VI. *On the Phænomena of* Newton's *Rings when formed between two transparent Substances of different refractive Powers. By* G. B. Airy, *M.A. F.R.A.S. F.G.S. Late Fellow of Trinity College, and Plumian Professor of Astronomy and Experimental Philosophy in the University of Cambridge**.

IN a paper communicated to this Society about four months since, I stated my expectation (founded on Fresnel's theory), that if a lens of a low-refracting substance were placed on a plane surface of a high-refracting substance, and if light polarized in the plane perpendicular to the plane of reflexion were incident upon it, then so long as the angle of incidence was less than the polarizing angle of the low-refracting substance, or greater than that of the high-refracting substance, Newton's rings would be seen with a black centre; but if the angle of incidence was greater than the first of these and less than the second, Newton's rings would be seen with a bright centre. I have now to announce the fulfilment of this anticipation.

Before describing the method by which I have succeeded in the examination of these phænomena, I think it right to give a theoretical calculation of the intensity of light in the rings; as without this, the necessity for some of the precautions will not be sufficiently evident.

Conceive two nearly parallel plates of different media to be separated by a plate of air whose thickness is T; and let the vibration in the plane of reflexion, of an incident stream of light within the first medium, be represented by $a \sin \frac{2\pi}{\lambda}(vt-x)$ where x is the equivalent in air to the actual distance of a particle from some fixed point, (the light being supposed polarized in a plane perpendicular to the plane of reflexion). Let ι be the angle of incidence on the last surface of the first medium; ι' the angle of refraction, which is the same as the angle of incidence on the first surface of the second medium; and ι'' the angle of refraction in the second medium. A part of the light will be reflected at the last surface of the first medium; a part will reach the first surface of the second medium, where it will be subdivided; and one portion will be reflected to the surface of the first medium, where it will be again divided; and one of its parts will enter in the same direction as that which was reflected at first. In this the phase of the undulation will be *behind* that which was first

* From the Transactions of the Cambridge Philosophical Society; before which body this paper was read, March 19, 1832, as noticed in the Philosophical Magazine and Journal of Science, vol. i. p. 400.

reflected by the quantity corresponding to the space $2\,T\cos\iota'$: or if $\frac{2\pi}{\lambda}(vt-x)$ be still taken as the measure of the phase of the ray first reflected, $\frac{2\pi}{\lambda}(vt-x)-\frac{4\pi}{\lambda}T\cos\iota'$ will be that of the ray which has been reflected at the surface of the second medium and then enters the first. The quantity $\frac{4\pi}{\lambda}T\cos\iota'$ we shall for abbreviation call V. Of the light which reaches the surface of the first medium, a part will be partially reflected at the surface of the second medium, and will partially enter the first medium: its phase will be $\frac{2\pi}{\lambda}(vt-x)-2\,V$; and so for succeeding reflexions.

Now suppose that at the last surface of the first medium, the coefficient of the incident vibration being 1, that of the reflected vibration is e, and that of the refracted f; at the first surface of the second medium, suppose the coefficient of the reflected vibration to be g; and for light incident from air on the surface of the first medium, suppose the coefficients of the reflected and refracted vibrations to be h and k. Then, the coefficient in the incident light being a,

That in the first reflected light is ....$a\,e$
that in the refracted light is $a\,f$
that in the light reflected at the second medium is $a\,f\,g$
and that in the light refracted into the first medium is ...$a\,f\,g\,k$
that in the light reflected from the first medium is $a\,f\,g\,h$
that in the light reflected from the second medium is $a\,f\,g^2\,h$
and that in the light refracted into the first medium is...$a\,f\,g^2\,h\,k$
and so on; the coefficients after the first following a geometrical progression whose ratio is $g\,h$. Thus it appears that the whole vibration will be $a\,.\,e\,.\sin\frac{2\pi}{\lambda}(vt-x)+a\,.f\,g\,k$

$$\left\{\sin\left(\frac{2\pi}{\lambda}(vt-x)-V\right)+g\,h\,.\sin\left(\frac{2\pi}{\lambda}(vt-x)-2\,V\right)+\&c.\right\},$$

or $a\,.\,e\,.\sin\frac{2\pi}{\lambda}(vt-x)+a\,.f\,g\,k\,.$

$$\frac{\sin\left(\frac{2\pi}{\lambda}(vt-x)-V\right)-g\,h\,.\sin\left(\frac{2\pi}{\lambda}(vt-x)\right)}{1-2\,g\,h\,.\cos V+g^2\,h^2}.$$

Now in Fresnel's expressions,

$$e=\frac{\tan(\iota-\iota')}{\tan(\iota+\iota')},$$

$$f = \frac{\cos \iota}{\cos \iota'} \left(1 - \frac{\tan (\iota - \iota')}{\tan (\iota + \iota')}\right),$$

$$g = \frac{\tan (\iota' - \iota'')}{\tan (\iota' + \iota'')},$$

$$h = \frac{\tan (\iota' - \iota)}{\tan (\iota' + \iota)},$$

$$k = \frac{\cos \iota'}{\cos \iota} \left(1 - \frac{\tan (\iota' - \iota)}{\tan (\iota' + \iota)}\right).$$

Hence $fk = 1 - e^2$, and $gh = -ge$; and the expression becomes

$$a\, e \sin \frac{2\pi}{\lambda}(v\,t - x) + a\, g\,(1 - e^2)\,.$$

$$\frac{\sin \left(\frac{2\pi}{\lambda}(v\,t - x) - V\right) + g\,e\,.\sin\left(\frac{2\pi}{\lambda}(v\,t - x)\right)}{1 + 2\,g\,e \cos V + g^2\, e^2}.$$

Resolving this into the form

$$P \sin \frac{2\pi}{\lambda}(v\,t - x) + Q \cos \frac{2\pi}{\lambda}(v\,t - x),$$

the intensity or $P^2 + Q^2$ becomes

$$a^2 \frac{g^2 + e^2 + 2\,g\,e \cos V}{1 + 2\,g\,e \cos V + g^2\, e^2}.$$

The maxima and minima of this correspond to the maxima and minima, or the contrary, of cos V. When $V = 0$, 2π, &c. that is when $T = 0$, or $= \frac{\lambda}{2 \cos \iota'}$, or $= \frac{2\lambda}{2 \cos \iota'}$, &c. the intensity of the reflected light is

$$a^2 \left(\frac{g + e}{1 + g\,e}\right)^2,$$

and when $T = \frac{\lambda}{4 \cos \iota}$, $\frac{3\lambda}{4 \cos \iota}$, &c. the intensity is

$$a^2 \left(\frac{g - e}{1 - g\,e}\right)^2,$$

and the excess of the latter above the former is

$$-a^2\,.\,\frac{4\,e\,g\,(1 - e^2)\,(1 - g^2)}{(1 - e^2\, g^2)^2}.$$

This is the difference of intensity of the brightest and of the darkest parts of the rings: and when it is positive, the centre of the rings is dark.

Now $\tan^2(\iota + \iota')$ is always greater than $\tan^2(\iota - \iota')$, and

$\tan^2(\iota' + \iota'')$ is always greater than $\tan^2(\iota' - \iota'')$: so that $(1 - e^2) \cdot (1 - g^2)$ is always positive. Consequently the central spot is black when e and g have different signs, and bright when they have the same sign. Or as $\tan(\iota - \iota')$ is always negative, and $\tan(\iota' - \iota'')$ always positive, the central spot is black when $\tan(\iota + \iota')$ and $\tan(\iota' + \iota'')$ have the same sign, and bright when they have different signs: that is, it is dark when $\iota + \iota'$ and $\iota' + \iota''$ are both less or both greater than 90°, and bright when $\iota + \iota'$ is less than 90° and $\iota' + \iota''$ greater than 90° (or vice versâ). From this it follows that while the angle of incidence is less than the polarizing angle of the first medium, the central spot is black: at that polarizing angle the rings disappear (as $e = 0$): from that angle to the polarizing angle of the second medium the central spot is bright: at the polarizing angle of the second medium the rings disappear (as $g = 0$); and beyond that, the central spot is again dark.

Now let us estimate the intensity of the light at the central spot when the first ring is black (the angle of incidence being between the two angles of polarization). If the first ring is black we have $\frac{g-e}{1-ge} = 0$, whence $g = e$: and the intensity in the central spot becomes $a^2 \cdot \left(\frac{2e}{1+e^2}\right)^2$. The condition $g = e$ gives

$$\frac{\tan(\iota'-\iota'')}{\tan(\iota'+\iota'')} = \frac{\tan(\iota-\iota')}{\tan(\iota+\iota')},$$

whence $\sin^2 2\iota' = \sin 2\iota \cdot \sin 2\iota''$:

$$\text{or } \cos^2\iota' = \frac{1}{m\,m'}\cos\iota \cdot \cos\iota''.$$

where m and m' are the refractive indices of the two media. Without attempting to solve this equation generally, suppose $m = 1{\cdot}53$ and $m' = 2{\cdot}45$ (which correspond nearly to plate glass and diamond). The values of ι' at the polarizing angles are 56° 49′ 54″ and 67° 47′ 48″; and the value of ι' which makes the first ring black is 63° 19′ 4″; the values of ι and ι'' corresponding to this are 35° 43′ 57″ and 21° 23′ 21″: whence $e = g = 0{,}083215$; and the intensity of the light at the central spot $= a^2 \times 0{\cdot}02732$.

But to obtain a practical idea of the import of this expression we must compare it with the intensity of light in the rings in some other position. Now when the incidence is perpendicular, the expressions above give for the difference of the light in the dark spot and bright rings, $a^2 \times 0{\cdot}28159$. Consequently the intensity of light in the rings seen between the two polarizing angles is less than one tenth of that in the rings seen at a nearly perpendicular incidence. As the latter are by no means vivid, we must expect the former to be faint.

The intensity of the rings which would be produced at the same angle of incidence by light polarized in the plane of reflexion, found in the same way, $\left(\text{putting } e' = \frac{\sin(\iota - \iota')}{\sin(\iota + \iota')} \text{ and } g' = \frac{\sin(\iota' - \iota'')}{\sin(\iota' + \iota')}\right)$ is $a^2 \times 0{\cdot}66487$; and is consequently about twenty-four times greater than that of the rings of which we are treating.

This shows that much care will be necessary to make the rings visible. Suppose for instance that the incident light is polarized by a plate of tourmaline, or (which amounts to the same thing) that the reflected light is examined by a tourmaline, with its axis perpendicular to the plane of reflexion. Few tourmalines are so perfect as to transmit no more than one twenty-fourth part of the light polarized perpendicular to their axis. If then the rings are examined with one of these, the rings of which we are in quest (whose centre is bright) will be mixed with rings produced by light polarized in the plane of reflexion (whose centre is black) of at least equal intensity: and their character will therefore be entirely destroyed. If instead of a tourmaline we use a doubly-refracting prism, with which both sets of rings are exhibited, separated from each other, there will be no fear of confusion of the rings, but a sheet of bright light (from the rays polarized in the plane of reflexion) will be spread over the faint rings that we are seeking, and will effectually make them invisible.

The plan which I have successfully adopted is, to combine a tourmaline and doubly-refracting prism. By means of the tourmaline (with axis perpendicular to the plane of reflexion) the brightness of the sheet of light, which would otherwise cover the rings that we have to examine, is so far diminished, that it offers no serious obstacle. At the same time the other set of rings is seen, and serves very well as an object of comparison.

To destroy the reflexion at the upper surface of the imposed lens is a matter of importance. I have used a plano-convex lens of 5·8 inches focal length with an obtuse-angled prism placed upon its plane side, the obtuse angle being over the centre of the lens. A drop of water was placed between them. Though its refractive index differs sensibly from that of the glass, yet the reflexion at the common surface of the prism and lens is almost totally destroyed, for the following reason. The surface of the lens is I suppose very slightly convex, and when the drop of water is interposed, and the airbubbles are rubbed out, Newton's rings are seen, very large though slightly irregular, with the black spot in the centre. The rings in question are seen through this black spot, and consequently are not injured by the effects of reflexion. The water seems to have the power of bringing the lens and prism into closer contact than is otherwise attainable*: for I am well convinced that no force that could be applied without injuring them would bring them so near together as to exhibit the central black.

For the denser medium I have used a diamond with a surface of about $\frac{1}{12}$ inch in diameter, mounted in a ring: for the use of which I am indebted to the politeness of William John Broderip, Esq. Vice-President of the Geological Society. When the lens and prism were placed on this, a small system of rings was seen perfectly distinct and well formed, the diameter of the fifth ring not exceeding $\frac{1}{2}$ of the diameter of the surface.

These rings were examined with the combination of tourmaline and doubly-refracting prism that I have described. When the angle of incidence was small, the rings formed by light polarized perpendicular to the plane of reflexion were seen sufficiently vivid, with black centre, accompanied by the other set of rings which were faint. When the angle of incidence reached the polarizing angle of the glass, the first set of rings disappeared. On increasing the angle, the first set of rings was again seen with centre white. In the most favourable state, the first set of rings was much more faint than the second, but not so faint that there could be the slightest doubt upon the fact of the existence of the rings and the whiteness of the centre, as I saw them repeatedly with every change in the arrangement of the apparatus, and saw a succession of several rings. The white spot appeared larger than the dark spot in the other set of rings, but this I imagine is owing merely to the undefined nature of the spots, and to the circumstance that, in appreciating their comparative extent, the eye always gives credit to the brightness for a greater surface than it can properly claim. In respect of dimensions of corresponding parts, I could see no difference. On increasing the angle

* I may here mention a curious circumstance which occurred to me in the use of this combination. After leaving the prism, with the lens hanging to its lower surface, for one or two days, the water contracted itself to a spot (having partly gone off, I suppose, by evaporation) of about $\frac{1}{4}$ inch in diameter, its outline following most accurately the course of one of the rings (I think the third) even in its deviations from symmetry. In this state I was not able to move the lens upon the prism, though I applied a force parallel to the surface of the prism sufficiently great to shiver large splinters from the lens. On dipping them into water they instantly dropped asunder.

of incidence, the first set of rings again disappeared, and reappeared in great brilliancy, the centre being now black.

I am willing to think these experiments important, because they bear immediately upon a part of Fresnel's theory which has always appeared to me most liable to objection, namely, the formulæ for the extent of vibration in reflected and refracted rays. On the truth of Fresnel's general theory as a mere geometrical representation, namely, that light consists of transversal vibrations, and that polarized light is light in which all the vibrations are perpendicular to the plane of polarization, I shall say nothing, because I do not think it will be doubted by any one who is well acquainted with the experiments and has examined their agreement with calculation. But on the theorems for intensity in reflected rays, &c. involving points of the greatest obscurity, and supported only by very forced suppositions, any one may I think with reason be sceptical. The phænomena described here and those described in a former paper (On a remarkable Modification, &c.) depend entirely, in theory, upon the changes of sign of certain quantities which enter into Fresnel's expressions for these intensities. With respect to the absolute measure of the intensities I can say nothing, except that the general appearance of the brightness is sufficiently in accordance with the law. On the whole I think that these experiments give great probability to the truth of the formula considered as a general law: and that they establish with certainty that part of it which implies that, after passing a certain angle, the direction of the vibration in the reflected ray (considered with respect to that in the incident ray) is reversed.

Observatory, Feb. 4, 1832. G. B. AIRY.

Postscript.—Since the above account was written I have (with a favourable sky) seen the white-centred rings many times, and several times with a doubly-refracting prism only, unassisted by a tourmaline. In examining one part of the phænomena, I find that there is a discordance of a most curious kind from what the strict theory had led me to expect.

When the light is incident at the polarizing angle of the glass, the rings, so far as I can see, vanish totally. Though I have looked several times with the most scrutinizing attention, I have not been able to see the least trace. If the angle of incidence is gradually increased till it exceeds the polarizing angle, the black-centred rings disappear gradually without altering their size (a considerable quantity of light being still reflected from the diamond) and white-centred rings of the same size appear in their place, without any intermediate stage except a total absence of rings. From the agreement of this with theory I conclude that the polarization of light at the inner surface of glass is (to the senses) complete. But at the polarizing angle of the diamond the case is perfectly different. On increasing the angle of incidence till it exceeds this angle, the white-centred rings do not disappear, but the first black ring contracts so as to leave no central white, and becomes itself the black centre. After this there is no material change: I find, however, that the black centre of the rings produced by light polarized perpendicular to the plane of reflexion is always (beyond tbe polarizing angle of the diamond) sensibly larger than the black centre of the rings produced by light polarized in the plane of reflexion.

The nature of this transition from rings of one character to rings of the opposite character appears to me to be, theoretically, extremely curious. As the rings do not disappear, it is plain that if light polarized perpendicular to the plane of incidence (or whose vibrations are entirely in that plane) is incident at what is called the maximum polarizing angle of the diamond, a portion of it is still reflected. Still, however, on increasing the angle of incidence the character of the rings is changed: and this takes place at an angle where (so far as we are entitled to conclude) there is nothing peculiar in the reflexion from the glass; and we are therefore compelled to admit, that the incident vibration being $a \cdot \sin \frac{2\pi}{\lambda}(vt - x)$, when the angle of incidence is increased so as to exceed that angle, the reflected vibration is changed from $+ p \cdot \sin \frac{2\pi}{\lambda}(vt - x)$ to $- q \cdot \sin \frac{2\pi}{\lambda}(vt - x)$. A similar change takes place at the polarizing angle of the glass: but there, as we have seen, the transition from $+ p$ to $- q$ is effected by passing through 0, or by the entire cessation of reflexion at one angle of incidence; which is not the case at the polarizing angle of the diamond. How then is the gradual change from $+ p \sin \frac{2\pi}{\lambda}(vt - x)$ to $- q \cdot \sin \frac{2\pi}{\lambda}(vt - x)$ to be explained? I answer that the phænomena prove that it follows from a *gradual change of phase*, while the coefficient is not much altered. In other words (neglecting the trifling alteration in the coefficient) the quantity $+ p \sin \frac{2\pi}{\lambda}(vt - x)$ is changed to $- p \sin \frac{2\pi}{\lambda}(vt - x)$, not by the disappearance of p, but by the expression assuming the form $p \sin \left\{\frac{2\pi}{\lambda}(vt - x) - \theta\right\}$, where θ increases from 0 to π. This may be popularly explained in the following manner. The common Newton's rings, formed between two lenses, are produced by the interference of the light reflected from the lower surface of the upper lens with that reflected from the upper surface of the lower lens. Now if the upper lens be raised a little, or the lower depressed a little, the rings contract. As the only immediate effect of depressing the lower lens is to cause the light reflected from it to describe a longer path, or to have its phases retarded, it appears that a contraction of the rings may be considered as the effect of a retardation in the phase of the light reflected from the lower surface. The contraction of the rings then in passing the polarizing angle of the diamond requires us to admit that the phase of the reflected light (the incident light being polarized perpendicular to the plane of the reflexion) is, on increasing the angle of incidence by a few degrees, retarded nearly 180°.

The retardation, however, is not quite 180°. For if it were, the character of the rings would be exactly changed, so that the proportion of the size of the central black spot to that of the first white ring would be the same as that of the central white spot (before the change) to the first black ring. But as the central black spot formed by rays polarized perpendicular to the plane of reflexion is distinctly larger than that formed by rays polarized in the plane of reflexion, it seems that the black ring has not contracted completely, or that the alteration of phase is not quite 180°. This reasoning it must be confessed is not certain, as the same thing would be explained by supposing a small alteration of phase in the light polarized in the plane of reflexion. I may mention here, that in the Newton's rings formed between two lenses of the same kind of glass, the central black spot in those formed by light polarized perpendicular to the plane of reflexion is larger than in those formed by the light polarized in the plane of reflexion.

If, while the white-centred rings are under examination, the tourmaline and doubly-refracting prism are turned round, the rings become faint, but do not disappear, and are changed into black-centred rings by the contraction of the rings. This is exactly similar to what takes place when a lens is placed on a metallic surface, and it proves that (as in the former paper), while the angle of incidence is a few degrees less than the maximum polarizing angle of the diamond, the phase of light polarized perpendicular to the plane of reflexion is more retarded than the phase of light polarized in that plane.

I have not found any variation in these results from changing the position of the plane of reflexion on the diamond surface.

The result of these experiments and reasonings may be thus stated.

1. When the angle of incidence is less than the maximum polarizing angle of the diamond, the nature of its reflexion is similar to that of metallic reflexion: the phase of vibrations in the plane of reflexion being more retarded than that of vibrations perpendicular to the plane of reflexion, but perhaps by a smaller quantity than in reflexion from metals.

2. In the neighbourhood of the polarizing angle, the nature of the reflexion is different from any that has hitherto been described. The vibrations in the plane of reflexion do not vanish, but on increasing the angle of incidence by three or four degrees the phase of vibration is gradually retarded by nearly 180°. In the reflexion of light whose vibrations are perpendicular to the plane of reflexion, there is no striking difference between the effects of diamond and those of glass.

3. For angles of incidence greater than the polarizing angle, there is no sensible difference between the effects of diamond and those of glass.

I may remark that the extent of vibration in the plane of reflexion may be represented thus (the formula being purely empirical and given only for illustration). The vibration in the incident light being $a \sin \frac{2\pi}{\lambda}(vt - x)$, that in the reflected light is

$$\frac{\tan(\iota' - \iota'')}{\tan(\iota' + \iota'')} a \sin \frac{2\pi}{\lambda}(vt - x) - b\, a \cos \frac{2\pi}{\lambda}(vt - x),$$

where b is always small but never $= 0$, and is perhaps constant.

The conclusions at which I have arrived are at variance with one of Sir David Brewster's (Phil. Trans. 1815). Sir David Brewster's character as an experimental philosopher stands deservedly so high, and my estimation of his accuracy (as observed by myself in the repetition of many of his experiments) is so great, that I think it necessary to point out distinctly the nature of this disagreement.

Sir David Brewster states that homogeneous light is completely polarized by the diamond at the proper angle. I have made no experiments here with homogeneous light, and I know that, on account of its extreme faintness, however obtained, little confidence can be placed in results which depend only on the evanescence of the reflected light. But the phænomena observed by me are entirely inconsistent with this supposition. If homogeneous light were used, then (on this supposition) the bright-centred rings would disappear and black-centred rings would succeed them as at the polarizing angle of the glass. If white light were used, the rings in the neighbourhood of the polarizing angle would be wholly coloured, and on changing the angle the intensity of the different colours in each ring would alter, but there would be nothing like contraction. Thus at a certain angle the brightest part of the red would be at the centre of the spot, and its faintest part would be in the first ring; while for the blue the places would be reversed: on increasing the angle the brightest parts of both would be in the first ring. Whereas in my experiments there was no discoverable alteration in the colours of the rings, there never was seen a bright red centre surrounded by a bright blue ring; but the rings, without changing their character as to colour, diminished steadily till the central spot was as it were squeezed out. Whether the only diamond which I have used may possess any peculiarity which distinguishes it from those used by Sir David Brewster, I cannot say. Meantime I may observe, that the singularity in the reflexion at the surface of the diamond makes it not improbable that there may be some singularity in the refraction also, and renders a more extended inquiry into the laws both of its reflexion and of its refraction highly desirable.

Observatory, Feb. 16, 1832. G. B. Airy.

VII. *On certain Defects in the British Almanac.* By B. Bevan, *Esq.*

To the Editors of the Philosophical Magazine and Journal.

Gentlemen,

IS it not worthy of remark, that an Almanac published under the patronage of so learned a Society as that established for the Diffusion of Useful Knowledge, should continue to be published without giving the *sun's declination?* It cannot surely be owing to a want of room, when two pages per month are appropriated to the calendar.

The patrons of this publication must be aware of the importance of this information to all persons who may wish to become acquainted with practical astronomy. The declination of the sun is independent of the latitude of the place, and therefore will serve for all the British dominions; whereas the table of "sunrise" and "sunset" can be true only for the particular latitude of the place for which it is calculated, and can be of little, if any practical use;—at least, a general table for