ASTRONOMISCHE NACHRICHTEN.
№ 520.

Schreiben des Herrn Baronets *Herschel* an den Herausgeber.

Collingwood Oct. 8. 1844.

My dear Sir,

I have read with much interest the curious paper by Mr. *Houzeau* in Nr. 496 and 498 of your Nachrichten, on the effect of aberration arising from proper motion in changing the apparent orbits of double stars by which he considers that a clue is afforded to an explanation of the irregularities exhibited by the recorded measures of distance in certain of there systems such as 61 Cygni and 70 Ophiuchi, and which effects applied with a contrary sign as a correction to the measured distances are stated by him to destroy, or greatly palliate those irregularities. Having always believed these discordancies to have arisen from errors of observation, the prospect so held out of a more satisfactory explanation from an hitherto unnoticed physical cause could not fail to engage my especial attention.

I hope I shall be excused if I profess myself unconvinced of the correctness of the view taken by M. *Houzeau* of this subject. On the contrary, it appears to me that no such effects as those ascribed by him to aberration arising from proper motion ought to take place. My reason for this opinion lies in a small compass and therefore, if erroneous, its errors can easily be pointed out with little aid from symbols or calculation.

Adopting (as M. *Houzeau* appears to do) the undulatory theory of light, the velocity of propagation will be the same in all directions, and must be absolutely independent of the velocity of the motion of the luminous body. If there be one consequence of the undulatory hypothesis more certain than another it is this — that light is propagated with equal velocity from a luminary at rest or in motion. In the corpuscular theory it is other wise. — Neither is it a less elementary and fundamental consequence of this (the undulatory) theory that a Spectator at rest sees a luminary in motion not along the line which actually joins his eye and the luminary at the moment when the impression is made on his retina, but along that which did join them at the moment when the light by which he sees it quitted the luminary. — Lastly it is equally certain on this theory that the aberration (properly so called) which arises in the eye of a spectator in motion by reason of his motion is identically the same whether the object seen be in motion or at rest. For distinction and because I think the want of some nominal distinction has often given rise to some confusion in this matter, I shall restrict the term aberration to this latter or subjective effect, applying to those apparent displacements which originate in the length of time occupied in the transmission of light from the luminary to the eye, the term equations of light, which may be of various kinds according to the dynamical conditions and dimensions of the system. If the terms subjective and objective aberration should be preferred I should not object to them, but some verbal distinction between classes of effects so essentially different seems indispensable.

Let us now consider a Binary Star consisting of two individuals A, B, at such a distance that light shall require T years to traverse it, or at a sidereal distance T taking the unit of sidereal distance to be that travelled over by light in 1 year. Suppose for simplicity the common proper motion of the center of gravity of AB to be in a direction perpendicular to the visual ray. It is obvious then that each of the two stars A and B, will be seen, independent of the other, at any given moment, not in the place which it occupies at that moment, but in that which it did occupy T years ago without regard to any change which may have taken place in its velocity or direction since as this is true of each individual A, B independently, it is true of both together regarded as forming a composend luminary $A+B$, the parts of which must therefore have with respect to each other and the spectator an apparent relative situation identical with the real relative situation which they had T' years ago without regard to their or either of their changes of velocity and direction since. We see therefore the compound object $A+B$ in the state in which it really did exist T years previously, and no way disfigured or distorted. This being true at every instant, it follows that in viewing such a system continuously for a series of years, we necessarily perceive its orbit in its true form and all the angles of position and distances in that orbit will be truly given by our measurements, unaffected by any optical illusion or distorsion whatever only as if for an epoch historically antecedent by T years. Consequently the total effect of the equation of light (neglecting the dimensions of the orbit as inconsiderable with respect to the distance of the star) will be taken account of by subtracting T years from the epoch in the elements of the orbit, or substituting $t+C-T$ for $t+C$ in the equations of the motion of the system.

Let us now take the case of a Binary Star having a proper motion not at right angles to the visual ray. In this case it will approach to or recede from us uniformely so that the time required to transmit its light to us, instead of a constant quantity T will be a variable one expressed by $T+kt$, k being a constant coefficient expressing the ratio of the velocity of the system from the eye, to the velocity of light. Therefore instead of $t+C$ in the equations of the orbitual motion we must substitute $t+C-(T+kt)$, which done, the mean motion $n(t+C)$ will become $n\{t+C-T-kt\}$ or

$$n(1-k)\left\{t+\frac{C-T}{1-k}\right\}.$$

A very curious consequence follows from this — viz: that the whole effect of the equation of light (when the dimensions of the orbit are neglected) falls on the Periodic time and the epoch. For the apparent period will be represented by $\frac{360^\circ}{n(1-k)}$ while the real period is $\frac{360^\circ}{n}$. If therefore the star be receding from our system in which case k is positive the apparent periodic time will be less than the true, and *vice versa*, a consequence at first sight paradoxical, but which a moment's consideration and tracing the star in imagination through a complete revolution while approaching to or receding from the eye shews to be perfectly correct and natural. The apparent axis of the orbit at any moment will be also affected and changed from a to $a.\frac{T}{T+kt}$, but the angles of position remaining the same, the distances corresponding will retain the same ratio to the varied axis as if the star were at rest and the period alone altered.

If we take account of the dimensions of the orbit so as to include in the total effect of the equation of light, that part of it to which M. *Savary* some years ago drew attention, we have only to add to the value of $n(t+C)$ altered as above, the minute term $-z$ where z in the coordinate of the companion star in the direction of the visual ray expressed *in units of sidereal distance*, and the equation for the excentric anomaly which in the real orbit is expressed by

$$n(t+C) = u - e.\sin u$$

will be converted into

$$n(1-k)\left(t+\frac{C-T}{1-k}\right)-nz = u - e.\sin u$$

in which the term nz being excessively minute may be treated as approximately known at any given instant

If I am wrong in these conclusions, I trust M. *Houzeau* will set me right by pointing out the particular flaw which vitiates my reasoning, which I profess myself unable to detect by my own penetration — and at all events that he will believe me to be actuated by no spirit of cavil in these remarks or in those which follow. To entitle me to make this request, however, he has a right to expect that I should on my part point out that particular step in *his* reasoning which I consider incorrect. It is that part of § 2, p. 244, Nr. 496 A. N. — where he says: „Il est clair que l'inégalité n'aura aucune action sur les y, et que les x seuls en seront affectés," according to which principle he decomposes the tangential velocity of B parallel to the axis of the x (assumed to be the direction of the proper motion) and applies than *part only* of the equation of light, or the objective aberration, which arises from this decomposed position, to the true place of B; rejecting altogether, (avowedly and purposely) that other portion which arises from the motion decomposed in the direction y, as having no influence in the question. The apparent place of B, is therefore necessarily carried *out* of the apparent ellipse, whereas had the other portion of the effect parallel to the y been taken into the accounts as it seems to me it ought, the apparent place of B would of equal necessity lie *in* that ellipse, and the whole apparent movement be performed according to the laws of elliptic motion.

This argument may be otherwise put as follows. The motion of B may be decomposed into two, viz: one equal and parallel to the uniform proper motion of A, the other orbitual, in the direction of a tangent to the relative orbit. The equation of light arising from the former of these decomposed portions, displaces B by exactly the same amount and in the same direction as it displaces A and will therefore affect neither their relative position nor apparent distance, which will be precisely the same as if there were no proper motion at all. So that the proper motion is altogether eliminated from the enquiry, and it remains only to take account of the effect of the tangential motion, the correction for which will obviously be to be applied in an opposite direction to that motion and therefore along the tangent and not as M. *Houzeau* contends along a line parallel to the proper motion. The equation of light due to the tangential motion alone is identical with M. *Savary's* equation already attended to.

Although I suppose no philosopher of the present day believes in the corpuscular theory of light as a matter of physical fact, yet it may not be amiss to mention that on that theory (which in its application requires us to suppose the velocity of light emitted from a moving luminary to be not the same in all directions) the equation of light will really contain periodical terms, which will arise and may readily be developed from the substitution for t in the equation $n(t+C) = u - e . \sin u$, of the following expression

$$t(1-k) - T - z + (T+kt)(v . \cos\varphi + v' . \cos\varphi')$$

where v is the velocity of proper motion common to the two stars and φ the angle which that motion makes with the visual ray, and where moreever v' is the tangential velocity of B in the relative orbit and φ' the angle which the tangent makes with the visual ray, which expression if we consider that $-v . \cos\varphi$ is the velocity of recess of the system in virtue of its proper motion, and therefore is the same thing which has been also represented by k, becomes, neglecting powers and products of k and v' and z

$$t(1-k) - T(1+k) - z + T . v' . \cos\varphi'.$$

The form of the periodical terms however arising from this substitution will of necessity be widely different from those of M. *Houzeau's* theory, for wich reason it is unnecessary to go farther with this view of the subject.

J. F. W. Herschel.

Ueber die Berechnung der wahren parabolischen Anomalie aus der Zeit, für Fälle, in welchen sie sich 180° nähert.

Von Herrn Geh. Rath *Bessel.*

Die Tafeln, durch deren Hülfe man, in dem Falle der parabolischen Bewegung eines Kometen, seine, gegebenen Zeiten (t) zugehörigen Anomalien (v) zu bestimmen pflegt, werden unvortheilhaft, wenn v sich 180° nähert; der merkwürdige Komet von 1843 ist, bei der äufsersten Kleinheit seiner Perihelentfernung (q), schon sehr bald nach seinem Durchgange durch das Perihel zu solchen Anomalien gelangt, und hat dadurch ein besonderes Hülfsmittel zu ihrer Bestimmung wünschenswerth erscheinen lassen. Ich werde hier eine Tafel mittheilen, welche, in ähnlichen Fällen, das Gewünschte leisten wird.

Die Relation zwischen q, t und v, welcher gemäfs die letzte dieser Gröfsen aufgesucht werden mufs wenn die beiden ersten gegeben sind, ist bekanntlich

$$\frac{kt}{\sqrt{2}\sqrt{q^3}} = tgt\tfrac{1}{2}v + \tfrac{1}{3}tgt\tfrac{1}{2}v^3$$

wo k die von *Gauss*, unter diesem Zeichen eingeführte Zahl

$$\log k = 8{,}23558\ 14526 - 10\ ^{*})$$

ist. Schreibt man rechts von dem Gleichheitszeichen

$$\tfrac{1}{3}\,tgt\tfrac{1}{2}v^3\left\{1+3\,Cotg\tfrac{1}{2}v^2\right\} = \tfrac{1}{3}\,tgt\tfrac{1}{2}v^3\,(1+Cotg\tfrac{1}{2}v^2)^3 . \frac{1+3\,Cotg\tfrac{1}{2}v^2}{(1+Cotg\tfrac{1}{2}v^2)^3}$$

oder

$$\frac{8}{3\sin v^3} . \frac{1+3\,Cotg\tfrac{1}{2}v^2}{(1+Cotg\tfrac{1}{2}v^2)^3}$$

so ist der zweite Factor nur um eine Gröfse von der Ordnung von $Cotg\tfrac{1}{2}v^4$ von 1 verschieden, also, immer wenn v sich 180° nähert und daher $Cotg\tfrac{1}{2}v$ eine kleine Gröfse ist, sehr nahe = 1. Wenn w der Werth ist, den v annimmt, indem 1 statt dieses Factors gesetzt wird, wenn daher der *stumpfe* Winkel w, nach der Formel

$$\sin w = \frac{2\sqrt{2}}{\sqrt[3]{6k}} . \frac{\sqrt{q}}{\sqrt[3]{t}}$$

aufgesucht wird, so ist w wenig von v verschieden, oder wenn man $v = w + \delta$ setzt, δ eine kleine Gröfse.

Zur Bestimmung von δ hat man die Gleichung:

$$\frac{8}{\sin w^3} = 3\,tgt\tfrac{1}{2}(w+\delta) + tgt\tfrac{1}{2}(w+\delta)^3$$

oder, wenn man $tgt\tfrac{1}{2}w = \theta$, $tgt\tfrac{1}{2}\delta = x$ setzt,

$$\frac{(1+\theta\theta)^3}{\theta^3} = 3\,\frac{\theta+x}{1-\theta x} + \frac{(\theta+x)^3}{(1-\theta x)^3}$$

welche, durch Multiplication mit $(1-\theta x)^3\,\theta^3$, in

$$1+3\theta\theta = 3\theta(1+4\theta^2+2\theta^4+\theta^6)x - 3\theta^2(1+4\theta^2+2\theta^4+\theta^6)x^2 + \theta^3(2+6\theta^2+3\theta^4+\theta^6)x^3$$

verwandelt wird, oder, durch Division mit dem Coefficienten von x, in:

$$\frac{1+3\theta\theta}{3\theta(1+4\theta^2+2\theta^4+\theta^6)} = x - \theta x^2 + \frac{\theta^2(2+6\theta^2+3\theta^4+\theta^6)}{3(1+4\theta^2+2\theta^4+\theta^6)}x^3.$$

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*) Dieser Werth von *log k* ist nicht genau der *Theoria Mot. C. C.* p. 2 angegebene, dessen drei letzte Decimalen 414 sind. Er entspricht dem siderischen Jahre = $365^T256374417$ (A. N. Nr. 133. S. 266) und dem Verhältnisse der Sonnenmasse zu der Erdmasse = 354890 : 1. Der Unterschied beider Werthe ist aber so unbedeutend, dafs er nicht beachtet zu werden braucht.