CLINICAL CALORIMETRY

TENTH PAPER

A FORMULA TO ESTIMATE THE APPROXIMATE SURFACE AREA IF HEIGHT AND WEIGHT BE KNOWN*

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Since the publication of Paper 5 of this series the so-called "Linear Formula" has been used in the study of a large number of individuals. Practically all of the subjects of respiration experiments in the Sage calorimeter have been measured in this way, and in addition Means[1] of Boston has used it as a factor in determining his normal base line of metabolism and the extent of the pathological variations. Means has found that the range of normal variation from the average is smaller and that the apparent depression of metabolism in obesity is much less marked when the linear formula, instead of Meeh's formula, is used to determine surface area.

The accuracy of the linear formula has been shown in Paper 9 of this series. In order to correct the slight error in the factor for the arms, and also in order to clear up a few points in the measurements which may cause confusion, it seems best to repeat the formula and show the bony landmarks by diagram (Fig. 1). Some difficulty has been experienced in locating the superior border of the great trochanter in fat subjects. This landmark is the starting point of the measurement "O" which represents the length of the thighs. If we employ another factor we can use the new measurement "W," the distance from lower border of the patella to the upper border of the pubes, a point already located in the measurement "L." In taking this measurement, however, one must be careful to have the legs straight and the knees, heels and great toes touching. It is better to take all measurements from a footboard with the subject lying down,[2] determining distance from soles of feet to lower border of patella, to upper border of

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1. Means, J. H.: Studies of the Basal Metabolism in Obesity and Pituitary Disease, Jour. Med. Research, 1915, xxxii, 121; Basal Metabolism and Body Surface, Jour. Biol. Chem., 1915, xxi, 263.

2. This is especially important with obsese patients.

pubes, to suprasternal notch and to top of the head.[3] In Table 1 a comparison is made of the old and new formulas for determining the surface of the thighs. It is seen that the average error is the same.

In the literature of the work on respiratory metabolism it has been customary to give only the age, weight and height. If, therefore, we

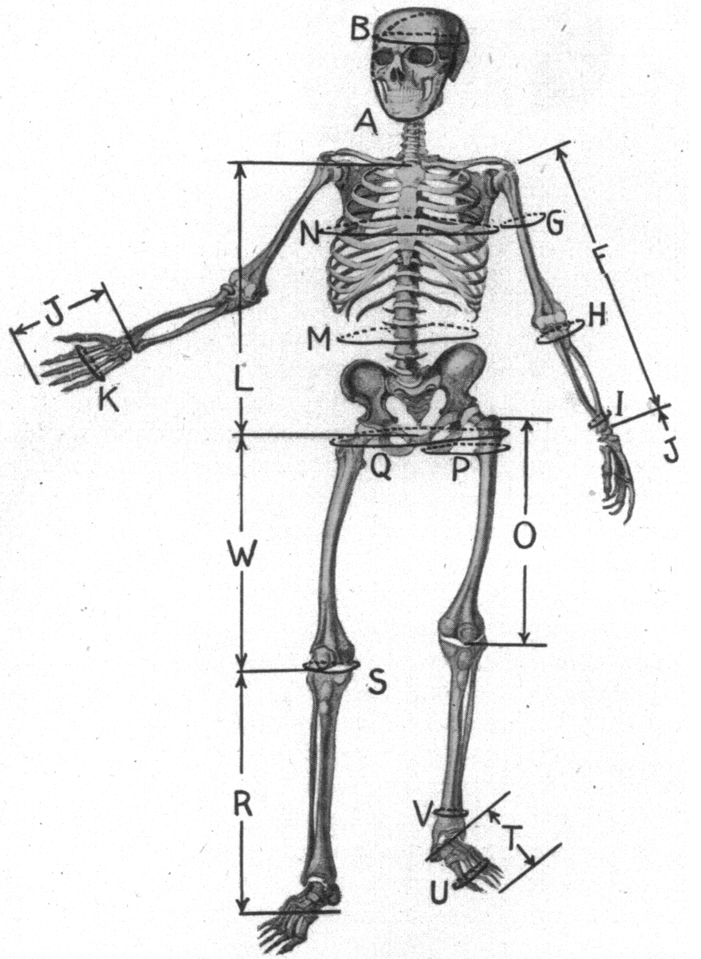

Fig. 1.—Measurements used in "Linear Formula."

are to recalculate previous work in an effort to get more accurate results than are furnished by Meeh's formula we must content ourselves with calculations based on height and weight. A formula such as Meeh's, based on weight alone, can easily give an error of 15 to 20 per cent.,

3. Dr. F. G. Benedict of Boston has called our attention to the fact that this determines the length rather than the height. We have found that as a rule the length is 1 or 2 centimeters greater than the height, but we must remember that height varies 1 to 3 centimeters during the day.

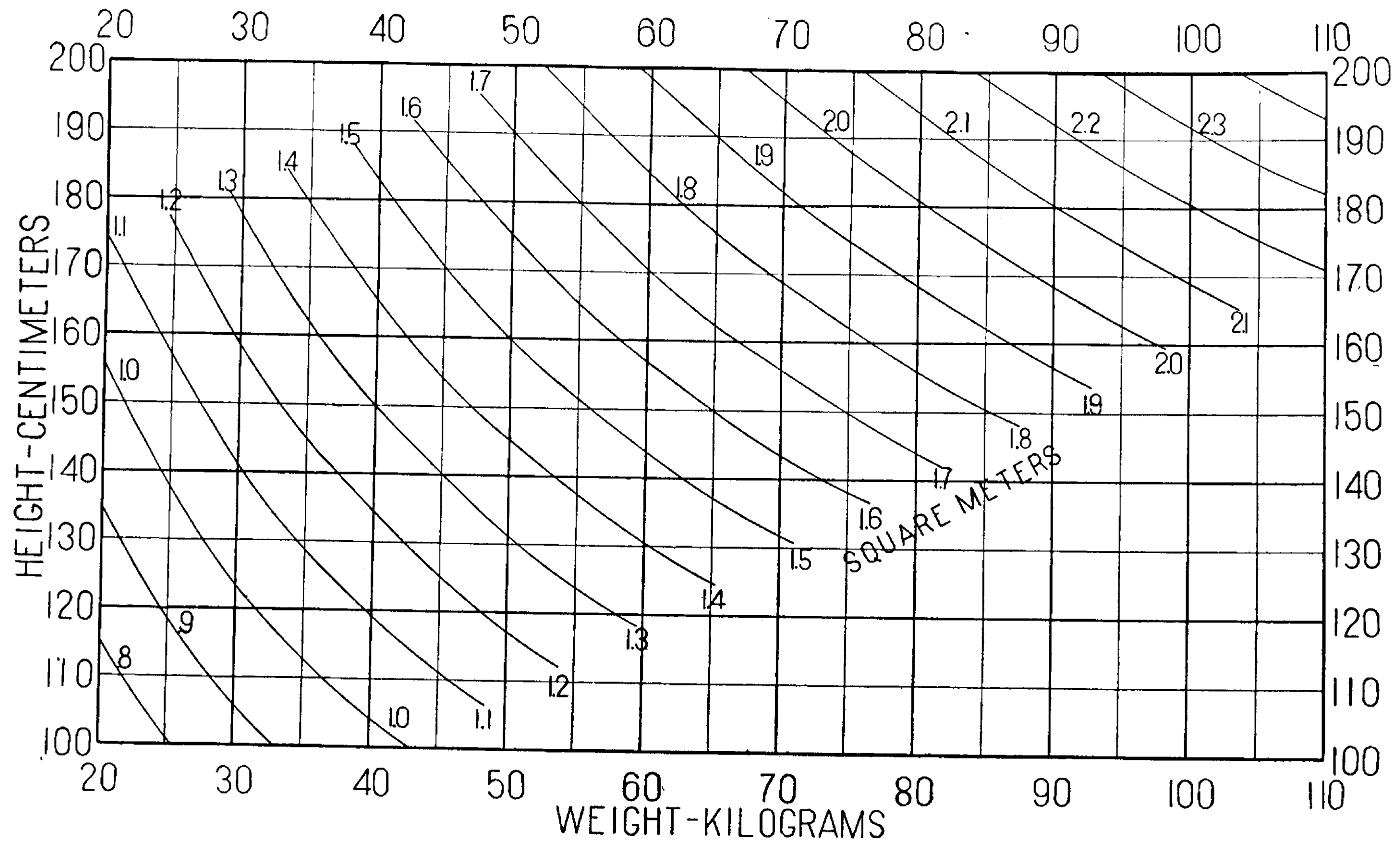

Fig. 2.—Chart for determining surface area of man in square meters from weight in kilograms (Wt.) and height in centimeters (Ht.) according to the formula: Area (Sq. Cm.) $= Wt.^{0.425} \times Ht.^{0.725} \times 71.84$.

but this error is greatly reduced by taking the height into consideration. With people of very unusual body shape there does not seem to be any accurate method simpler than the linear formula with its nineteen measurements. The reason why a consideration of the height does not entirely correct the calculations based on weight becomes apparent when we consider the circumference of the body at various levels. For instance, in the case of R. H. H. the average circumference of the legs was 30.0 cm. and of the thighs 43.9 cm. An increase of 10 cm. in the length below the knees would mean an increase of 600 sq. cm. in surface area, but if the length of the thighs were increased 10 cm. it would mean a gain of 878 sq. cm. Variations in the arms would not affect the height at all.

TABLE 1.—Comparison of Old and New Formulas for Determining Surface Areas of Thighs

Name	Thigh Measurement*		Surface Thighs as Meas., Sq. Cm.	Surface Calc. O(P+Q) 0.508	Error, %	Surface Calc. W(P+Q) 0.552	Error, %
	"O" Cm.	"W" Cm.					
Benny L.	26.4	22.4	1284	1294	+1	1195	− 7
Morris S.	41.7	38.8	3022	3207	+6	3251	+ 7
R. H. H.	47.0	46.0	3712	3512	−5	3745	+ 1
E. F. D. B.	46.3	46.3	3820	3655	−4	3981	+ 4
Mrs. McK.	40.0	32.2	3500	3594	+3	3152	−11
Anna M.	16.0	14.4	478	488	+2	479	+ 0
Gerald S.	44.7	44.9	3002	2677	−11	2927	− 3
Emma W.	45.7	41.7	3324	3448	+4	3425	+ 3
R. H. S.	50.3	46.3	3175	3457	+9	3464	+ 9
Average........	....	....	....	....	±5	....	± 5

* Old measurement "O," superior border of great trochanter to lower border of patella. New measurement "W," superior border of pubes to lower border of patella.

A formula to express surface area must naturally be a bi-dimensional formula, as surface involves two dimensions. If we assume that weight is proportional to volume, it is obvious that three dimensions are involved in any expression for weight. Height is, of course, a single dimension. If we attempt to construct a formula for surface area (A) based on weight (W) and height (H), it is obvious that a simple formula such as $A = W \times H \times C$ (C being a constant depending on the units used and the subject to which the formula is to apply) is not logical. In this formula one side, A, is bidimensional and the other side, $W \times H \times C$, involves four dimensions, three from W and one from H. If W is tridimensional, it is obvious that the cube root of W ($=\sqrt[3]{W}$ or $W^{1/3}$) is undimensional and a formula $A = W^{1/3} \times H \times C$ is logical in that it is bidimensional on both sides. Another bidimensional expression involving W and H would be the square root of $W \times H$ ($\sqrt{W \times H}$ or $W^{1/2} \times H^{1/2}$) because $W \times H$, being four-dimensional, is reduced to a bidimensional expression on taking the square root. A formula based on this method of reduction would be $A = W^{1/2} \times H^{1/2} \times C$.

TABLE 2.—Measurements and Constants for Linear Formula (Measurements Taken with Subject Lying on a Flat Surface)

Head: AB 0.308.
- A—Around vertex and point of chin.
- B—Coronal circumference around occiput and forehead, just above eyebrows.

Arms: F(G + H + I) 0.611.*
- F—Tip of acromial process to lower border of radius, measured with forearm extended.
- G—Circumference at level of upper border of axilla.
- H—Largest circumference of forearm (just below elbow).
- I—Smallest circumference of forearm (just above head of ulna).

Hands: JK 2.22.
- J—Lower posterior border of radius to tip of second finger.
- K—Circumference of open hand at the meta-carpo-phalangeal joints.

Trunk (Including neck and external genitals in the male, breasts in female): L(M + N) 0.703.
- L—Suprasternal notch to upper border of pubes.
- M—Circumference of abdomen at level of umbilicus.
- N—Circumference of thorax at level of nipples in the male and just above breasts in the female.

Thighs: O(P + Q) 0.508.
- O—Superior border of great trochanter to the lower border of the patella.
- P—Circumference of thigh just below the level of perineum.
- Q—Circumference of hips and buttocks at the level of the great trochanters.

Or:—Thighs: W(P + Q) 0.552.
- W—Upper border of pubes to lower border patella (measured with legs straight and feet pointed anteroposteriorly).
- P—As above.
- Q—As above.

Legs: RS 1.40.
- R—From sole of foot to lower border of patella.
- S—Circumference at level of lower border of patella.

Feet: T(U + V) 1.04.
- T—Length of foot including great toe.
- U—Circumference of foot at base of little toe.
- V—Smallest circumference of ankle (just above malleoli).

* Factor 0.558 if F is measured over olecranon with forearm flexed.

Note.—The constants for arms, thighs, etc., when multiplied by the measurements of one side give the surface area for both sides. To find total surface area add the seven parts.

TABLE 3.—Comparison of Various Formulas

Name	Weight, Kg.	Height or Length, Cm.	Measured or Determined by Linear Formula, Area, Sq. Cm.	$\frac{Area}{Ht.\sqrt[3]{Wt.}} = C$		$\frac{Area}{\sqrt{Ht. \times Wt.}} = C$		$\frac{Area}{Wt.^{\frac{1}{2.35}} \times Ht.^{\frac{1}{1.38}}} = C$	
				Factor C	Variation from 25.6, %	Factor C	Variation from 167.2, %	Factor C	Variation from 71.84, %
Measured by Molds:									
Benny L.	24.2	110.3	8473	26.7	+4.3	164.0	—2.0	72.30	+0.7
Morris S.	64.0	164.3	16720	25.5	—0.5	163.0	—2.5	70.65	—1.6
R. H. H.	64.1	178.0	18375	25.8	+0.8	171.5	+2.6	73.22	+1.9
E. F. D. B. ..	74.1	179.2	19000	25.3	—1.2	164.7	—1.5	70.85	—1.3
Mrs. McK. ...	93.0	149.7	18592	27.2	+6.2	157.5	—5.8	71.70	—0.2
Gerald S.	45.2	171.8	14901	24.4	—4.7	169.2	+1.2	70.36	—2.0
Fab. S.	32.7	141.5	(11869)*	25.5	(—0.5)*	174.2	(+4.0)*	74.37	(+3.5)*
Anna M.	6.27	73.2	3699	27.4	+7.1	172.0	+3.0	75.54	+5.1
Emma W. ...	57.6	164.8	16451	25.8	+0.8	169.0	+1.1	72.56	+1.0
R. H. S.	63.0	184.2	17981	24.5	—4.3	167.0	0.0	70.58	—1.7
Average....	....	.....	.....	....	+3.3	.....	+2.2	.....	+1.7
Measured by Linear Formula									
Edw. B.	62.3	174.0	17270	25.0	—2.3	165.2	—1.2	70.75	—1.5
John K.	65.4	176.0	17610	24.9	—2.7	164.2	—1.7	70.83	—1.4
Alb. S.	66.4	162.2	16720	25.5	—0.5	161.0	—3.7	70.20	—2.3
Wm. S.	44.6	179.0	15450	23.7	—7.4	172.8	+3.3	71.53	—0.4
A. F. C.	69.6	179.4	17960	24.4	—4.7	160.5	—4.0	68.71	—4.4
Wm. A.	63.4	180.0	17940	24.9	—2.7	168.2	+0.6	71.22	—0.8
Mart. C.	44.0	166.8	14370	24.5	—4.3	167.5	+ .1	70.44	—1.9
Jos. U.	40.1	179.0	14520	23.7	—7.4	171.2	+2.3	70.36	—2.1
Wm. Shee....	63.8	171.0	16070	23.5	—8.2	154.8	—7.5	66.06	—8.0
Arthur V. ...	58.3	155.0	15560	25.8	+0.8	163.3	—2.4	73.02	+1.6
Armon W. ...	60.8	161.0	15500	23.6	—7.8	156.6	—6.4	67.96	—5.4
Annie T.	26.3	137.0	10460	25.6	0.0	174.0	+4.1	73.60	+2.4
Fred D.	49.3	157.0	13870	24.1	—5.8	157.2	—6.0	67.69	—5.7
Fred D.	40.0	157.0	12960	24.2	—5.5	163.5	—2.3	69.12	—3.8
Edw. T.	50.4	168.0	14830	23.4	—8.6	161.0	—3.7	68.27	—5.0
J. McE.	41.8	166.0	13260	23.1	—9.7	159.0	—4.8	66.67	—7.2
Bart D.	43.5	156.0	13850	25.3	—1.2	168.1	+0.5	71.62	—0.3
Burr Ph.	70.7	169.0	17390	24.9	—2.7	159.0	—4.8	69.01	—3.9
J. D. D. B. ...	34.5	152.8	12240	24.7	—3.5	168.6	+0.7	70.90	—1.3

TABLE 3.—(*Continued*)

Name	Weight, Kg.	Height or Length, Cm.	Measured or Determined by Linear Formula, Area, Sq. Cm.	$\frac{\text{Area}}{\text{Ht.}\sqrt[3]{\text{Wt.}}}=C$		$\frac{\text{Area}}{\sqrt{\text{Ht.}\times\text{Wt.}}}=C$		$\frac{\text{Area}}{\text{Wt.}^{\frac{1}{2.35}}\times\text{Ht.}^{\frac{1}{1.38}}}=C$	
				Factor C	Variation from 25.6, %	Factor C	Variation from 167.2, %	Factor C	Variation from 71.84, %
Ray M.	30.4	140.5	10840	24.8	—3.1	165.8	—0.8	70.42	—1.9
Harry B.	36.5	146.0	12320	25.4	—0.8	168.6	+0.8	72.01	+0.3
Harry K.	35.9	148.2	12240	25.0	—2.3	167.8	+0.2	71.30	—0.8
Arthur A. ...	30.6	146.6	11260	24.6	—3.9	167.0	—0.1	70.71	—1.6
Leslie B.	28.5	140.8	10500	24.5	—4.3	166.5	—0.5	70.00	—2.6
Peter N.	63.4	187.7	17950	24.0	—6.2	164.2	—1.8	69.09	—3.8
Max W.	73.2	173.7	18540	25.5	—0.5	164.5	—1.7	71.07	—1.1
Dan O'C.	60.0	167.0	16570	25.4	—0.8	165.7	—1.0	71.12	—1.0
Jack O'C. ...	31.4	162.8	12040	23.5	—8.2	168.2	+0.6	69.35	—3.5
A. F.	52.6	159.0	14870	25.0	—2.3	162.5	—2.9	69.95	—2.6
G. L.	79.2	175.5	20300	26.9	—5.1	172.2	+3.0	74.70	+4.0
F. C. G.	56.5	173.9	16440	24.7	—3.5	167.4	0.0	70.29	—2.1
L. M.	59.5	170.6	16340	24.6	—3.9	162.1	—3.0	69.30	—3.5
F. G. B.	87.1	182.8	20760	25.8	—0.8	164.5	—1.1	71.22	—0.9

* Measured by adhesive plaster method which gives results about 3.3 per cent. too high. The plus variations would be reduced by this amount.

Comparing the two formulas $A = W^{1/3} \times H \times C$ and $A = W^{1/2} H^{1/2} \times C$, it will be seen that they differ in the relative importance given to W and H. In the former W has less importance and H more importance than in the latter. Meeh's formula $A = W^{2/3} \times C$, failed because H was neglected entirely. Adding H to the formula makes it more nearly applicable to subjects of the same general shape but differing somewhat in relative dimensions, and the best formula involving only W and H will be the one which gives a certain best relative importance to W and H.

Both of the above formulas were carefully investigated by applying them to the nine subjects that had been measured in the laboratory. For these subjects W, H[4] and A are known. In testing a formula the procedure was to solve for C (the only unknown) for each of the ten cases and then to assume the correct constant for the formula to be the average value of the C's so found. The merit of the

4. With about half the subjects this was determined standing. No attempt has been made to correct for the difference between height and length. (See Footnote 3, p. 864.) The largest difference we have found would cause a change of about 1.5 per cent. in the surface area and reading. This is within the limit of accuracy claimed for the Height-Weight Chart.

formula was then judged by the percentage variation of the factors C, as found for the individual cases, from the constant chosen. This percentage variation would also be the percentage error in area in the individual cases if the formula were applied using the chosen constant.

The formulas with H^1 and $H^{1/2}$ both gave rather good results, but it was noticed in a number of cases that the percentage error for the same subject differed in sign for the two formulas. This would indicate that some formula would be better than either of these two if H were raised to some power between ½ and 1.

The formula $A = W^{1/3} \times H \times C$ can also be written $A = W^{1/3} \times H^{1/1} \times C$, bringing it into the same form as $A = W^{1/2} \times H^{1/2} \times C$ and the general form of this formula can be written $A = W^{1/a} \times H^{1/b} \times C$. In order that the expression $W^{1/a} \times H^{1/b} \times C$ may remain bidimensional it is only necessary that $3/a \times 1/b = 2$, as it does in the two cases considered. For an intermediate equation it is obvious that (b) must be greater than 1 but less than 2. A value of $b = 1.25$ would give $a = 2.5$ and the formula would be $A = W^{1/2.5} \times H^{1/1.25} \times C$. This formula when tested gave very much better results than either of the others, but to find the best values of "a" and "b" it was necessary to explore formulas having a number of other combinations of "a" and "b" and then to interpolate graphically.

The best values of "a" and "b" were found to be $a = 2.35$ and $b = 1.38$ giving the formula the final form of $A = {}^{1/2.35} \times H^{1/1.38} \times C$ or $A = W^{0.425} \times H^{0.725} \times 71.84$. This formula can be solved by logarithms as follows:

$$\text{Log. } A = \text{Log. } W \times 0.425 + \text{Log. } H \times 0.725 + 1.8564.$$

1.8564 is a constant equal to Log. C.

In order to make this somewhat complicated formula easy of application a chart has been constructed (Fig. 2). By means of this it is possible to find the approximate surface area at a glance. The ordinates represent the height in centimeters, the abscissae the weight in kilograms. The point of intersection of these lines is found for any given subject and the surface area in square meters read off on the curved lines by interpolation. The second decimal place, which is never accurate, is estimated by the distance of the point from the nearest curved line. For instance, if the man were 150 centimeters tall and weighed 60 kilograms, the approximate surface area would be 1.55 square meters.

The large plus error in the constant employed by Meeh[5] has been established by the previously quoted works of Bouchard, Lissauer, Sicheff, Lassabliere and ourselves. According to our calculations the

5. Meeh: Ztschr. f. Biol., 1879, xv, 425.

average error in Meeh's constant is about 15 per cent. Instead of a uniform figure of 12.312, the "constant" should average about 10.5, varying between 12.3 for the greatly emaciated and 9.0 for the very stout. We must remember that figures for the calories per square meters of body surface will average 15 per cent. smaller when Meeh's formula is used then when the linear formula or the new Height Weight Formula" is employed.

SUMMARY AND CONCLUSIONS

The method of calculating the surface area from the so-called "Linear Formula" is given with a slight correction in the factor for the arms and an alternative measurement for the thighs. A simpler "Height-Weight Formula" has been devised to estimate the surface of subjects if only their height and weight be known. This is expressed in the terms $A = W^{0.425} \times H^{0.725} \times C$. A being the surface area in square centimeters, H the height in centimeters, W the weight in kilograms and C the constant, 71.84. A chart has been plotted from this formula so that the approximate surface area may be determined at a glance.

We may estimate the errors in the various formulas as follows: "Linear Formula" and "Height-Weight Formula," maximum $\pm$ 5 per cent., average $\pm$ 1.5 per cent., Meeh's Formula, maximum + 30 per cent., average + 15 per cent. In general the maximum figures apply only to those of unusual shape, while with those of average body form the average error will seldom be exceeded.

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