THE MECHANICAL PRINCIPLES OF BRENNAN'S MONO-RAIL CAR.*

BY

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[This paper is an endeavor to develop by brief elementary methods the mathematics and physics of the gyroscope, and their application to Brennan's Mono-rail Car. The writer is of the opinion that anyone who is familiar with elementary mechanics will be able to understand this paper and use the equations in practical design. The relations developed are so simple and the proof so plain that it is hoped that they will assist in perfecting the design of various sorts of balancing devices by revealing clearly the secret of the mechanical principles involved in them. Outline: (1) Account of trials of Brennan car. (2) Analogy of centrifugal and gyroscopic action. (3) Equilibrium of the car under unbalanced loads. (4) Equilibrium of the car running on a curve. (5) Limits of stability of the car. (6) Nutation during the establishment of equilibrium.]

EARLY in 1907, Louis Brennan, the inventor of the Brennan torpedo, exhibited before the Royal Society of England a working-model of a mono-rail car which was able to preserve its equilibrium perfectly while in motion or at rest, while travelling around sharp curves, while running on a swinging cable, or subjected to sudden shifting of load from side to side. The apparatus by which automatic balance is preserved in this remarkable manner consists of a pair of heavy fly-wheels mounted on the car which rotate at high speed in opposite directions about axes

* Since the time when Foucault, in 1852, applied the gyroscope to demonstrate ocularly the rotation of the earth on its axis and the precessional motion of the latter, various efforts have been made to utilize gyroscopic action for practical purposes. The first, and until within recent years, the only attempt in this direction of which there is any authentic record, is that of Prof. Piazzi Smyth who, in 1856, devised a telescope stand provided with a gyroscopic apparatus calculated to maintain it level while supported on the constantly changing plane of the deck of a vessel at sea. The apparatus was tested by Prof. Smyth on board a naval vessel in the course of a sea voyage and, according to authoritative reports of the experiment, was found to serve its purpose. It appears, however, that the mechanism was too complicated for regular use. About 1897, an Austrian engineer, M. Obry, applied the principle of the gyroscope, namely the tendency of the fast-revolving wheel to maintain its given direction in space, for the purpose of automatically steering a torpedo. His invention was subsequently perfected and successfully applied by the Whitehead Torpedo

transverse to the car. The gyroscopic effect of these rotating wheels prevents the car from toppling over, somewhat as a top is prevented from falling while spinning.

On November 11, 1909, a further demonstration was given with a full-sized car in the War Office grounds of Great Britain, at Chatham, England. Accounts state that the car which was there operated with entire success was 40 feet long, 10 feet wide and 13 feet high, and weighed 22 tons. The car ran on a single rail around a circular track 220 yards in circumference at a speed of 25 miles an hour, carrying a load of 40 passengers on its platform where they had entire freedom of movement. The balance was automatically preserved by the action of two gyroscopes 3½ feet in diameter, weighing ¾ of a ton each, and revolving at the rate of 3000 revolutions per minute. The gyroscopes run in a vacuum to reduce friction. A gasolene engine was used to propel the car and keep up the speed of the gyroscopes.

It is reported that a German mono-rail car is soon to be exhibited embodying the same principle, of which the motive power will be electrical. Indeed, synchronous motors would apparently furnish an ideal means of keeping the two gyroscopes in step with each other, and the trolley would be a natural means of propulsion for such a car.

Models to illustrate the working of the Brennan car constructed in this country from the designs of Professors Crew and Tatnall, Northwestern University, are accessible to the public,

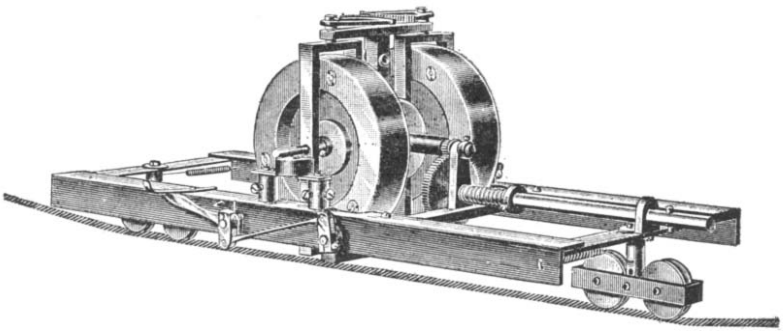
Co. In 1904, Herr Otto Schlick brought out before the British Institute of Naval Architects a gyroscopic mechanism which was subsequently developed into an apparatus to prevent the rolling of ships at sea. This appliance was successfully demonstrated in 1906 in experiments on a German torpedo-boat, and in 1908 was practically applied on an English steamship by the Neptune Works at Newcastle. About the same time, according to report by U. S. Consul-General Wright, from Munich, April, 1905, another German inventor, Dr. Anschütz-Kämpfe, produced an apparatus in which the gyroscopic principle of maintenance of direction in space is applied as a compass. Experiments with this instrument on board a German warship appear to have proven it capable of replacing the magnetic needle under conditions wherein the latter is unreliable. The latest practical application of the gyroscope as represented by the Mono-rail Car of Mr. Louis Brennan was the subject of a lecture by Prof. W. S. Franklin of Lehigh University before the Institute at its stated meeting in February last and the following paper affords a lucid demonstration of the principle involved.—ED.

as shown in Fig. 1. It is said by the makers, to embody the following essential points, which are here quoted from their published statement, with a few emendations and amplifications:

A. The use of two gyroscopes or fly-wheels, mounted side by side in frames, as shown in Fig. 1, and rotating about a horizontal axis in opposite directions with equal speed. The precessional couple about an axis lengthwise of the car, due to the turning of the car about a vertical axis in rounding a curve, is equal and opposite for the two gyroscopes, hence the upsetting torque arising from this source vanishes.

B. The employment of a pair of shelves or plates as friction surfaces, attached to the sides of the car, one on each side. When

FIG. 1.



the car begins to tip over to one side, one of these shelves is raised and begins to press against the rotating end of the gyroscope axle, which projects over it. This pressure causes the gyroscope to turn, or precess, about a vertical axis. The frame of the second gyroscope is geared to that of the first in such a way that it is forced to turn equally in the opposite direction, as shown at the top of Fig. 1. The axle, on account of friction on the shelf along which it tries to roll has a force exerted on it whose moment tends to increase the precession. The moment of this frictional force about a vertical axis has the effect, not to increase the precession already taking place, but instead, to cause precession about the longitudinal axis. The opposite moments acting on oppositely rotating gyroscopes conspire to cause precession in the same direction about the longitudinal axis of the car. This rights the car and tips it slightly to the other side, to be brought back again by the second shelf. The centre of gravity of the whole car is thus kept oscillating very slightly on either side of the line of upward thrust of the rail.

C. The use of a second pair of shelves next to the first pair, each engaging an idle roller whose friction is negligible, attached to the gyroscope frame, and serving to return the frame, after displacement, to its normal position, transverse to the car.

D. The use of a single central frame in which both the gyroscope frames turn, or precess, which is itself solidly mounted on an axis lengthwise of the car, and on the same level with the gyroscope axes, so that this frame and the gyroscopes it carries can together tip to the right or left with reference to the car, or the car can be tipped with reference to them, to an amount determined by the clearance between the shelves and the gyroscope axes over them.

This description of the car and its action will become more clear after following the developments in the following pages.

Gyroscopic action has up to the present time not been regarded as one of the available devices for securing mechanical effects in machine design, but rather as one to be avoided, and where unavoidable, one to be guarded against in the same manner as is done in most cases of centrifugal action. For example, the centrifugal force tending to burst a fly-wheel must be carefully guarded against in design and operation, but on the other hand centrifugal action is boldly made use of as the essential element of design of engine and turbine governors of all sorts. In the same way the gyroscopic action of armatures on shipboard must be carefully provided against. But as appears from this invention, we must, hereafter, be prepared to make use of the peculiar properties of gyroscopic action to produce mechanical effects that are otherwise unattainable.

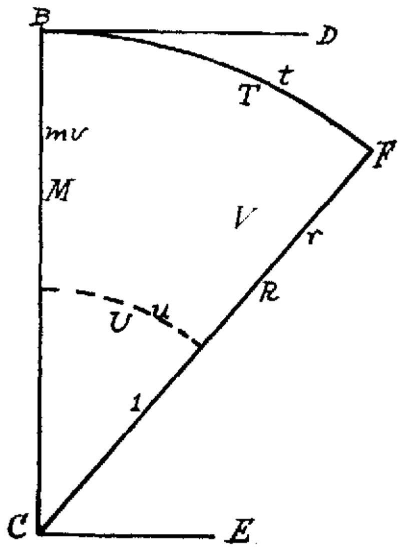
To this end, it is the purpose of the present paper to set forth the principles of gyroscopic action in general and, in particular, to describe their application to the mono-rail car, in such an elementary and geometrical form as to make the action intelligible and calculable without recourse to complicated mathematics. This may be done most readily by help of the close analogy, or say identity, which exists between the mathematics of centrifugal action and that of gyroscopic action.

This we now proceed to develop. Suppose a body of mass m revolve in the horizontal plane of Fig. 2 about C to which it is attached by a string of length r. If the linear velocity of m along the circle of radius be v, then it is well known that the tension of the string

$$t = \frac{mv^2}{r} = mu^2r = mvu \tag{1}$$

in which the deviating (or centrifugal) force t continually draws m away from the straight path it would otherwise follow along

FIG. 2.



a tangent to the circle, and u is the angular velocity of revolution, *i.e.*, the linear velocity of a point on r at the distance of unity from C.

Equation (1) may be written in the form

$$l : mv :: u : t \tag{2}$$

from which it appears that if the distance $r = CB = CF$ be taken on some arbitrary scale to represent graphically the numerical amount of the momentum mv, and it be laid off at right angles to the direction of the momentum along BF, viz., at successive instants along r in its successive positions, then on the same scale the length of the arc BF at right angles to the successive positions of r measures the impulse of the deviating force, which is numerically the same as the tension t, the tension at any instant being at right angles to the arc, just as mv at any instant is at right angles to the position of r at that instant.

The elementary considerations by which equation (1) is established will not be here developed, as they may be found in any treatise on the mechanics of motion. Equations (1) and (2) express a necessary relationship of the mechanical and physical quantities concerned. For example, the tension t can be increased only by some suitable change in one of the other quantities, such as using either a greater mass m, or imparting to it a higher velocity v, or making r, the radius of the circle of motion, smaller.

Now in order to develop the corresponding equations for gyroscopic action, let us assume that a gyroscope or fly-wheel whose moment of inertia about its axis of rotation is M is rotating with its axis initially horizontal at CB. It is to be noticed that the actual linear motion of its particles in space is at right angles to CB, as was the linear motion of m previously treated. Let the axis have a point of support at C only, and let the centre of gravity of the total weight W supported at C be situated at B, a distance of $CB = R$ from C, so that it requires a couple of torque $T = WR$, combined with a vertical force at $C = W$ (the weight acting at B) in order to keep W from falling. Let the gyroscopic wheel revolve about CB with an angular velocity V, then its constant angular momentum about CB is MV.

On some arbitrary scale let $R = CB = CF$ be taken to represent graphically the numerical amount of the angular momentum MV. On the same scale the arc $BF = T$ may be taken to represent the deviating angular impulse of the torque T, which is numerically the same as the torque T. This angular impulse continuously generated by W at right angles to the angular momentum MV is to be compounded with it in precisely the same manner that the linear impulse generated by the deviating tension t is compounded with the linear momentum mv, instant by instant, during its generation, in order to produce a uniform deviation in a circle.

The torque T due to the weight W at B acts always at right angles to the axis of the angular momentum MV just as the tension t at B acts always at right angles to the linear momentum mv, hence MV and T must be compounded according to the same laws as mv and t.

Therefore

$$1 : MV :: V : T \tag{3}$$

or

$$T = MVU = \frac{MV^2}{R} = MU^2R \tag{4}$$

in which U is the angular velocity of R about C in the horizontal plane and the gyroscope is said to precess or turn at the rate U. The apparently paradoxical action of the gyroscope consists in this, that while rotating about one axis a torque acting on it apparently tending to turn it about a second axis at right angles to the first, actually does turn it about a third axis at right angles to the other two, and this is its only effect.

Equation (4) may also be written:

$$W = \frac{T}{R} = \frac{MVU}{R} = \frac{MV^2}{R^2} = MU^2 \tag{5}$$

These equations (3), (4) and (5) express physical relations that must necessarily exist in gyroscopic action during dynamic stability of motion. They show, for example, that a decrease in the rotary velocity V through friction or otherwise would require a more rapid precession U in order to support the given weight W, a fact readily observed; or again, V remaining unchanged, any increase in W will produce a corresponding increase in U, as appears at once experimentally.

In order to comprehend clearly how the direction of precession is related to the rotation and torque, suppose the rotation of the gyroscopes appear clockwise to an observer looking from B towards C, then to an observer looking from D to B gravity tends to make the axis BC rotate about C, so that the impulse of T is clockwise also, about a horizontal axis EC, which is parallel to the tangent at B, to an observer looking from E to C.

This angular impulse of T, generated instant by instant, is continuously compounded with the constant angular momentum MV. Since it acts always at right angles to MV, its effect is not to increase or decrease MV but solely to change its direction, so that the axis of rotation R is steadily moved forward to successive positions intermediate between BC and EC and thus at the end of a unit of time it occupies some position CF. It will evidently move to a position between BC and EC whenever both are clockwise, which will therefore also make the precession U clockwise when the observer looks down on C.

Careful attention should be given to the relative directions of rotation just indicated for the three quantities T U V, which are arranged in alphabetical order, are mutually at right angles, and are so situated with respect to each other that they follow each other cyclically clockwise to an observer looking towards C from any point in the solid angle between them, and they each indicate clockwise rotation to an observer looking along either of them towards C. Considered in this way it is not difficult to analyze gyroscopic effects into clockwise actions, since every rotation is clockwise for one aspect or the other. It further appears that a force applied at right angles to W in an attempt forcibly to increase or decrease the precession U would fail to do so, but would instead produce a precession in a vertical plane, thereby causing W to rise or fall according to the above rule. It would greatly assist the student of the mono-rail car to use a pair of graphical models made of corks into which three pins of different kinds are stuck at right angles, one along the axis of the cork to represent V or MV, and two others at right angles for T and U respectively, placed in the order mentioned above.

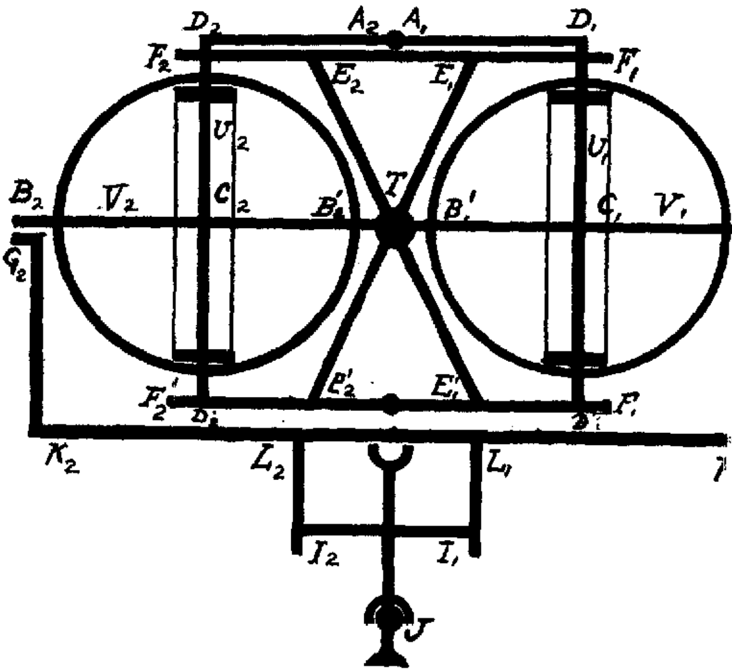
In all this discussion of centrifugal action and gyroscopic action it will be noticed that we have carefully confined our attention to the relation between the external applied forces and the motions ensuing, and have not considered the equal and opposite internal forces that are developed in the form of reactions in equilibrium with the applied forces. This has been done for the sake of simplicity and to avoid all possible confusion as to cyclic order, etc. Its correctness as a method can no more be questioned than can its application to the case of motion of a falling body where the external force of gravitation acts and motion ensues, and no regard need be had to the fact that by its inertia the body develops an internal reaction equal and opposed to the force of gravitation.

A vertical transverse section of the gyroscopic equilibrating parts of Brennan's mono-rail car may be represented in diagram as in Fig. 3, and in horizontal section or ground plan in Fig in which the same letters refer, so far as possible to correspc ing parts in both figures at once.

The arrangement consists of a pair of gyroscopes revolv on horizontal axes B_1C_1, B_2C_2 at the same speed $V_1 = V$ opposite directions, so that a spectator looking along either from B_1 to C_1, or B_2 to C_2 will see the wheels rotating clockw

This speed is to be maintained by applying so much pc

FIG. 3.



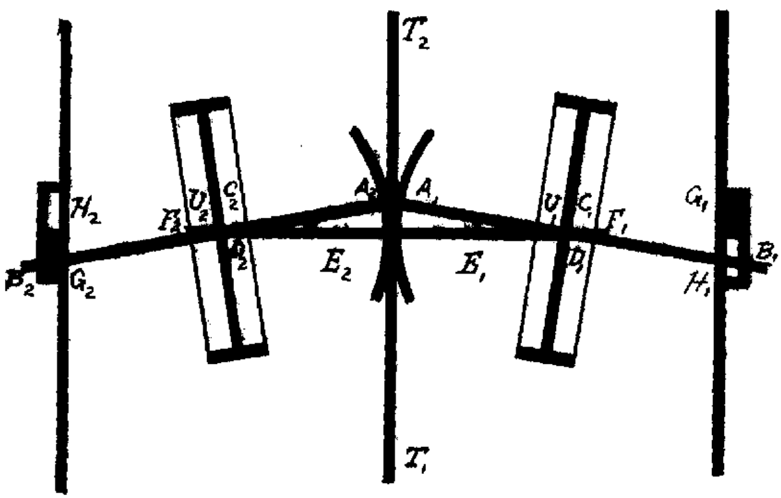
Elevation

as may be necessary to overcome friction, preferably b chronous mot rs upon the axes BC. The axes BB are in bearings ir the vertical frames which are shown in Fi the form of ircular rings. These rings swing about axes $D_1D'_1$, $D_2D'_2$ which in turn are carried in bearing vertical fram work EF rigidly fixed to an axle TT. Tl TT which li s lengthwise of the car, which is represe cross section by GKL, is solidly supported by bearings in the car, vertically above the rail J and the car wheel, which last revolves about the horizontal axis I_1I_2.

In Fig. 4 the gyroscopes are represented in a position which they would assume after having suffered a small deviation from their mean normal position (in which BB and TT are mutually at right angles) such as would occur after a small precessional rotation had taken place.

The gyroscopes are kept in step with each other in their motion about their vertical axes DD as well as in their motion about their horizontal axes BB, so that their rate and amount of

FIG. 4.



Plan

precession about DD is always equal and opposite. This first result is effected by means of two toothed sectors A_1 and A_2, which engage each other and which are rigidly attached to D_1 and D_2 respectively by arms AD which may be regarded as right angled prolongations of D_1 and D_2. Thus the arms AD turn with the rings about the vertical axes DD and we thus have $U_1 = -U_2$, as well as $V_1 = -V_2$, so that the rates of precession are equal and opposite as well as the rotary velocities.

The fact that both gyroscopes are carried by the rigid frame EF requires, however, that any turning of the gyroscopes about the axis T_1T_2 which is perpendicular to both BB and DD, shall be the same for both gyroscopes and in the same direction, not in opposite directions. Let this condition be expressed by the equation $U'_1 = U'_2$, in which U'_1 and U'_2 are precessions about T_1T_2.

In Fig. 3 the car is represented as in perfect balance in a horizontal position, but this is not the case in Fig. 4. Suppose an unbalanced load be placed upon the right side of the car between L_1 and K_1. That would tip the car and bring the plate at G_2 into contact with the rotating prolongation of the axis B_2C_2 at B_2.

This plate consists of two parts, a rough part G_2 intended to act as a friction brake on B_2 and a smooth part H_2 on which the axis B_2 can rotate without friction. As soon as this plate exerts an upward pressure upon the axis at B_2 the unbalanced load is transmitted to the gyroscopes in the form of a torque about T_1T_2 and the action is the same as though a pair of equal weights were hung upon the right hand half of each gyroscopic axis whose combined torque is the same as that of the unbalanced load. As already explained the effect produced upon the gyroscopes is not to cause any revolution about the axes about which T acts, but instead to produce precession about the vertical axes DD.

The right hand gyroscope then has its three clockwise actions TUV situated as already described in our treatment of the gyroscope, Fig. 2, and its precession is clockwise as we look upon it from above as represented in Fig. 4; while the left hand gyroscope having a clockwise rotation V_2 observed from B_2 and a clockwise T looking from T_1 to A_2 must have a clockwise precession U_2 viewed from below, thus producing the precession represented in the left gyroscope, Fig. 4.

The sectors A_1A_2 are therefore unnecessary so far as this action is concerned. But it is otherwise with respect to the action of the friction plate G_2. For, as the axis at B_2 rolls upon G_2 a horizontal force is exerted on B_2 which exerts a torque T'_2 say, about the vertical axis D'_2D_2, clockwise when viewed from below, in its effort to increase the rate of precession U_1. This it is unable to change. Instead of causing an increase in the precession U, the torque T'_2 will cause a precession U'_2 about an axis at right angles both to MV_2 and U'_2, *i.e.*, about T_1 T_2.

Let T' designate the torque of any couple produced by friction tending to turn either gyroscope or both about a vertical axis in a clockwise direction, then by equation (4),

$$T' = MVU' \tag{6}$$

gives the value of U', the precessional angular velocity produced about the horizontal axis T_1T_2, to right the car and bring its centre of gravity to a position vertically over the rail. That this statement is correct is evident upon considering the necessary relative position of $T'_2U'_2V_2$ for the left gyroscope, since they may be regarded as acting independently of TUV previously treated. The graphical model will show that the precession U'_2 is such as to right the car by giving it a precession opposite to the torque T of the unbalanced load, which precession will diminish and cease as the car is righted.

We have here considered the left gyroscope as if it alone were effective in righting the car. In fact both gyroscopes conspire to produce the same effect, for one-half of the torque T' is transmitted by the toothed sectors AA to the right gyroscope in which T' is, therefore, clockwise seen from above and U'_1 is clockwise looking from T_2 to A_1. This action therefore brings a mechanical stress on the toothed sectors to transmit part of T' from one gyroscope to the other, and in equation (6) M is the sum of the moments of inertia of both gyroscopes.

It is evident since the car as shown in Fig. 3 is in unstable equilibrium that left to itself it must immediately incline to the one side or the other. We have just shown how the gyroscopes act in case of an unbalanced load to return the car to the position of unstable equilibrium for car and load together.

Suppose now that the car tips still further in the direction in which the gyroscopes have been operating to right it, so that from unstable equilibrium it begins to tip over and down to the left, while the axes of the gyroscopes have stopped in the position shown in Fig. 4, as they would, since there is no unbalanced load to maintain precession. The first effect of this initial inclination to the left would be to bring the smooth plate H_1 into contact with the right axis at B_1. Whatever might be the force which H_1 exerted to press B_1 upward, its only effect would be to produce a precession about the vertical axes DD in the opposite direction from that which was produced when G_2 pressed B_2 upward. This precession about DD will serve to restore the axes BB of both gyroscopes to their normal positions transverse to T_1T_2, but it will not be accompanied by any precession about T_1T_2 until B_1 reaches G_1.

The effect of the plates G and H combined with the very small clearance between them and the axes is ordinarily to limit the deviations of the axes BB from their normal transverse position to very small amounts, for any tipping is almost instantaneously corrected and relatively large unbalanced loads may be applied suddenly without large disturbance of the gyroscopes. We shall return to this matter later.

The discussion thus far has considered what occurs while the car stands still or moves upon a straight track or cable. A single gyroscope would have been sufficient to produce all the results reached so far. The two have, however, been shown to act in unison so far, and the two become necessary only by reason of the forces brought into play in rounding a curve in the track.

So far as the centrifugal force which is developed in running the car around a curve is concerned, it acts precisely like an unbalanced load in applying an angular impulse about T_1T_2 to tip the car off the track. We have already explained in detail how the gyroscopes operate to balance the car on the rail in such circumstances. It should be remembered that in rounding a curve the car is in balance not when its centre of gravity is vertically over the rail, but instead, when it lies in the line of the resultant of the centrifugal force and gravity, just as it must also stand inclined in case of sidewise wind-pressure.

The one thing, however, which we have not yet considered is the torque developed in each gyroscope about the longitudinal axis T_1T_2 by reason of the common precession of both gyroscopes in the same sense with the car itself about a vertical axis in rounding a curve in the track. Were it not for the toothed sectors A_1A_2, the axes B_1B_2 would both maintain their original direction in space while the car turned the curve, but, unable to do this by reason of A_1A_2, they are both forced to precess with the car. Suppose the car moves in the direction T_1T_2 around a curve that turns to the right as A_1 does, then the precession is clockwise viewed from above, and the torque developed in the right gyroscope is formed by placing our cork model of TUV with V to the right, U vertical and T horizontal, so that

T acts about T_1T_2 and clockwise looking from T_1 to T_2, while the torque in the left gyroscope is about T_1T_2 but clockwise looking from T_2 to T_1, as will be readily perceived by comparing the cork model before mentioned with U upward in the two while V in one case is opposite to that in the other.

The two gyroscopes, therefore, develop equal and opposite torques about T_1T_2 which are transmitted to the rigid frame EF through the bearings DD and they hold each other in equilibrium by means of internal stresses induced in the frame. While running around a curve to the right a tension is caused in F_1F_2 and an equal compression in $F'_1F'_2$.

The gyroscopes are both needed in this case in order that each may equilibrate and neutralize the torque of the other, which would otherwise infallibly overturn the car on attempting to run it round a curve. Furthermore, it is evident that in order to secure the existence of this dynamic equilibrium it is necessary that the two axial velocities V be at all times equal and opposite.

It is not, however, necessary that V remain, at all times, constantly of the same value. The high speed of 3000 r.p.m. proposed by the inventor is for the purpose of ensuring that the product MV on which the stability of the car depends shall be large without making the size and weight of the gyroscopes large. He estimates that for ample stability their weight may then be only 4 per cent. of the weight of the car, or less.

Under those conditions the gyroscopes would store so large an amount of rotary energy (which is measured by $\frac{1}{2} MV^2$) that the fluctuations of velocity V, due to the expenditure by friction on G_1G_2 and otherwise, would be practically imperceptible in case the power applied by the synchronous motors to maintain V is sufficient in the long run to make good all losses by friction. In any case the power needed by the gyroscopes would be inconsiderable compared with that required for the propulsion of the car.

The foregoing discussion has all depended upon equation (4) and its applicability to the gyroscopic interactions actually occurring in the proposed arrangement. Equation (4) assumes that the axes BB are in their normal transverse positions at right angles to T_1T_2. But owing to the precessions about DD we are compelled to consider what modifications are necessary in order to take into account the effect of such deviations in the positions of BB as may take place. Let d designate the amount of this angular deviation from the normal position at any instant. Then the angular velocity V may be regarded as made up of two components, viz., $V\ sin\ d$ about an axis parallel to T_1T_2 and $V\ cos\ d$ at right angles to T_1T_2, of which the latter component alone is effective in equation (4) which may be written:

$$T = MUV\ cos\ d \qquad (4)$$

From this it is seen that with a given torque T the precession U increases with the deviation d, though the deviation is in fact usually confined to such small values that the precession is but slightly varied from this cause.

The practical effect so far as balancing is concerned is the same as if the gyroscopes were in normal position with axes at right angles to TT, and rotating with velocities $V\ cos\ d$, and besides that there was an additional pair of gyroscopes exactly like these mounted on axes parallel to T_1T_2 and rotating with velocities $V\ sin\ d$. The effect of these latter, were they in action, will be readily seen to be nil so far as unbalanced loads on the car are concerned, as well as their effect on the centrifugal action in rounding a curve; but not so in regard the precessional effect in rounding a curve, which precession, as previously remarked, consists in a turning of the whole car including the gyroscopes about a vertical axis.

In case the gyroscopes stand at a deviation d while rounding a curve the stresses before described as produced in FF are diminished in the ratio of V to $V\ cos\ d$, the latter being the component of the angular velocity of the gyroscopes in the plane of the frame. Besides this stress, a twisting stress is induced in the frame by the angular component $V\ sin\ d$, the component rotation parallel to $T_1\ T_2$, which exerts a torque about an axis perpendicular both to the axis of precession which is vertical and the axis of rotation TT, *i.e.*, about CC. This torque, therefore, consists of equal forces parallel to T_1T_2 applied to the frame at the four points D_1, D'_1, D_2, D'_2, such that those at D_1 and D'_2 act in one direction and those at D_2 and D'_1 in the opposite direction, thus causing a bending stress in the frame EF so that the torque exerted by one gyroscope is neutralized by the other, and the effect upon the car is nil.

It thus appears that no disturbance of the balance occurs by reason of the deviation caused by the precession occurring during the righting of the car. The sole effects are some slight variations in the rate of the precession and in the stresses to which the frame is subjected, matters which have no influence upon the action of the gyroscopes in righting the car.

It is furthermore evident that the gradual or instantaneous elevation of one end of the car above the other, such as might occur in running on a slack cable would, since it is a precessional rotation about an axis parallel to *CC*, have no effect, at least while the axes *BB* are in their normal position perpendicular to *TT*. The almost impossible case of such precession taking place during the continuance of a deviation *d* of perceptible magnitude could also be readily discussed, but is practically negligible since the deviation *d* actually flutters back and forth each side of zero in almost infinitesimal instantaneous excursions that check every attempted lapse from an upright position at its very beginning, and simultaneously with the attempt, the only lag in action being the time required for the movement necessary to tip the car far enough to reduce the clearance between the axis at *B* and the plate *GH* to zero. That clearance may be made as small as desired.

The question that naturally arises respecting this car, in the mind both of the engineer and the layman, is as to the limits of its stability. How much of an upsetting torque is it able successfully to withstand? It is evident that the larger the moment of inertia and rotary velocity of the gyroscopes the greater the stability, as appears also from equation (5). Brennan has stated that the weight of the gyroscopes need not exceed some 4 or 5 per cent. of that of the car. In his experimental car their weight is nearly 7 per cent. of the unloaded car, but perhaps may not exceed 4 or 5 per cent. of its total weight when loaded.

The point of danger in the apparatus is reached when the precession *U* has been so long operating as to carry the deviating axle B_2 (Fig. 4) to the end of the friction plate G_2. The car must be righted before the deviation reaches this point or the car will upset. A deviation of more than 30 degrees would apparently be unpracticable, and in practice 10 degrees or 15 degrees is probably nearer the limit.

In order that the car be righted with such promptness as to avoid this danger, U' in equation (6) must be large, which in turn requires T' to be large. Now T' is the torque about a vertical axis due to the friction of the axle B_2 as it rolls and slides upon G_2. This torque T' depends upon the product of the pressure of B_2 upon G_2 and the coefficient of friction between these. It has already been shown that the precession U is unaffected by T'. It is U' alone that is affected by T'. Now it would seem that the only thing necessary to make U' of any desired amount would be some means of increasing T' *ad libitum* before the deviation reaches its limit. To ensure ultimate safety either the coefficient of friction should in some manner be made to increase enormously before the limiting deviation is reached, or the pressure between B_2 and G_2 should be enormously increased in some automatic manner before reaching that limit. It would appear quite practicable to so arrange magnetic coils about B_2 as to have an electric current pass through more and more of them as B_2 approaches its limit of deviation and by that method automatically increase the magnetic pressure between B_2 and G_2 to any desired extent as the limit is approached, thus making T' and consequently U' both increase enormously at the limit. It would seem as if some such device must be adopted if occasional overturning from high winds and other accidental causes are to be avoided. The inertia of the gyroscopes in Brennan's experimental car is not known as yet, but computations based on assumed data make it probable that a very high wind causing a pressure of 30 pounds per square foot against the sides of a car of ordinary construction might cause a total precessional deviation of 30 degrees in a fraction of a second, from which the suddenness with which the car must be tilted to oppose the wind may be inferred and the necessity for some such automatic safety clutch is evident.

There is an action of the gyroscope still remaining which it is important to consider in connection with mono-rail car, which action is included under the head of 'nutation,' so called. The actions treated thus far in this paper have not taken into consideration the inertia effects which are brought into play about the vertical and longitudinal axes of the car by reason of the sudden or gradual application of unbalanced loads. Consider the effect of a weight W suddenly applied at B in Fig. 2. The discussion heretofore given treated the precessional motion as already established and its steady continuance was ensured by the moment of W acting about C. In suddenly applying a weight W at B we have to consider the phenomena occurring during the establishment of the precession due to W. This precession, consisting in a constant, angular velocity about a vertical axis through C, must necessarily arise by the application of an angular acceleration (constant or variable) about the same axis which is resisted by a corresponding couple (constant or variable) due to the inertia of the gyroscope.

During the establishment of the precession due to W there is therefore a resisting couple about the vertical axis due to inertia, which acts in addition to everything previously taken into account. Such a couple produces precession about CE Fig. 2 in such a way as to allow W to begin to move downwards. This is clearly what should occur, since the precession does not become large enough to support W until the precession reaches the value required by equation (5).

The motion just described is the beginning of the nutation. The precession about the vertical will gradually increase until it is sufficient to support W were it at rest, but by that time W has acquired a velocity and momentum such as to carry it still further downward before it can be stopped. In thus being stopped in its course, W by its inertia exerts a couple which continues to increase the precession beyond the amount necessary to simply support W, so that by the time W has ceased to move downwards the precession is so large as to give W an upward velocity during the time necessary to reduce the precession. It may readily be shown that W will be carried upward to the level from which it started, while the precession will also be diminished to the value it had to begin with. This oscillation of W up and down in unison with the increase and decrease of precession occurs harmonically, and constitutes the nutation.

In case the weight W is applied uniformly and gradually during the nutation, so that the whole of it is applied in course of the downward motion, it is evident that it will have half as much effect in causing nutation as if applied suddenly.

The relations just stated would enable us to investigate the amplitude and period of the oscillation were it important to do this. But as will now be shown such oscillations should not arise in the mono-rail car and hence we omit the investigation.

In the mono-rail car the friction plates are designed for the very purpose of producing a couple about the vertical axis opposed to the inertia couple which causes nutation. If properly designed the friction couple neutralizes and greatly exceeds the inertia couple due to any load likely to come upon the car. It will then happen that the car will not give way under an imposed load at all but will begin at once to bring the centre of gravity of load and car over the rail without any preliminary nutation, which would be a motion in the opposite direction. It might be possible, however, for a weight falling from a sufficient height to set up a small oscillation.

X-RAY AS DENTAL DETECTIVE. (*Popular Electricity,* ii, 12.)—A Cleveland oculist had a patient whose right eye was unaccountably inflamed, and the inflammation refused to yield to local treatment, which was evidence that the cause was not in the eye itself. The oculist sent the patient to a dentist, who, after close search, found a slight abscess at the root of the left eyetooth. On treating this abscess in the usual way the eye gradually improved. A year later the other eye was troubled in a similar manner, but no corresponding abscess was externally discoverable in the right eyetooth. However, the dentist drilled down to the root of the tooth and found a slight abscess. By curing this, the eye also was cured.

It occurred to the dentist that such cases might occur again, and that it would not always be desirable to bore into sound teeth on the chance of locating the trouble. So he tried if the X-rays would locate the seat of trouble and found them effective.

A NEW NITRIDE OF CARBON. (*Bull. Soc. d'Encour.*, 113, 3.)—Cyanogen, C_2N_2, discovered by Gay Lussac in 1815 is a nitride of carbon. MM. Ch. Moureu and J. Ch. Bongrand, at the meeting of the Academy of Sciences on Jan. 24, 1910, announced the isolation of a sub-nitride with the formula C_4N_2, by the separation of water from the butine-diamide, thus.—

$$CONH_2—C—C—CONH_2—2H_2O=$$
$$N—C—C—C—C—N \text{ or } C_4N_2$$

The product obtained, which is dicyanacetylene, can be recognized as a carbon cyanide C_2 (C_2N). The product appears as fine white needles, melting at 21° C. It boils at 76° C., its odor, its irritating properties and its vapor resemble those of cyanogen. It burns easily, its vapor ignites spontaneously in the air at about 130° C.; that of carbon disulphide ignites at about 150° C.