THE FLOW OF LIQUIDS THROUGH CAPILLARY SPACES¹

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MOVEMENT OF LIQUIDS THROUGH CAPILLARY TUBES

In a former publication² an empirical equation was proposed to describe the movement of water and of aqueous solutions through such media as blotting-paper and soils. This equation is

$$y^n = \mathbf{K}t,\tag{1}$$

where y represents the distance through which the liquid has moved in the time t, and n and K are constants depending on the temperature and the specific substances.

Theoretical Discussion.—The movement of a liquid from one level to another through a capillary tube has been studied by Poiseuille.⁸ In his experiments the ends of the capillary tube opened into large reservoirs, and the length of the capillary tube through which the liquid passed was constant. The results show that the amount of water which passes through the tube in a given time is proportional to the difference in level between the liquid in the two reservoirs, proportional to the fourth power of the diameter of the tube, and inversely proportional to the length of the tube, provided the length is very long in comparison with the diameter. This is expressed by the equation

$$Q = C \frac{HD^4}{L}, \qquad (2)$$

where Q is the amount of liquid which passes through in unit time, H is the difference in level or "head," D and L are the diameter and length of the capillary tube respectively, and C is a constant. This formula may be applied to the advance

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² Bull. No. 30, Bureau of Soils, U. S. Department of Agriculture (1905).

³ Ann. Chim. Phys. (3) 7, 50 (1843).

or retreat of water in a capillary tube, only when the assumption be made that the energy gained or lost with change of velocity is negligible in comparison with the energy gained or lost in the change of level of the water or in the friction against the walls of the tube.

For a constant difference in level and for a tube of uniform bore, equation (2) may be simplified to the form

$$Q = \frac{C}{L}.$$
 (3)

Now, the amount of water which passes through the tube in unit time is a measure of the velocity, and if we are measuring the advance or retreat of a liquid, we may write $\frac{dL}{dt}$ instead of Q and equation (3) becomes

$$\frac{d\mathbf{L}}{dt} = \frac{\mathbf{K}}{\mathbf{L}},\tag{4}$$

which, when integrated between the limits L_1 , L_2 , becomes

$$L_2^2 - L_1^2 = K(t_2 - t_1).$$
 (5)

Returning now to equation (1), when n = 2, the equation becomes

$$y^2 = Kt, \tag{6}$$

and if y_1 , t_1 and y_2 , t_2 be corresponding values of the variables, we obtain

$$y_2^2 - y_1^2 = K(t_2 - t_1),$$
 (7)

which is identical with equation (5).

In equation (5) the constant K being proportional to C_1 of equation (3) varies directly as the difference in level or "head," and directly as the fourth power of the diameter of the tube.

Experiments.—Capillary tubes of about 1 mm diameter were laid on a millimeter scale on a horizontal table. A length of about 10 or 12 cm of the tube was bent down over the end of the table and this end dipped into the liquid. The tube was filled with the liquid by applying suction at the open end and the rate of retreat of the liquid was observed. The value of the quantity H of equation (2) may be found from the difference in level between the horizontal capillary tube and the natural position of the liquid in the capillary tube, which in the case of water, alcohol or benzene will be considerably above the free surface of the liquid into which the tube dips. The quantity L is the length of the capillary tube filled with liquid, including, of course, the vertical part of the tube. In the following tables x is the length of the horizontal part of the thread, l is the length of the vertical portion of the tube and t is the time taken for the liquid to move to any given point of the tube.

Equation (5) becomes

$$(x_2+l)^2 - (x_1+l)^2 = K(t_2-t_1).$$
(8)

The following tables give the results of several experiments upon the movement of water, alcohol, or benzene. In these tables K has been calculated from the above formula (8),

$$\mathbf{K} = \frac{(x_2 + l)^2 - (x_1 + l)^2}{t_2 - t_1}.$$

The values x_1 , t_1 , are taken from the table. The temperature did not vary over 0.2° C during any experiment.

Discussion of Results.—There are several sources of error in the results which have been obtained in the above manner. Where the value of x is small and consequently the velocity of retreat of the liquid is great, the possible error of reading is somewhat large. The tube may not have been absolutely uniform in bore over its whole length, and as the amount of liquid which passes through a tube is proportional to the fourth power of the diameter, very small variations in the diameter will have large effects on the velocity of flow. The surface tension of the liquid changes with the change of velocity and it is possible that this change may have had an appreciable influence on the results. Another source of error is involved in an assumption in the original deduction of the formula, viz., that the changing velocity or acceleration of a liquid flowing through the tube is negligible in comparison

	water, $t = 9.0$ $x_1 = 114.3$ $t_1 = 370$					
-	Time, <i>t</i> Seconds	Distance, x obs. cm	Distance, x Calc. (K = 40.9) cm	$K = \frac{(x+9)^2 - (123.3)^2}{t - 370}$		
	0	0.0	0	40.9		
	IO	13.0	13.0	40.9		
•	20	21.0	21.0	40.9		
	35	29.0	29.6	41.1		
	50	36.2	37.0	41.1		
	70	44.8	45.2	41.0		
	100	54.9	55.5	41.2		
	130	64.3	64.3	40.9		
	ıĞo	72.0	71.7	41.1		
	190	79.6	79.6	40.9		
	220	85.9	86.3	41.3		
	250	92.3	92.5	41.2		
	280	97.8	98.4	42. I		
	310	103.7	103.9	41.8		
	340	108.8	109.2	44.I		
	370	114.3				
	400	119.2	119.2	41.2		
	430	124.6	123.9	38.6		
	460	128.8	128.4	42.2		
	520	138.0	137.1	42.7		
		I. 1				

TABLE I Water l = 0.0 $x_t = 114.3$ t = 37

TABLE II

Time, t Seconds Distance, x obs. cm Calc. $(K = 60.5)$ $K = \frac{150^2 - (x + 10)^2}{370 - t}$	
0 0.0 I.4 59.8	
10 16.5 16.9 59.9	
20 26.2 26.3 59.9	
30 33.4 33.9 60.0	
40 39.3 40.3 60.2	
55 48.9 48.7 60.0	
70 56.3 55.9 59.8	
100 68.4 68.5 60.0	
130 79.4 79.3 59.9	
160 88.3 89.0 60.6	
190 97.2 97.8 60 7	
220 105.0 106.0 61.4	
250 112.9 113.4 61.2	
280 119.8 120.6 62.3	
310 126.9 127.4 60.5	
340 133.4 134.1 64.1	
370 140.0	

Water, l = 10.0, $x_1 = 140.0$, $t_1 = 370.0$

TABLE III

	Time <i>t</i> Seconds	Distance, x obs. cm	Distance, x Calc. (K = 69.8) cm	$\mathbf{K} = \frac{137.5^2 - (x+7)^2}{270 - t}$
	о	0.0	0.0	69.8
	10	20.3	20.5	69.8
	20	31.8	31.1	69.6
	30	39.7	39.4	70.1
	45	49.5	49.6	69.8
	60	57.4	58.2	70.3
	75	65.8	65.8	69. 8
	90	72.8	72.7	69.7
	105	7 8 .8	78.9	70.0
	120	84.7	84.8	70.0
•	150	96.0	95.6	69.1
	180	105.3	105.4	69.9
	210	114.4	114.3	69.5
	240	122.3	122.7	72.9
	270	130.5	_	

Alcohol, l = 7.0, $x_1 = 130.5$, $t_1 = 270$

TABLE IV

		· -	
Time, t Seconds	Distance, x obs. cm	Distance, x Calc. (K = 133.1) cm	$K = \frac{140^2 - (x + 12)^2}{147 - t}$
0	0.0	0.0	133.1
2	10.0	5.3	131.8
12	30.0	28.4	132.1
22	43.0	42.4	132.6
32	53.3	53.5	133.3
42	62.6	63.0	133.7
52	72.0	71.4	132.0
62	80.0	79.0	131.0
72	87.0	85.0	130.6
82	93.2	92.8	131.3
92	98.8	98.8	133.1
102	104.4	105.5	134.4
I I 2	110.2	I 10.2	133.4
122	115.7	115.6	131.7
132	120.7	120.7	132.7
147	128.0		[

Benzene, l = 12.0, $x_1 = 128$, $t_1 = 147$

with the other forces at work, *viz.*, the surface tension, the friction, and the "head."

In spite of all these sources of error the results are well described by the equation derived from Poiseuille's formula. MOVEMENT OF LIQUIDS THROUGH POROUS MEDIA

It is not possible to measure the rate of movement of a liquid through porous substances such as blotting-paper and soils in the same manner as this movement through capillary tubes has been measured. In the case of a liquid running into a clean glass tube, the glass becomes wet in advance of the main body of liquid, and there is also the same phenomenon in the case of porous material. In this latter case, however, there is no way of distinguishing between that portion of the substance which is merely wet by a capillary. film of liquid over the grains or fibers, and that portion of the substance whose interstices are filled with liquid. In the following experiments the distance which had become wet by the liquid was read at various times. Strips of blotting-paper were cut 3 cm wide and about 20 cm long and then scaled by marking with a pencil. At the beginning of the experiment one end of the paper was immersed in the liquid. In the case of the soils, tubes of 1 cm diameter were filled with dry soil and were packed as nearly as possible in the same way by tapping the end of the tube. At the beginning of the experiment the tubes were lowered to touch the surface of the liquid. No precaution was taken to preserve a constant temperature, but temperature readings did not vary more than 0.2° C during any experiment. The effect of gravity is negligible in these cases for the same results have been obtained when the paper is held in a horizontal position as when it stands in a vertical position. The capillary forces are much greater than the gravitational forces where the distance to which the water has risen is small in comparison with the distance to which it may ultimately rise.

The following tables give the results of experiments upon the rate of flow of water and of a solution of potassium chloride through blotting-paper and upon the rate of flow of water through columns of dry soil. The columns of the tables headed by "K" give the values of this constant calculated from the equation

 $v^n = \mathbf{K}t.$

The value of *n* was found graphically. Taking logarithms of both sides of this equation we have

$$n \log y = \log K + \log t,$$

and when $\log y$ and $\log t$ are plotted, all the points are found to lie on a straight line, showing that the original equation describes the facts accurately. It will be observed that the slope of this line is n and that one of the intercepts is log K.

Distined		
Time, t Min.	Height, y cm	$\binom{\mathrm{K}}{(n=2.3)}$
0.0	o .o	
0.25	1.51	10.4
0.5	2.29	· · ·
0.75	2.4I	10.2
1.0	2.72	10. 2
1.25	2.97	10.0
1.5	3.20	9.9
1.75	3.44	I O. I
2.0	3.59	9.7
2.5	3.95	9.7
3	4.28	9.7
	4.87	10.0
4 5	5.39	10.0
7	6.28	10,1
10	7.33	10.1
13	8.11	9.9
20	9.85	10.1
25	10.8	10.1
30	1 1.6	10.0
40	13.1	9.8
50	14.2	9.4
60	15.4	9.5

TABLE V Distilled water in blotting-paper

TABLE VI

Time, tHeight, y K Min. cm (n = 2.35)ο 0 0.25 1.41 9.0 8.6 2.05 0.5 0.75 2.40 10.4 I 2.77 11.0 1.5 3.33 11.3 2 II.I 3.74 4.54 3 11.7 5.13 4 11.7 5 5.69 11.8 7 6.52 11.2 7.62 11.8 10 8.46 13 11.6 16 9.23 11.6

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Potassium chloride solution in blotting-paper.

TABLE VII

10.2

11.3

11.8

11.8

11.9

10.9

Distilled water in columns of Penn loam soil ($T = 18^\circ$)

Time, t Min.	Height, y cm	$\frac{\mathrm{K}}{(n=2.05)}$
0	0	
I	0.77	0.58
2	1.15	0.66
3	1.58	0.85
	1.87	0.90
4 5 7	2.29	1.10
7	2.56	0.97
10	2.95	0.92
15	3.64	0.94
20	4.15	0.93
30	5.03	0.93
40	5.96	0.97
50	6.59	0.96
60	7.18	0.95
75	7.90	0.93
90	8.69	0.94
105	9.29	0.92

Disti	Distined water in Fein io		$\lim \operatorname{son}\left(\mathbf{I} - 2\mathbf{I}\right)$	
	Time, <i>t</i> Min.	Height, y cm	$\binom{\mathrm{K}}{(n=2.25)}$	
	0	0		
	I	1.15	1.37	
	2	1.54	1.33	
	3	1.85	1.33	
`		2.08	1.30	
	4 5	2.28	1.28	
	7	2.59	I.21	
	10	2.97	1.16	
	15	3.47	1.10	
	20	3.90	1.07	
	30	4.67	1.06	
	40	5.39	1.11	
·	50	5.90	1.09	
	60	6.47	I.I2	
	75	7.20	1.13	
	90	8.03	I.2I	
	105	8.72	1.25	

TABLE VIII Distilled water in Penn loam soil (T = 21°)

The values of K in these tables are constant within the errors of experiment, and consequently the formula proposed is accurate within the limits of the experimental errors.

In the following table there have been calculated the results of Loughridge¹ upon the passage of water through an alluvial soil from the Gila River. The figures have been taken from a diagram and are therefore approximate only.

TABLE IX Alluvial Soil, Gila River				
Time, t Min.	Height, y Inches	$\begin{array}{c} \mathrm{K} \\ (n = 1.86) \end{array}$		
2	1.5	1.05		
5	2.4	1.02		
IO	3.6	1.08		
15	4.3	1.01		
30 60	6.3	1.05		
60	9.2	1.07		

¹ Report Agr. Exp. Stat. Univ. California, 1893-94, p. 93.

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After 125 days water rose in this soil to a height of 47 inches. Other results have been given in tabular form in the same paper but it is not possible to obtain from the diagram sufficiently close approximations to the figures, to test the formula. In the majority of cases described in the literature the times of observations are so long, and the heights observed are so great that gravitational and possibly other effects are introduced, which materially alter the results, and consequently the formula does not hold for such cases. It does not seem worth while to cite further cases in illustration of the formula, as the specific data do not in themselves possess any special value in this connection.

SEPARATIONS BY ABSORBING MEDIA

When a solution passes through a porous material there is very often a separation effected, either of different solutes from each other, or the solution into some of the pure solvent and a stronger solution, or both effects may be superimposed. The separation of a solution into some of the pure solvent and a stronger solution has been observed by J. J. Thomson¹ in the case of potassium permanganate solution rising through filter-paper. The paper is wet by the solvent, water, for a considerable distance in advance of the color produced by the permanganate. Trey² has observed that when a solution containing copper and cadmium salts is allowed to run into filter-paper, the cadmium salt moves faster than the copper and thus a partial separation is effected. Goppelsröder³ has taken advantage of the different absorptive power of paper for different dyes and has succeeded in separating a rose-red colored substance, probably fuchsine, from commercial azuline, the azuline thereby losing its violet tint and acquiring a purer blue color.

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¹ Applications of Dynamics to Physics and Chemistry, 1888, p. 191.

² Zeit. anal. Chem. 37, 743 (1898).

⁸ Verhandlung d. Naturforsch. Ges. in Basel, **3**, 263 (1861–63); Mitteil. d. techn. Gewerbemuseums, Wien (1889).

The separation of solute and solvent has been made a velocity study and here two rates have been measured, the rate at which the paper becomes wet and the rate of the movement of the colored solution through the paper, and some experiments have been made upon the rate of absorption of solutions of eosine by blotting-paper.

Solution containing 5.82 grams per liter				
Time Min.	Height Wet cm	$\binom{\mathrm{K}}{(n=2.2)}$	Height Colored cm	$\binom{\mathrm{K}}{(n=2.2)}$
0	0		о	
0.25	1.28	6.9	1.08	· · · · ·
0.5	1.72	6.6	1.41	4.3
0.75	2.15	7.2	1.67	4.I
I	2.54	7.7	1.92	4.2
1.5	2.97	7.3	2.44	4.7
2	3.36	7.3	2.69	4.4
2.5	3.74	7.3	2.97	4.4
3	4.05	7.2	3.23	4.4
4	4.59	. 7.1	3.69	4.4
5	5.10	7.2	4.10	4.5
7	6.03	7.4	4.75	4.4
10	7.05	7.3	5.54	4.3
20	9.54	7.1	7.62	4.4
25	10.4	6.9	· 8.87	4.8
40	12.6	6.6	9.98	4.0

TABLE XSodium eosine solution in blotting-paperSolution containing 5.82 grams per liter

In Figure 1, Curve 1 shows the height to which pure water rose at different times. Curve 2a shows the height which had become wet, and Curve 2b shows the height which was reddened. The original solution in this case contained 5.82 grams per liter. A strip of paper was dipped into a solution containing twice the amount of the dye per liter. Curve 3a represents the rate at which the paper became wet and Curve 3b the rate at which it became red.

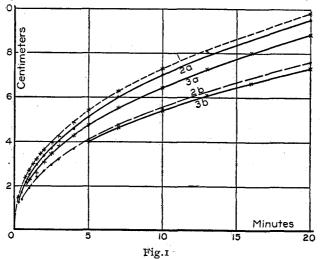
It will be seen from the curves that the rate at which water travels is decreased by the presence of sodium eosine, that the greater the concentration the less is the rate, and that

TABLE	XI
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Sodium eosine solution in blotting-paper Solution containing 11.64 grams per liter

the separation of water and eosine is more marked for the lower concentration.



The effect of potassium chloride on the rate at which sodium eosine is absorbed was determined at 25° C, and the results given in Tables VI and XII are plotted in Figure 2.

TABLE XII

Solution containing sodium eosine and potassium chloride in blotting-paper. Solution containing 50 grams KCl and 5.83 grams sodium eosine per liter

Time Min.	Height Wet cm	$\binom{\mathrm{K}}{(n=2.3)}$	Height Colored cm	$\begin{pmatrix} \mathbf{K} \\ (n=2.9) \end{pmatrix}$
0	0	_	о	
0.5	1.80	7.6	0.77	0.94
I .O	2.56	7.9	1.03	1.09
1.5	3.08	8.9	1.28	1.36
2	3.51	8.9	1.4 I	1.35
3 4 5 7	4.28	9.4	1.67	1.48
4	4.87	9.5	1.80	1.37
5	5.39	9.7	2.05	1.61
	6.23	9.6	2.39	1.78
IO	7.23	9.5	2.56	1.52
13	18.08	9.4	2.69	1.36
16	8.87	9.3	2.87	1.33
20	9.61	9.2	3.08	1.31
25	10.51	9.0	3.33	1.31
30	11.5	9.2	3.54	1.30
40	13.3	9.6	3.98	1.37
Centimeters				
2	*			Minutes

It is quite evident that the effect of the potassium chloride is to decrease markedly the rate at which the dye travels through the paper. We have thus shown that the solutes do not act independently of one another, but the nature and extent of this dependence have yet to be determined.

The rate at which certain dye solutions rise in columns of dry soil has been measured and the general results are quite similar to those obtained for the rise of such solutions through paper. Two dyes were first tried which were markedly absorbed by paper, that is, the color rose only a small distance through the paper.

$= 10^{-10}$						
Time Min.	Height wet by safranine solution containing 2 grams per liter cm	$\underset{(n=2.05)}{\mathrm{K}}$	Height wet by methylene blue solution contain- ing 2 grams per liter cm	K (n = 2.05)		
Ο	о		0			
I	0.90	0.80	0.90	0.8 0		
2	1.28	0.82	1.54	I.2I		
3	1.67	0.95	1.87	I.2I		
4	1.92	0.95	2.13	1.18		
4 5 7	2.13	0.94	2.26	1.07		
7	2.44	0.89	2.69	1.09		
IO	2.82	0.94	3.20	1.08		
15	3.41	0.83	3.98	1.13		
20	3.93	0.83	4.70	1.19		
30	4.67	0.80	5.51	I.I2		
40	5.51	0.83	6.39	I.12		
50	6.03	0.80	6.98	1.08		
60	6.67	0.82	7.59	1.0б		
75	7.36	0.80	8.74	I.14		
90	8.03	0.80	9.05	I.02		
105	8.85	0.83	9.67	I.00		
120	9.36	0.83	10,0	0.94		

TABLE XIII Dve solution in columns of Penn loam soil ($T = 18^\circ$)

It will be observed that the dyes moved into the soils too slowly for any measurements and that the rate at which distilled water rose was intermediate between the rates at which the soils became moist from the two dye solutions. These differences are, however, not great enough to draw any general conclusions, for they might be attributed to slight differences in the packing of the soil in the tubes.

Two other dyes were tried, coralline and indigo carmine, which move through paper rather readily. It was found very difficult to read the height to which the dye had risen because it rose to a considerable height on one side of the tube and would scarcely rise at all on the other side, in some cases the difference being as great as 4 cm. However, a rough estimate was made of the average height of the dye in the tube when viewed from all directions. It was not found so difficult to measure the height to which the water rose, for in this case the differences of level were never more than 1 cm. The following results were obtained for the same kind of soil as before, Penn loam, and at the same time the rate at which coralline rose in Podunk fine sandy loam was measured. The temperature was 21° C.

TABLE XIV

Indigo carmine in Penn loam soil (T =	21°)
Solution containing 2 grams per liter	

Time Min.	Height Wet cm	$\binom{\mathrm{K}}{(n=2.25)}$	Height Colored cm.	$ \begin{array}{c} K \\ (n = 2.25) \end{array} $
0	0		0 .	
I	1.28	1.75	0.64	0.37
2	1.67	1.59	0.90	0.39
3	2.05	1.68	-	
4	2.26	1.56	—	·
5	2.49	1.56	I.02	0.21
7	2.74	1.38		
IO	3.20	1.40		<u> </u>
15	3.72	1.29		
20	4.28	1.32	I.92	0.22
30	5.10	1.31	<u> </u>	
40	5.77	1.29	2.69	0.23
50	6.41	1.26	3.20	0.28
60	6.90	1.29		
75	7.46	1.23	·	
90	8.74	1.46	3. 5 9	0.20
105	9.00	1.33	- .	

TABLE XV

Coralline solution in Penn loam soil ($T = 21^{\circ}$) Solution containing 2 grams per liter

Time Min	Height Wet cm	$\binom{\mathrm{K}}{(n=2.25)}$	Height Colored cm	$\binom{K}{(n=2.25)}$
0	о		0	
I	I.02	1.05	<u> </u>	
2	1.36	1.00		
3	1.61	0.98		
4	1.92	1.08		
5	2,10	1.06		
7	2.39	1.01		—
IO	2.74	0.98	·	
15	3.49	I.II		
20	4.05	1.16		
30	4.67	I .07	0.51	0.007
40	5.46	1.14	0.69	0.011
50	6.06	1.15		
60	6,62	1.17	0.77	0.009
75	7.10	1.10		
90	8.00	I.20	0.90	0.009
105	8.46	1.16]

TABLE XVI

Coralline solution in Podunk fine sandy loam $(T = 21^{\circ})$ Solution containing 2 grams per liter

Time Min.	Height Wet cm	$\binom{K}{(n=2.05)}$	Height Colored cm	_	
0.0	0				
0.5	3.46	25.5			
I.0	5.03	27.5	·		
1.5	6.41	30.2			
2	7.44	30.7	1.54		
3	9.00	30.2			
4	10.5	31.0	—		
4 5	11.6	29.6	2.3I		
7	13.6	30.2			
10	15.9	29.0	2.56		
15	19.0	28.0	2.74		
20	21.2	26.2			
25	23.0	24.8	3.08		

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The above tables indicate that the same empirical formula will describe the rate of flow when a solution is used which undergoes a change while moving through the medium. Both the measurable heights, *viz.*, the height wet by the solvent and the height colored by the dye, when plotted against the time, give the same kind of parabolic curves. In all these cases the value of n lies above 2 which is the value of n when a liquid is moving through a capillary tube.

Conclusion

In this paper it has been shown that:

1. The movement of water through a capillary tube is accurately described by the equation $y^2 = Kt$, an equation derivable from Poiseuille's equation by making the assumption that the acceleration is negligible in comparison with the velocity. The same restrictions regarding the size of the tube hold in this case as in Poiseuille's experiments, *viz.*, the length of the tube must be great as compared with the diameter.

2. An empirical equation, $y^n = Kt$, describes the movement of water and solutions through porous materials, such as filter-paper and dry soils. The value of *n* lies above 2 in most cases.

3. The empirical equation, $y^n = Kt$, describes the movement through soils and paper, of solutions which suffer change during the process. Both the advance of the solute and of water in the medium are described by the formula.

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