lines are a *result of the motion*, and that they correspond with the *veins* of glaciers; the lines incline most when the surface is steepest, as at h, fig. 3, and are very faint and nearly horizontal at i, where the surface of the stream is nearly so too. I left Gateshead without having an opportunity of getting a sectional view of this stream. I can get no *real* Stockholm pitch in Glasgow, else I should have made the experiment you have incited me to attempt here.

"I am, &c.,

"Glasgow, January 31, 1845." "LEWIS GORDON."

XXVIII. *On* Jacobi's *Elliptic Functions, in reply to* the Rev. Brice Bronwin; *and on Quaternions. By* ARTHUR CAYLEY, *Esq., B.A., F.C.P.S., Fellow of Trinity College, Cambridge.*

To the Editors of the Philosophical Magazine and Journal.

GENTLEMEN,

AS my last paper on Elliptic Integrals does not appear to have met with Mr. Bronwin's approbation, I will, with your permission, say a few more words on the question, and these will be the last I shall trouble you with on the subject. As Mr. Bronwin's last paper hardly professes to bring forward any new arguments, and complains of my not having condescended to reply to his previous ones, I shall endeavour at present to repair that omission. Mr. Bronwin says (Phil. Mag. S. 3. vol. xxiii. p. 90), "Moreover, he [Jacobi] has set out from an assumed equation $1-y=f(x)$, p. 39, from which all the rest of the formulæ are derived [and correctly derived, for Mr. Bronwin says afterwards, that "he supposes each particular step to be quite correct"]. In this assumed equation he has not actually determined the constants, but only assumed them. If he had actually determined them, it might appear that they are not susceptible of that generality which the author and Mr. Cayley suppose."

Jacobi is, of course, entitled to assume y any function of x that he pleases, and he might, if he had thought proper, have made ω perfectly determinate in his assumption, *e. g.* $\omega = \frac{K' i}{n}$. He then proves that this assumed value of y gives

$$\frac{dy}{\sqrt{1-y^2\,1-h^2y^2}} = \frac{dx}{M\sqrt{1-x^2}\sqrt{1-k^2x^2}}.$$

If then

$$x = \mathrm{s}\,a\,(u, k), \quad y = \mathrm{s}\,a\,(v, \lambda),$$

it is proved that

$$d\,v = \frac{1}{M}\,d\,u, \quad \text{or} \quad v = \frac{u}{M}.$$

Admitting all this, how can a person deny it immediately, and say that the proof only applies to particular forms of ω? [Of course the proof assumes that $\omega = \dfrac{m\,K + m'\,K'\,\iota}{n}$, the reason of this being in order that the equation $s\,a\,.\,(u + 4\,n\,\omega) = s\,a\,.\,u$ may be satisfied; and this is the case for any value of ω included under the above form.]

But the real ground of Mr. Bronwin's objection is evidently the different results to which his own reasonings lead him. In the Mathematical Journal, p. 124, he says, as the foundation of his formulæ, "Make $\underline{\omega} = \dfrac{K}{n}$. Moreover, when $u = 0, \omega, 2\,\omega, \dots$ suppose $v = 0$, H, 2 H, ..." (clearly meaning H to be analogous to K). And he then assumes a certain expression for $s\,a\,.\,v$, not to mention another one for $c\,.\,a\,.\,v$, the identity of which with the former is not satisfactorily proved (at least to me, but I am not pressing upon this at present). I believe in this case the two suppositions of the correspondence of values of u, v, and the assumed form of $s\,a\,.\,v$ are correct; still one ought to be deduced from the other, as Mr. Bronwin appears to admit (Phil. Mag.), "there is certainly room for discussion whether the quantities p, p' in the equation $\dfrac{u}{M} = p\,H + p'\,H'\,\iota$ are to be assumed or determined." This was my original objection, that they ought to be determined, and moreover, that in the cases Mr. Bronwin objects to, his assumption was incorrect. I had overlooked an equation in Jacobi (p. 59) which tends to confirm this; it is for the case of the impossible transformation $\omega = \dfrac{K'\,\iota}{n}$, viz. the formula $\Lambda'_{\prime} = \dfrac{K'}{n\,M_{\prime}}$, *i. e.* $\iota\,\Lambda'_{\prime} = \dfrac{\omega}{M_{\prime}}$, so that when $u = \omega$, v or $\dfrac{u}{M_{\prime}} = \Lambda'\,i$ (instead of Λ as Mr. Bronwin supposes [Λ is Jacobi's letter corresponding to H]). Of course I am not quoting this as proving the point; it is only that it enables me to retort Mr. Bronwin's challenge about the above transformation. Let him begin with the assumptions $\omega = \dfrac{K'\,\iota}{n}$, $u = 0, \omega, 2\,\omega, \dots$ $v = 0$, $H'\,\iota$, $2\,H'\,\iota$, ... and see what his theory will lead him to. I cannot undertake to do it myself, for I do not understand it; *I have worked out the particular case* $\omega = \dfrac{K'\,\iota}{3}$ *by* Jacobi's method, beginning, as I suppose

Mr. Bronwin wishes me to do, by expressing y by means of the complementary functions, and find that the process agrees step by step with Jacobi's, the only difference being that the transformation to the complementary functions is there made at the end.

An extraordinary assertion is the following one:—"It is sufficient to observe that the first form of ω only will satisfy the conditions $\mathrm{s}\,a\,.\,u = 0$, $\mathrm{s}\,a\,.\,u = 1$, required by Jacobi's theory, pages 40 and 41." If u is a misprint for v, and these equations are to be satisfied for $u = 0$, Jacobi's form of $\mathrm{s}\,a\,.\,v$ certainly satisfies them in any case; the first, because of the factor $s\,.\,a\,.\,u$; the second, because M is just determined by this very condition. If this is not the meaning, the true one has escaped me. One word on my preceding paper: the principal thing gained in it seems to be, its being likely to lead to the complete determination of the values of Λ, Λ' in the general case, a question which Jacobi has not examined; the principle is very clear, and one that is immediately suggested by Abel's formulæ, but I have no wish to force it upon Mr. Bronwin. I remain, Gentlemen,

Your obedient Servant,

Cambridge, January 16, 1845. A. CAYLEY.

P.S. *On Quaternions.*

It is possible to form an analogous theory with seven imaginary roots of (-1) (? with $\nu = 2^n - 1$ roots when ν is a prime number). Thus if these be $\iota_1, \iota_2, \iota_3, \iota_4, \iota_5, \iota_6, \iota_7$, which group together according to the types

$$123,\ 145,\ 624,\ 653,\ 725,\ 734,\ 176,$$

i. e. the type 123 denotes the system of equations

$$\iota_1\iota_2 = \iota_3,\quad \iota_2\iota_3 = \iota_1,\quad \iota_3\iota_1 = \iota_2,$$
$$\iota_2\iota_1 = -\iota_3,\quad \iota_3\iota_2 = -\iota_1,\quad \iota_1\iota_3 = -\iota_2,$$

&c. We have the following expression for the product of two factors:

$$\begin{aligned}
&(X_0 + X_1\iota_1 + \ldots X_7\iota_7)(X'_0 + X'_1\iota_1 + \ldots X'_7\iota_7) \\
&= X_0X'_0 - X_1X'_1 - X_2X'_2 \ldots - X_7X'_7. \\
&+ [\overline{23} + \overline{45} + \overline{76} + (01)]\iota_1 \qquad \text{where } (01) = X_0X'_1 + X_1X'_0 \\
&+ [\overline{31} + \overline{46} + \overline{57} + (02)]\iota_2 \qquad \vdots \\
&+ [\overline{12} + \overline{65} + \overline{47} + (03)]\iota_3 \qquad \overline{12} = X_1X'_2 - X_2X'_1 \\
&+ [\overline{51} + \overline{62} + \overline{47} + (04)]\iota_4 \qquad \text{\&c.} \\
&+ [\overline{14} + \overline{36} + \overline{72} + (05)]\iota_5 \\
&+ [\overline{24} + \overline{53} + \overline{17} + (06)]\iota_6 \\
&+ [\overline{25} + \overline{34} + \overline{61} + (07)]\iota_7
\end{aligned}$$

And the modulus of this expression is the product of the moduli of the factors. The above system of types requires some care in writing down, and not only with respect to the combinations of the letters, but also their order, it would be vitiated, *e. g.* by writing 716 instead of 176. A theorem analogous to that which I gave before, for quaternions, is the following:— If $\Lambda = 1 + \lambda_1 \iota_1 .. + \lambda_7 \iota_7$, $X = x_1 \iota_1 .. + x_7 \iota_7$. It is immediately shown that the possible part of $\Lambda^{-1} X \Lambda$ vanishes, and that the coefficients of $\iota_1, .. \iota_7$ are linear functions of $x_1 .. x_7$. The modulus of the above expression is evidently the modulus of X; hence "we may determine seven linear functions of $x_1 .. x_7$, the sum of whose squares is equal to $x_1^2 + .. + x_7^2$." The number of arbitrary quantities is however only seven, instead of twenty-one, as it should be.

XXIX. *Observations on the Decomposition of Metallic Salts by an Electric Current. By* Mr. James Napier*.

IN the paper I had the honour of reading before the Chemical Society at the close of the last session upon this subject, I stated my intention of bringing under their notice, in a series of short papers, such phænomena as seemed interesting which might present themselves in my daily avocations. In fulfilling this promise, I must beg leave to remind the Society that they are phænomena observed during the practical application of electro-metallurgy on the large scale, most of which are apparently at variance with many of the prescribed fundamental laws of electro-chemical decomposition, such as that referred to in my last communication, the non-transference of the base of an electrolyte, a subject I intend to discuss in a separate paper at an early opportunity. Some of the experiments were made on a small scale with different electrolytes from those used on the large scale, in order both to verify the results of practice and to ascertain if they were confined to the electrolytes there employed, which are the double cyanides of gold, silver and copper, with potassium. Some of these experiments will be given in detail.

It is laid down as a fundamental law in electro-decomposition, that there can be no inequality of force in any part of a voltaic current, and that the decompositions dependent upon the current are always in definite proportions, so that the amount of any element or salt radical liberated at one elec-

* Communicated by the Chemical Society; having been read January 6, 1845.