A fuller discussion of these facts may with advantage be deferred until some further experiments on the scattered radiation, at present in progress, are completed.

A striking point is specially emphasized in the case of silver. It was pointed out in an earlier paper that the speed with which the corpuscles emerge from a radiator depended only upon the hardness of the exciting radiation, and not at all upon the material of which the radiator was composed. Consequently the speed does not depend at all upon the penetrating power of the Röntgen radiation characteristic of the tertiary radiator, even though the emission of the corpuscular radiation is evidently very intimately connected with the emission of this characteristic Röntgen radiation.

When the exciting beam is from tin (λ in Al $=4{\cdot}33$) both the soft and the hard characteristic radiations are emitted by silver. The ionization produced by the corpuscles accompanying the emission of hard tertiary X radiation is approximately equal to that produced by the corpuscles accompanying the soft X radiation. The curves giving the values of the absorption coefficient of the corpuscular radiation, consisting of these two systems of corpuscles, clearly indicated that the corpuscles emerged with the same velocity in either case, this velocity being the same as that with which corpuscles emerge from any of the other tertiary radiators when excited by the secondary radiation from tin.

In conclusion, I wish to thank Professor Wilberforce for the interest he has shown throughout these investigations.

I wish also to thank Mr. Mesham, M.Sc., for his assistance in carrying out some of the evperiments.

The George Holt Physics Laboratories,
June 30, 1911.

XLIV. *Non-Newtonian Mechanics :—The Direction of Force and Acceleration.* *By* RICHARD C. TOLMAN, *Ph.D., Instructor in Physical Chemistry at the University of Michigan* *.

IF force is defined as the rate of increase of momentum, the equation

$$\mathbf{F}=\frac{d}{dt}(m\mathbf{u})=m\frac{d\mathbf{u}}{dt}+\frac{dm}{dt}\mathbf{u} \quad . \quad . \quad . \quad . \quad (1)$$

allows for a change in mass as well as a change in

* Communicated by the Author.

velocity. This is the fundamental equation of non-Newtonian mechanics *.

It has been shown from the principle of relativity † that the mass of a moving body is given by the equation

$$m=\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}},$$

where m_0 is the mass of the body at rest and c is the velocity of light. Substituting in equation (1) we obtain

$$\mathbf{F}=\frac{d}{dt}\left(\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}\mathbf{u}\right)=\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}\frac{d\mathbf{u}}{dt}+\frac{d}{dt}\left(\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}\right).\mathbf{u}. \quad (2)$$

From an inspection of equations (1) and (2) it is evident that the force acting on a body is equal to the sum of two vectors, one of which is in the direction of the acceleration $d\mathbf{u}/dt$ and the other in the direction of the existing velocity $\mathbf{u}$, so that *in general the force and the acceleration it produces are not in the same direction.* If the force which does produce acceleration in a given direction be resolved perpendicular and parallel to the acceleration, it may be shown that the two components are connected by a definite relation.

* This definition of force was first used by Lewis (Phil. Mag. xvi. p. 705 (1908)). In Einstein's later treatment of the principle of relativity, *Jahrbuch der Radioaktivität*, iv. p. 411 (1907), he defines force by the equations

$$F_x=\frac{d}{dt}\left\{\frac{m_0u_x}{\sqrt{1-\frac{u^2}{c^2}}}\right\}, \quad F_y=\frac{d}{dt}\left\{\frac{m_0u_y}{\sqrt{1-\frac{u^2}{c^2}}}\right\},$$

$$F_z=\frac{d}{dt}\left\{\frac{m_0u_z}{\sqrt{1-\frac{u^2}{c^2}}}\right\}.$$

He there states that this definition has in general no physical meaning. We see, however, that these are merely the scalar equations corresponding to equation (2) above and hence derivable from equation (1), which is an obvious definition of force and has a physical meaning. In further support of this definition of force, it has recently been pointed out by the writer, Phil. Mag. xxi. p. 296 (1911), that, combined with the principle of relativity, it leads to a derivation of the fifth fundamental equation of electromagnetic theory in its exact form

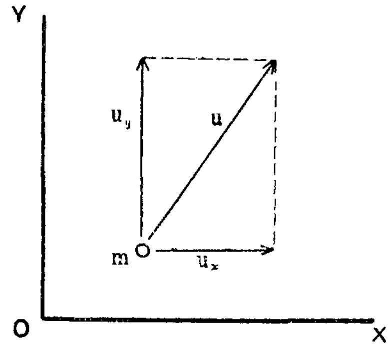
$$\mathbf{F}=\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{H},$$

there being no necessity for distinguishing between longitudinal and transverse mass.

† Lewis & Tolman, Proc. Amer. Acad. xliv. p. 711 (1909); Phil. Mag. xviii. p. 510 (1909).

Relation between the Components of Force Parallel and Perpendicular to the Acceleration.

Fig. 1.



Consider a body (fig. 1) moving with the velocity

$$\mathbf{u} = u_x \mathbf{i} + u_y \mathbf{j}.$$

Let it be accelerated in the Y direction by the action of the component forces F_y and F_x.

From equation (2) we have

$$F_x = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}} \frac{du_x}{dt} + \frac{d}{dt}\left(\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}\right) . u_x \quad . \quad . \quad (3)$$

$$F_y = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}} \frac{du_y}{dt} + \frac{d}{dt}\left(\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}\right) . u_y \quad . \quad . \quad (4)$$

Introducing the condition that there is no acceleration in the X direction, which makes $du_x/dt = 0$, further noting that $u^2 = u_x^2 + u_y^2$, by the division of equation (3) by (4) we obtain

$$\frac{F_x}{F_y} = \frac{u_x u_y}{c^2 - u_x^2},$$

$$F_x = \frac{u_x u_y}{c^2 - u_x^2} F_y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Hence in order to accelerate a body in a given direction, we may apply any force F_y in the desired direction, but must at the same time apply at right angles another force F_x whose magnitude is given by equation (5).

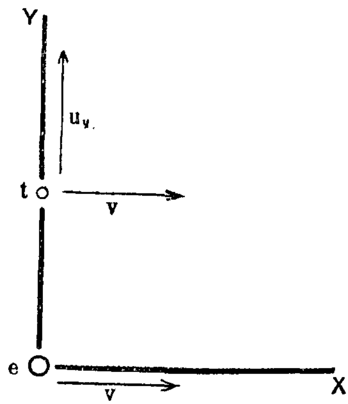
From a qualitative consideration, it is also possible to see the necessity of a component of force, perpendicular to the desired acceleration. Referring again to fig. 1, since the body is being accelerated in the Y direction, its total velocity and hence its mass are increasing. This increasing mass is accompanied by increasing momentum in the X direction even when the velocity in that direction remains constant. The component force F_x is necessary for the production of this increase in X-momentum.

In predicting the path of moving electrons with the help of the fifth equation of electromagnetic theory, $\mathbf{F}=\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{H}$, we find an interesting application of equation (5).

Application in Electromagnetic Theory.

Consider a charge ϵ constrained to move in the X direction with the velocity v and let it be the origin of a system of moving coordinates YϵX (fig. 2). Suppose now a test electron t, of unit charge, situated at the point $x=0$, $y=y$,

Fig. 2.



moving in the X direction with the same velocity v as the charge ϵ, and also having a component velocity in the Y direction u_y. Let us predict the nature of its motion under the influence of the charge ϵ.

The moving charge ϵ will be surrounded by electric and magnetic fields whose intensities at any point are given by the following expressions *, obtained by integrating Maxwell's

* Abraham, *Theorie der Elektrizität,* vol. ii. p. 86 *et seq.* (B. G. Teubner, Leipzig and Berlin, 1908).

four field equations, for the case of a moving point charge,—

$$\mathbf{E}=\left(1-\frac{v^2}{c^2}\right)\frac{\epsilon\mathbf{R}}{\mathrm{R}^3\left(1-\frac{v^2}{c^2}\sin^2\psi\right)^{\frac{3}{2}}} \quad . \quad . \quad . \quad . \quad (6)$$

$$\mathbf{H}=\frac{1}{c}\mathbf{v}\times\mathbf{E}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where $\mathbf{R}$ is the radius vector connecting the moving charge with the point in question and ψ is the angle between $\mathbf{R}$ and $\mathbf{v}$.

For the field acting on the test electron t, situated at the point $x=0$, $y=y$, we may substitute $\mathbf{R}=y\mathbf{j}$ and $\sin\psi=1$, giving us,

$$\mathbf{E}=\frac{\epsilon}{y^2\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}\mathbf{j} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and

$$\mathbf{H}=\frac{v}{c}\frac{\epsilon}{y^2\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}\mathbf{k}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

substituting into the fifth fundamental equation of electromagnetic theory,

$$\mathbf{F}=\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{H} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

we obtain the force acting on the unit test electron t.

[Note in the above equation that $\mathbf{v}$, the velocity of the electron, is for our case $v\mathbf{i}+u_y\mathbf{j}$.]

$$\mathbf{F}=\frac{\epsilon}{y^2\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}\mathbf{j}-\frac{1}{c^2}\frac{v^2\epsilon}{y^2\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}\mathbf{j}+\frac{1}{c^2}\frac{vu_y\epsilon}{y^2\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}\mathbf{i}, \quad (11)$$

or,

$$\mathrm{F}_x=\frac{\epsilon}{y^2\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}\frac{vu_y}{c^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\mathrm{F}_y=\frac{\epsilon}{y^2\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}\left(1-\frac{v^2}{c^2}\right) . \quad . \quad . \quad . \quad . \quad (13)$$

Under the action of the component force F_x we might at first sight expect the electron t to aquire an acceleration in the X direction. Such a condition, however, would not be in agreement with the *principle of relativity*, since from the point of view of an observer who is moving along with the charge ϵ, the phenomenon is merely one of ordinary *electrostatic* repulsion and the test electron should experience no change in velocity in the X direction but should be accelerated merely in the Y direction.

If, however, we divide equation (12) by (13) we obtain

$$F_x = \frac{vu_y}{c^2 - v^2} F_y, \quad \ldots \ldots \quad (14)$$

which agrees with equation (5), the necessary relation for zero acceleration in the X direction. The application of equation (5) thus removes a discrepancy which could not be accounted for in any system of mechanics in which force and acceleration are in the same direction.

Summary.

For non-Newtonian mechanics, it has been pointed out that force and the acceleration it produces are not in general in the same direction. A definite relation (equation 5) has been derived connecting the components of force parallel and perpendicular to the acceleration. For a special problem, the application of this relation has removed an apparent discrepancy between the predictions based on the electromagnetic theory and on the principle of relativity.

Ann Arbor, Mich.
March 25th, 1911.

XLV. *Notices respecting New Books.*

The Principles of Electric Wave Telegraphy and Telephony. By Prof. J. A. FLEMING, *D.Sc.*, *F.R.S.* Second Edition. Pp. xviii +906. With Illustrations. (London: Longmans, Green & Co. 1910.)

ALTHOUGH it is only five years since the first edition of this book appeared, yet, so vast have been the developments since its appearance, that this new edition is almost a new book. Those who are acquainted with Professor Fleming's previous treatises will be prepared for a masterly exposition of electric wave telegraphy; but he surpasses himself in his presentation of this subject. No one is better qualified than he for the task. Not only is he intimate with the mathematical and theoretical side of the subject; but he has for many years been in close contact with and has himself contributed largely to the rapid developments which have taken place both on the theoretic and experimental sides. It is impossible to do justice to this book in the space at our disposal — the enumeration of the headings of the chapters would almost exhaust it. The characteristic feature is the happy combination of theory with experiment, each being dealt with in an encyclopædic manner. For example, twelve pages deal with