv. pp. 408, 409. And on this supposition we find for the distance of the required from the given point an expression of the form

$$A+B\sqrt{-1}\ \text{(or, more generally, } A+B\sqrt{-1}+Cj);$$

him in the opening of his "Abstract of a Paper on Algebraic Triplets," &c. in part 1 of vol. iii. of the Proceedings of the Royal Irish Academy, where he establishes similar relations between the moduli and amplitudes of couplets.

On the Derivation of the word Theodolite.

Professor De Morgan has (see Phil. Mag. S. 3. vol. xxviii. pp. 287–289) upon this subject offered some remarks which attracted my attention to the point, and induced me to make a conjecture as to the derivation, which I communicated to the Mechanics' Magazine*. Mr. De Morgan's remarks have given an interest to the topic; and to this Journal, as the most fitting place, I have ventured to transmit these comments upon his remarks.

The question whether the word *theodolite* is derived from *alidade* entails upon us (not necessarily perhaps, but sufficiently) the consideration of the point (1) whether the instrument called *alidade* or *alhidada* is *peculiar* to the theodolite; and this latter point again conducts to the further question (2), whether the alhidada differs from what is termed in the *Pantometria* the "index with sightes."

It will be convenient to consider the latter question first. Let us turn to "The. 22. Chapter." of the "fyrst Booke" (*Longimetra*) of the *Pantometria* (edition of 1571). This chapter treats of "The making of an Instrument named the Geometricall square." And, among other instructions, we are told ".., forget not to haue an index, not with commune sightes, but thus, .." Then follow directions for their construction, and we are informed respecting the index that "it hath place in the cêtre, and there made to tarry, so that with ease it may be turned from the first to any pointe." This is the "index with sightes" as it is afterwards expressly called in chapter 29 of the *Longimetra*. And, if there be any difference between this *index with sightes* and an *alidade*, it must consist in this,—that the index has *one extremity fixed to the centre* of the circle of which a quadrant is employed, while the alidade, which may be regarded as a *double index*, has its centre fixed to the centre of the circle to which it appertains. In effect, if not precisely in form, the index must be a revolving *radius*, and the alidade a revolving *diameter*; each furnished with "sightes." Mr. De Morgan, in his paper above

and if $A^2+B^2=a^2$ (or a^2-C^2, as the case may be), we see that the point so determined will be situate at a distance a from the given one, though *not in the* AXE. But, this last condition not being essential to the inquiry, the virtual solution is complete. This is the theory of the Virtual Solution.

* See pp. 159, 160 of vol. xlv. (No. 1201) of that work.

referred to, applies the term alidade to a revolving diameter. At least, such seems to be the effect of his statement, that "A ruler with sights, travelling upon a graduated circle, was a constituent part of various astronomical instruments imported into Europe from the East, and was accompanied by the Arabic term *alhidada* to express it:" combined with a remark in the paragraph preceding that statement. But it will be observed that it is only by implication (if, indeed, at all) that Mr. De Morgan excludes the application of the term *alidade* to a revolving *radius*, and, consequently, without contradicting his high authority, I may express a doubt whether any more is essential to the definition of an alidade than that it is a ruler with sights travelling on a circle *or portion of a circle*, and that it is as applicable to a revolving radius as to a revolving diameter. It is true that, in chapter 29 of *Longimetra*, the term "index with sightes" is used with reference to the "square Geometricall," and "Alhidada" with reference to "Theodelitus;" but then the words "er* index with sightes" immediately follow the word "alhidada," and would rather seem to show that the expressions are synonymous. Why, then, are they apparently distinguished in the manner just mentioned? It may be said, because the alhidada is a double index, whose diversity from the single one is manifested by the first plate to chap. 29, where the alidade occupies the right, and the index the left-hand side. But on the other side it may be urged, and perhaps not without effect, that, although the word "Theodelitus" occurs before that chapter, yet the term "alhidada" does *not*; and further, that the more complicated nature of the "instrument Topographicall" described in chap. 29 renders the use of the word *alhidada* necessary, in order to distinguish the index of the theodolite from that of the square, and that it is only for convenience and distinctness that it is there used.

The view contained in the preceding paragraph appears to be confirmed by a consideration arising upon the first of the points above alluded to. It will be found that the chapter 27 of *Longimetra* ("The composition of the instrument called Theodelitus") contains no mention whatever of the term alidade. We are told that "The index of that instrument with the sightes, &c. are not unlike to that whiche the square hath: . ." No *peculiarity* of its index is adverted to either in that or in the succeeding (28th) chapter, in which the "index" is mentioned no less than three times. Add to this, that the *theodelitus* may (*Long*. chap. 27 and 28) be *semicircular*, in which case a *single index* would be used, and the alidade (even if it meant a *double* index) would cease to be identified

* *Sic* in original. Probably a misprint for *or*?

with the instrument; and that other instruments into whose construction circles entered—such as Astrolabes—would also be furnished with *alidades*, and we may perhaps be justified in entertaining some doubt as to the connexion of the words theodolite and alidade.

It will be noticed that Bourne's "Treasure for Travailers," cited by Mr. De Morgan, was published in 1578, seven years after the first edition of the *Pantometria*, in which, so far as I am aware, the term *athelida* does not occur. Might not the term *theodelitus*, which was prior in point of publication, suggest *athelida*?

Adopting, as I do, Professor De Morgan's view, that *theodelitus* is an adjective or a participle, I think that fact favourable to my own derivation. The *graduation* of the instrument appears to be its principal feature in the eye of the writer of the *Pantometria*. "It is," he says, "but a circle divided," . . "or a semicircle parted.." Now if we pass from ὀβελὸς, ὀβελίζω to the Doric or Æolic forms ὀδελὸς, ὀδελίζω, we have in the word *odelited* the very expression of a *graduated* instrument. The prefix *The* may either be a redundancy, or connected with θεάομαι, in which latter case the instrument would be described as a *graduated seer*, or, what would be equivalent, a *seer of graduations or angles*. The graduation may, as a criterion, be liable to as much ambiguity as the being furnished with an alidade, but it provides us with a singular approximation in sound and meaning.

On Light under the action of Magnetism.

At pp. 469–477 of vol. xxviii. of the present series of this Journal, the Astronomer Royal has given equations applying to light under the action of magnetism. Some time since I made a communication, on the subject of Mr. Airy's investigations, to a cotemporary Journal*, but the following observations may not be misplaced here.

Of the species of force to which that alluded to by Mr. Airy in the last paragraph but one of his letter (Phil. Mag. S. 3. vol. xxviii. p. 477) belongs, we have a striking instance in *centrifugal force*. Conceive a particle attached to a fixed point by an inextensible string, and revolving round the point. The tension of the string depends upon the velocity of the particle in the direction of its motion—which is always *perpendicular* to the string. This induced me to enter into some investigations on the subject, which I discontinued; for I found that some researches of Mr. O'Brien (in this Journal, vol. xxv. pp. 326–334) might be adapted to the same end. If (Ib. p. 331) we

* See Mechanics' Magazine, vol. xlvii. pp. 575, 576.

make T=0, and suppose the polarization circular, we readily arrive, after some necessary substitutions, at the Astronomer Royal's results. I hope that I am not too rash in adding, that it would seem that those results exclude the idea of there being anything like *friction* between the particles of the luminiferous æther and those of the glass. At least they would do so if the equations held for any considerable portion of the path of the ray—if they did not hold they might, it seems to me, be made to furnish a measure of the friction, and so aid us in forming a mechanical theory.

2 Pump Court, Temple,
February 16, 1850.

XXXVI. *On a new Equation in Hydrodynamics, in Reply to* Professor P. Tardy. *By the* Rev. J. CHALLIS, *M.A., F.R.S., F.R.A.S., Plumian Professor of Astronomy in the University of Cambridge**.

THE observations communicated by Professor P. Tardy of Messina to the March Number of the Philosophical Magazine, on a new equation which I have asserted to be necessary to complete the analytical principles of hydrodynamics, proceed evidently from a mathematician who is well able to discuss this difficult question, and have received from me the most careful consideration. I shall endeavour to reply to them as nearly as possible in the order in which they occur in Professor Tardy's communication.

In the first place, I fully admit that Professor Tardy has proved that the equation

$$\frac{d\rho}{dt}+\frac{d.\rho\mathrm{V}}{ds}+\rho\mathrm{V}\left(\frac{1}{r}+\frac{1}{r^1}\right)=0 \quad . \quad . \quad . \quad . \quad (1.)$$

may be obtained without making use of my new equation, viz.

$$\frac{d\psi}{dt}+\lambda\left(\frac{d\psi^2}{dx^2}+\frac{d\psi^2}{dy^2}+\frac{d\psi^2}{dz^2}\right)=0. \quad . \quad . \quad . \quad (2.)$$

I arrived at (1.) after first eliminating λ by means of (2.) from the equations

$$u=\lambda\frac{d\psi}{dx},\quad v=\lambda\frac{d\psi}{dy},\quad w=\lambda\frac{d\psi}{dz};$$

but I did not remark, what indeed the process sufficiently indicated, that the result was independent of the value of λ employed, and I concluded erroneously that (2.) was necessary

* Communicated by the Author.