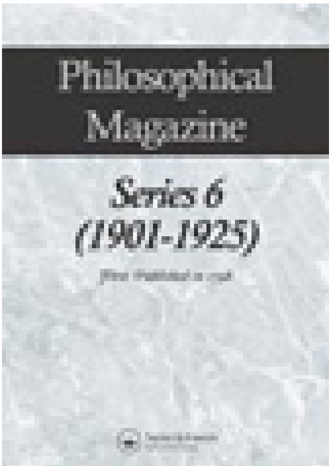

Philosophical Magazine Series 6
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tphm17

LXXVI. The probability variations in the distribution of α particles

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Published online: 21 Apr 2009.

the scintillations were as bright if not brighter than those from a thin film of uranium. Boltwood has found that the range of the α particle from ionium is 2·8 cms., so that it appeared probable that the range of the α particles from uranium had been overestimated. This conclusion was confirmed by finding that the α rays from a thin film of uranium were more readily absorbed by aluminium than those from ionium. By a special method, the range of the α particle from uranium has been measured and found to be about 2·7 cms., while the range of the α particle from ionium is a millimetre or two longer. Further experiments are in progress to determine the range of the α particle from uranium accurately, and to examine carefully whether two sets of α particles of different range can be detected.

University of Manchester,
July 1910.

LXXVI. *The Probability Variations in the Distribution of α Particles. By* Professor E. RUTHERFORD, *F.R.S., and* H. GEIGER, *Ph.D. With a Note by* H. BATEMAN *.

IN counting the α particles emitted from radioactive substances either by the scintillation or electric method, it is observed that, while the average number of particles from a steady source is nearly constant, when a large number is counted, the number appearing in a given short interval is subject to wide fluctuations. These variations are especially noticeable when only a few scintillations appear per minute. For example, during a considerable interval it may happen that no α particle appears; then follows a group of α particles in rapid succession; then an occasional α particle, and so on. It is of importance to settle whether these variations in distribution are in agreement with the laws of probability, *i. e.* whether the distribution of α particles on an average is that to be anticipated if the α particles are expelled at random both in regard to space and time. It might be conceived, for example, that the emission of an α particle might precipitate the disintegration of neighbouring atoms, and so lead to a distribution of α particles at variance with the simple probability law.

The magnitude of the probability variations in the number of α particles was first drawn attention to by E. v. Schweidler †. He showed that the average error from the mean number of α particles was $\sqrt{N \cdot t}$, where N was the number of particles emitted per second and t the interval under consideration. This conclusion has been experimentally verified by several

* Communicated by the Authors.

† v. Schweidler, Congrès Internationale de Radiologie, Liège, 1905.

observers, including Kohlrausch *, Meyer and Regener †, and H. Geiger ‡, by noticing the fluctuations when the ionization currents due to two sources of α rays were balanced against each other. The results obtained have been shown to be in good agreement with the theoretical predictions of von Schweidler.

The development of the scintillation method of counting α particles by Regener, and of the electric method by Rutherford and Geiger, has afforded a more direct method of testing the probability variations. Examples of the distribution of α particles in time have been given by Regener § and also by Rutherford and Geiger ‖. It was the intention of the authors initially to determine the distribution of α particles in time by the electric method, using a string electrometer of quick period as the detecting instrument. Experiments were made in this direction, and photographs of the throws of the instrument were readily obtained on a revolving film; but it was found to be a long and tedious matter to obtain records of the large number of α particles required. It was considered simpler, if not quite so accurate, to count the α particles by the scintillation method.

Experimental Arrangement.

The source of radiation was a small disk coated with polonium, which was placed inside an exhausted tube, closed at one end by a zinc sulphide screen. The scintillations were counted in the usual way by means of a microscope on an area of about one sq. mm. of screen. During the time of counting (5 days), in order to correct for the decay, the polonium was moved daily closer to the screen in order that the average number of α particles impinging on the screen should be nearly constant. The scintillations were recorded on a chronograph tape by closing an electric circuit by hand at the instant of each scintillation. Time-marks at intervals of one half-minute were also automatically recorded on the same tape.

After the eye was rested, scintillations were counted from 3 to 5 minutes. The motor running the tape was then stopped and the eye rested for several minutes; then another interval of counting, and so on. It was found possible to count 2000 scintillations a day, and in all 10,000 were recorded. The records on the tape were then systematically

* Kohlrausch, *Wiener Akad.* cxv. p. 673 (1906).

† Meyer and Regener, *Ann. d. Phys.* xxv. p. 757 (1907).

‡ Geiger, Phil. Mag. xv. p. 539 (1908).

§ Regener, *Verh. d. D. Phys. Ges.* x. p. 78 (1908); *Sitz. Ber. d. K. Preuss. Akad. Wiss.* xxxviii. p. 948 (1909).

‖ Rutherford and Geiger, Proc. Roy. Soc. A. lxxxi. p. 141 (1908).

examined. The length of tape corresponding to half-minute marks was subdivided into four equal parts by means of a celluloid film marked with five parallel lines at equal distances. By slanting the film at different angles, the outside lines were made to pass through the time-marks, and the number of scintillations between the lines corresponding to 1/8 minute intervals were counted through the film. By this method correction was made for slow variations in the speed of the motor during the long interval required by the observations.

In an experiment of this kind the probability variations are independent of the imperfections of the zinc sulphide screen. The main source of error is the possibility of missing some of the scintillations. The following example is an illustration of the result obtained. The numbers, given in the horizontal lines, correspond to the number of scintillations for successive intervals of 7·5 seconds.

									Total per minute.
1st minute :	3	7	4	4	2	3	2	0	25
2nd ,,	5	2	5	4	3	5	4	2	30
3rd ,,	5	4	1	3	3	1	5	2	24
4th ,,	8	2	2	2	3	4	2	6	31
5th ,,	7	4	2	6	4	5	10	4	42
					Average for 5 minutes ...				30·4
					True average				31·0

The length of tape was about 14 cms. for one minute interval. The average number of particles deduced from counting 10,000 scintillations was 31·0 per minute. It will be seen that for the 1/8 minute intervals the number of scintillations varied between 0 and 10 ; for one minute intervals between 25 and 42.

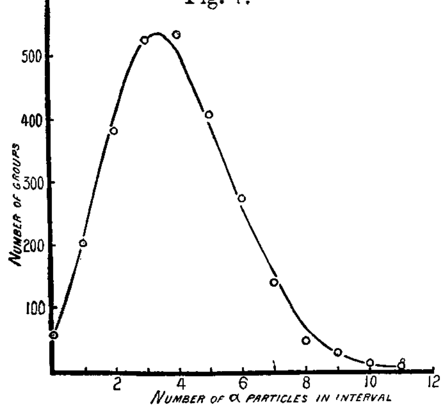
The distribution of α particles according to the law of probability was kindly worked out for us by Mr. Bateman. The mathematical theory is appended as a note to this paper. Mr. Bateman has shown that if x be the true average number of particles for any given interval falling on the screen from a constant source, the probability that n α particles are observed in the same interval is given by $\frac{x^n}{n!}e^{-x}$. n is here a whole number, which may have all positive values from 0 to ∞. The value of x is determined by counting a large number of scintillations and dividing by the number of intervals involved. The probability for n α particles in the given interval can then at once be calculated from the theory. The following table contains the results of an examination of the groups of α particles occurring in 1/8 minute interval.

Number of α particles.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Number of α particles.	Number of intervals.	Average number.
I.	15	56	106	152	170	122	88	50	17	12	3	0	0	1	0	3179	792	4·01
II.	17	39	88	116	120	98	63	37	4	9	4	1	0	0	0	2334	596	3·92
III.	15	56	97	139	118	96	60	26	18	3	3	1	0	0	0	2373	632	3·75
IV.	10	52	92	118	124	92	62	26	6	3	0	2	0	0	1	2211	588	3·76
Sum	57	203	383	525	532	408	273	139	45	27	10	4	0	1	1	10097	2608	3·87
Theoretical values.	54	210	407	525	508	394	254	140	68	29	11	4	1	4	1			

For convenience the tape was measured up in four parts, the results of which are given separately in horizontal columns I. to IV.

For example (see column I.), out of 792 intervals of 1/8 minute, in which 3179 α particles were counted, the number of intervals giving 3 α particles was 152. Combining the four columns, it is seen that out of 2608 intervals containing 10,097 particles, the number of times that 3 α particles were observed was 525. The number calculated from the equation was the same, viz. 525. It will be seen that, on the whole, theory and experiment are in excellent accord. The difference is most marked for four α particles, where the observed number is nearly 5 per cent. larger than the theoretical. The number of α particles counted was far too small to fix with certainty the number of groups to be expected for a large value of n, where the probability of the occurrence is very small. It will be observed that the agreement between theory and experiment is good even for 10 and 11 particles, where the probability of the occurrence of the latter number in an interval is less than 1 part in 600. The closeness of the agreement is no doubt accidental. The relation between theory and experiment is shown in fig. 1 for the results given in Table I., where the o represent observed points and the broken line the theoretical curve.

Fig. 1.



The results have also been analysed for 1/4 minute intervals. This has been done in two ways, which give two different sets of numbers. For example, let A, B, C, D, E represent the number of α particles observed in successive 1/8 minute intervals. One set of results, given in Table A, is obtained by adding A+B, C+D, &c.; the other set, given in Table B,

TABLE A.

Number of particles.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	Whole number of scintillations.	Whole number of intervals.	Average number in one interval.
I.	0	3	4	7	17	35	42	60	71	49	46	22	19	11	2	5	1	2	0	0	0	0	3182	396	8·04
II.	0	2	3	6	19	33	34	56	32	34	27	24	11	7	5	3	1	1	0	0	0	0	2330	298	7·82
III.	0	0	6	8	25	39	39	51	42	38	26	17	11	6	5	2	1	0	0	0	0	0	2373	316	7·51
IV.	0	1	7	11	14	30	40	47	48	36	27	18	8	4	1	0	1	1	0	0	1	0	2214	294	7·53
Sum	0	6	20	32	75	137	155	214	193	157	126	81	49	28	13	10	4	4	0	0	1	0	10099	1304	7·74

TABLE B.

Number of particles.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	Whole number of scintillations.	Whole number of intervals.	Average number in one interval.
I.	0	2	4	9	21	35	46	56	68	48	35	30	15	12	6	8	0	0	0	0	1	0	3180	396	8·03
II.	0	1	5	7	21	27	38	46	40	38	28	16	12	9	3	5	0	1	0	1	0	0	2333	298	7·83
III.	0	0	5	12	32	38	32	41	36	50	32	14	11	6	5	1	0	0	0	0	0	1	2371	316	7·50
IV.	0	0	3	18	25	26	35	44	36	37	36	15	6	8	2	0	1	0	2	0	0	0	2210	294	7·52
Sum	0	3	17	46	99	126	151	187	180	173	131	75	44	35	16	14	1	1	2	1	1	1	10094	1304	7·74
Sum of Tables A & B	0	9	37	78	174	263	306	401	373	330	257	156	93	63	29	24	5	5	2	1	2	1	20193	2608	7·74
Theoretical values.	1·1	9	34	88	170	263	339	372	363	312	242	170	110	65	36	19	9	4	1·8	·72	·28	·10			

by starting 1/8 minute later and adding B + C, D + E, &c. The results are given in the appended Tables. In the final horizontal columns are given the sum of the occurrences in Tables A and B and the corresponding theoretical values.

In the cases for 1/4 minute intervals, the agreement between theory and experiment is not so good as in the first experiment with 1/8 minute interval. It is clear that the number of intervals during which particles were counted was not nearly large enough to give the correct average even for the maximum parts of the probability curve, and much less for the initial and final parts of the curve, where the probability of an occurrence is small. However, taking the results as a whole for the 1/8 minute and the 1/4 minute intervals, there is a substantial agreement between theory and experiment, and the errors are not greater than would be anticipated, considering the comparatively small number of intervals over which the α particles were counted. We may consequently conclude that the distribution of α particles in time is in agreement with the laws of probability and that the α particles are emitted at random. As far as the experiments have gone, there is no evidence that the variation in number of α particles from interval to interval is greater than would be expected in a random distribution.

Apart from their bearing on radioactive problems, these results are of interest as an example of a method of testing the laws of probability by observing the variations in quantities involved in a spontaneous material process.

University of Manchester,
July 22nd, 1910.

Note.

On the Probability Distribution of α Particles. *By* H. Bateman.

Let λdt be the chance that an α particle hits the screen in a small interval of time dt. If the intervals of time under consideration are small compared with the time period of the radioactive substance, we may assume that λ is independent of t. Now let $W_n(t)$ denote the chance that n α particles hit the screen in an interval of time t, then the chance that $(n+1)$ particles strike the screen in an interval $t+dt$ is the sum of two chances. In the first place, $n+1$ α particles may strike the screen in the interval t and none in the interval dt. The chance that this may occur is $(1-\lambda dt)W_{n+1}(t)$. Secondly, n α particles may strike the screen in the interval t

and one in the interval dt; the chance that this may occur is $\lambda dt W_n(t)$. Hence

$$W_{n+1}(t+dt) = (1-\lambda dt)W_{n+1}(t) + \lambda dt W_n(t).$$

Proceeding to the limit, we have

$$\frac{dW_{n+1}}{dt} = \lambda(W_n - W_{n+1}).$$

Putting $n=0, 1, 2 \ldots$ in succession we have the system of equations:

$$\frac{dW_0}{dt} = -\lambda W_0,$$

$$\frac{dW_1}{dt} = \lambda(W_0 - W_1),$$

$$\frac{dW_2}{dt} = \lambda(W_1 - W_2),$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

which are of exactly the same form as those occurring in the theory of radioactive transformations *, except that the time-periods of the transformations would have to be assumed to be all equal.

The equations may be solved by multiplying each of them by $e^{\lambda t}$ and integrating. Since $W_0(0)=1$, $W_n(0)=0$, we have in succession:

$$W_0 = e^{-\lambda t},$$

$$\frac{d}{dt}(W_1 e^{\lambda t}) = \lambda, \qquad \therefore \quad W_1 = \lambda t e^{-\lambda t},$$

$$\frac{d}{dt}(W_2 e^{\lambda t}) = \lambda^2 t, \qquad \therefore \quad W_2 = \frac{(\lambda t)^2}{2!} e^{-\lambda t},$$

and so on. Finally, we get

$$W_n = \frac{(\lambda t)^n}{n!} e^{-\lambda t}.$$

The *average* number of α particles which strike the screen in the interval t is λt. Putting this equal to x, we see that the chance that n α particles strike the screen in this interval is

$$W_n = \frac{x^n}{n!} e^{-x}.$$

* Rutherford, 'Radioactivity,' 2nd edition, p. 330. The chance that an atom suffers n disintegrations in an interval of time t is equal to the ratio of the amount of the nth product present at the end of the interval to the amount of the primary substance present at the commencement.

The particular case in which $n=0$ has been known for some time (Whitworth's 'Choice and Chance,' 4th ed. Prop. 51).

If we use the above analogy with radioactive transformation, the theorem simply tells us that the amount of primary substance remaining after an interval of time t is $e^{-\lambda t}$ if a unit quantity was present at the commencement.

The *probable* number of α particles striking the screen in the given interval is

$$p=\sum_{n=1}^{\infty} n\mathrm{W}_n = xe^{-x}\sum_{n=1}^{\infty}\frac{x^{n-1}}{(n-1)\,!}=x.$$

The *most probable* number is obtained by finding the maximum value of W_n.

Since $\frac{\mathrm{W}_n}{\mathrm{W}_{n-1}}=\frac{x}{n}$, this ratio will be greater than 1 so long as $n<x$. Hence if $n \lesseqgtr x$,

$$\mathrm{W}_n \gtrless \mathrm{W}_{n-1};$$

if $n=x$, $\mathrm{W}_n=\mathrm{W}_{n-1}$. The most probable value of n is therefore the integer next greater than x; if, however, x is an integer, the numbers $x-1$ and x are equally probable, and more probable than all the others.

The value of λ which is calculated by counting the total number of α particles which strike the screen in a large interval of time T, will not generally be the true value of λ. The mean deviation from the true value of λ is calculated by finding the mean deviation of the total number N of α particles observed in time T from the true average number $\lambda\mathrm{T}$. This mean deviation D (mittlerer Fehler) is, according to the definition of Bessel and Gauss, the square root of the probable value of the square of the difference $\mathrm{N}-\lambda\mathrm{T}$, and so is given by the series

$$\mathrm{D}^2=\sum_{n=0}^{\infty}(\mathrm{N}-\lambda\mathrm{T})^2\frac{(\lambda\mathrm{T})^{\mathrm{N}}}{\mathrm{N}\,!}e^{-\lambda\mathrm{T}}$$

$$=e^{-\lambda\mathrm{T}}\sum_{\mathrm{N}=0}^{\infty}\left[\frac{(\lambda\mathrm{T})^{\mathrm{N}}}{(\mathrm{N}-2)\,!}+\frac{(\lambda\mathrm{T})^{\mathrm{N}}}{(\mathrm{N}-1)\,!}-2\frac{(\lambda\mathrm{T})^{\mathrm{N}+1}}{(\mathrm{N}-1)\,!}+\frac{(\lambda\mathrm{T})^{\mathrm{N}+2}}{(\mathrm{N})\,!}\right]=\lambda\mathrm{T}.$$

Hence $\mathrm{D}=\sqrt{\lambda\mathrm{T}}$, and the mean deviation from the value of λ is accordingly

$$\frac{D}{T} = \sqrt{\frac{\lambda}{T}};$$

it thus varies inversely as the square root of the length of the interval of time. This result is of the same form as the classical one used by E. v. Schweidler in the paper referred to earlier.

The probable value of $|N-\lambda T|$ (der durchschnittlicher Fehler) is much more difficult to calculate.

LXXVII. *A New Radiant Emission from the Spark.* By R. W. Wood, *Professor of Experimental Physics in the Johns Hopkins University* *.

[Plate XIV.]

I SCARCELY know how to designate the peculiar type of radiation referred to in the present paper, which was first discovered over two years ago in the course of some experiments made with a view of ascertaining whether the Schumann waves from the spark gave rise to any fluorescence of the air by which they were absorbed. It is now known that there is a feeble ultra-violet luminosity of air or nitrogen gas surrounding a small mass of radium, in other words the radium renders the gas luminescent. To test for a fluorescence due to the absorption of very short light-waves, the condenser spark between aluminium electrodes was passed behind and very close to a vertical strip of metal which completely concealed the spark, but which enabled observation, either visual or photographic, of the air in its immediate vicinity. If the air in the room was free from dust and smoke absolutely nothing could be seen with the eye, even after prolonged resting in the dark. A photograph, however, made with a small camera provided with a quartz lens, showed that the air around the spark was a source of a powerful actinic radiation, which was completely stopped by the intervention of a glass plate between the camera and the spark. The first photograph of the phenomenon which was obtained is reproduced on Pl. XIV. fig. 6. The narrow strip of metal between the two wider strips was about 1 cm. in width; the spark discharge was concealed behind this.

Two hypotheses immediately presented themselves: (*a*) we are dealing with a scattering of the shortest waves by the

* Communicated by the Author.