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Mapping on Complex Neutrosophic Soft Expert Sets

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Abstract. We introduce the mapping on complex neutrosophic soft expert sets. Further, we investigated the basic operations and other related properties of complex neutrosophic soft expert image and complex neutrosophic soft expert inverse image of complex neutrosophic soft expert sets.

INTRODUCTION

Smarandache [1] firstly proposed the theory of neutrosophic set as a generalization of fuzzy set [2] and intuitionistic fuzzy set [3]. Unlike goal programming [4-8], neutrosophic set can deal with uncertain, indeterminate and incongruous information where the indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are completely independent. Molodtsov [9] introduced the notion of soft set which is free from the inadequacy of the parameterized tools of the former theories. It was extended to many variants including vague soft sets [10-17], Q-fuzzy soft sets [18-24] and genetic algorithms [25-27]. Maji [28] introduced the notion of neutrosophic soft set to combine the strengths of both of the soft sets and neutrosophic sets. Soft set and neutrosophic soft set are then extended to soft expert set [29] and neutrosophic soft expert set [30], where the user can know the opinions of all the experts in these models without any operations.

The uncertainty sets that have been mentioned above are further extended to the complex field. The introduction of fuzzy sets was followed by their extension to the complex fuzzy set [31], intuitionistic fuzzy set also extended to complex intuitionistic fuzzy set [32]. Following in this direction, Ali and Smarandache [33] introduced complex neutrosophic set as a generalization of neutrosophic set. Complex neutrosophic set is characterized by complex-valued truth membership function, complex-valued indeterminate membership function, and complex-valued falsehood membership function such that each membership function associates with a phase term.

The main purpose of this paper is to continue investigating complex neutrosophic set, soft expert set and their combinations. Kharal and Ahmad [34] introduced the notions of a mapping on the classes of soft sets and studied the properties of soft images and soft inverse images. They [35] also presented the concept of a mapping on classes of fuzzy soft sets. In the neutrosophic environment, Alkhazaleh and Marei [36] studied the notion of mapping on neutrosophic soft classes, followed by a mapping on neutrosophic soft expert sets by Broumi et al.[37]. We define the notion of complex neutrosophic soft expert sets (CNSES) and its basic operations, which will then lead to the concept of mapping on classes of complex neutrosophic soft expert sets. We also provide verifications of some fundamental operations and properties of this concept. This will further extend studies on neutrosophic soft expert set [38-40]. It enriches current studies on fuzzy sets [41-46] compared to those in certain environments [47-50].

PRELIMINARIES

In this section, we recapitulate the concepts of neutrosophic and complex neutrosophic sets and present an overview of the operations structures of the complex neutrosophic model that are relevant to the work in this paper. The complex neutrosophic soft expert set (CNSES) and its basic operations are also introduced.

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We begin by recalling the definition of neutrosophic set.

Definition 1. (See [1]) Let U be a universe of discourse. A neutrosophic set N in U is defined as $N = \{<u; T_N(u); I_N(u); F_N(u)>; u \in U\}$, where $T_N(u), I_N(u)$ and $F_N(u)$ are the truth membership function, the indeterminacy membership function and the falsity membership function, respectively, such that T ; I ; $F : X \to]^{-}0;1^{+}[$ and $^{-}0 \le T_N(u) + I_N(u) + F_N(u) \le 3^{+}$.

Ali and Smarandache [33] conceptualized complex neutrosophic set and gave the basic operations in the following two definitions.

Definition 2. (See [33]) Let a universe of discourse U. A complex neutrosophic set S in U is characterized by a truth membership function $T_S(u)$, an indeterminacy membership function $I_S(u)$ and a falsity membership function $F_S(u)$ that assigns a complex-valued grade for each of these membership functions in S for any $u \in U$. The values $T_S(u)$, $I_S(u)$, $F_S(u)$ and their sum may all be within the unit circle in the complex plane and are of the form, $T_S(u) = p_S(u).e^{j\mu_S(u)}$, $I_S(u) = q_S(u).e^{j\nu_S(u)}$ and $F_S(u) = r_S(u).e^{j\omega_S(u)}$, each of $p_S(u), q_S(u), r_S(u)$ and $\mu_S(u), \nu_S(u), \omega_S(u)$ are, respectively, real valued and $p_S(u), q_S(u), r_S(u) \in [0,1]$ such that $0^{-} \le p_S(u) + q_S(u) + r_S(u) \le 3^{+}$.

Definition 3. (See [33]) Let A and B be two complex neutrosophic sets on the universe U, where A is characterized by a truth membership function $T_A(u) = p_A(u).e^{j\mu_A(u)}$, an indeterminacy membership function $I_A(u) = q_A(u).e^{j\nu_A(u)}$ and a falsity membership function $F_A(u) = r_A(u).e^{j\omega_A(u)}$ and B is characterized by a truth membership function $T_B(u) = p_B(u).e^{j\mu_B(u)}$, an indeterminacy membership function $I_B(u) = q_B(u).e^{j\nu_B(u)}$ and a falsity membership function $F_B(u) = r_B(u).e^{j\omega_B(u)}$.

We define the subset, union and intersection operations as follows.

1. A set A is said to be complex neutrosophic subset of B ($A \subseteq B$) if and only if the following conditions are satisfied:
   (a) $T_A(u) \le T_B(u)$ such that $p_A(u) \le p_B(u)$ and $\mu_A(u) \le \mu_B(u)$.
   (b) $I_A(u) \ge I_B(u)$ such that $q_A(u) \ge q_B(u)$ and $\nu_A(u) \ge \nu_B(u)$.
   (c) $F_A(u) \ge F_B(u)$ such that $r_A(u) \ge r_B(u)$ and $\omega_A(u) \ge \omega_B(u)$.

2. The union(intersection) of A and B, denoted as $A \cup (\cap) B$ and the truth membership function $T_{A\cup(\cap)B}(u)$, the indeterminacy membership function $I_{A\cup(\cap)B}(u)$ and the falsity membership function $F_{A\cup(\cap)B}(u)$ are defined as follows:

$$T_{A\cup(\cap)B}(u) = [(p_A(u) \vee (\wedge) p_B(u))].e^{j(\mu_A(u) \vee (\wedge) \mu_B(u))},$$
$$I_{A\cup(\cap)B}(u) = [(q_A(u) \wedge (\vee) q_B(u))].e^{j(\nu_A(u) \wedge (\vee) \nu_B(u))} \text{ and}$$
$$F_{A\cup(\cap)B}(u) = [(r_A(u) \wedge (\vee) r_B(u))].e^{j(\omega_A(u) \wedge (\vee) \omega_B(u))},$$

where $\vee = \max$ and $\wedge = \min$.

We will now give a definition of complex neutrosophic soft expert set below, by extending the concept of complex neutrosophic set defined earlier and give the definitions of empty and absolute CNSES.

Definition 4. Let U be a universe, E a set of parameters, X a set of experts (agents) and $O = \{1 = agree, 0 = disagree\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$. A pair (H, A) is called a complex neutrosophic soft expert set (CNSES) over U, where H is a mapping given by

$$H : A \rightarrow CN^U,$$

where CN^U denotes the power complex neutrosophic set of *U*.

Definition 5. Let (*H*, *A*) be a CNSES over *U*. Then,

1. (H, A) is said to be empty CNSES, denoted by $\tilde{\Phi}$ if $T_{H(\alpha)}(u) = 0, I_{H(\alpha)}(u) = 1, F_{H(\alpha)}(u) = 1, \forall \alpha \in A$ and $u \in U$.
2. (H, A) is said to be absolute CNSES, denoted by $\tilde{\Psi}$ if $T_{H(\alpha)}(u) = 1,\ I_{H(\alpha)}(u) = 0,\ F_{H(\alpha)}(u) = 0,$ $\forall \alpha \in A$ and $u \in U$.

In the following, we define the subset, union and intersection operations of two CNSESs.

Definition 6. For two CNSESs (H, A) and (G, B) over *U*, (H, A) is called a CNSE subset of (G, B) if
1. $A \subseteq B$,
2. $\forall \epsilon \in A, H(\epsilon)$ is complex neutrosophic subset of $G(\epsilon)$.

Definition 7. The union of two CNSESs (H, A) and (G, B) over a universe *U*, is a CNSES (K, C), denoted by $(H, A) \cup (G, B)$, such that $C = A \cup B$ and $\forall \epsilon \in C, u \in U$,

$$(K, C) = \begin{cases} H(\varepsilon), & \text{if } \varepsilon \in A - B; \\ G(\varepsilon), & \text{if } \varepsilon \in B - A; \\ H(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B; \end{cases}$$

where $\cup$ denotes the complex neutrosophic union.

Definition 8. The intersection of two CNSESs (H, A) and (G, B) over a universe *U*, is a CNSES (K, C), denoted by $(H, A) \cap (G, B)$, such that $C = A \cup B$ and $\forall \epsilon \in C, u \in U$,

$$(K, C) = \begin{cases} H(\varepsilon), & \text{if } \varepsilon \in A - B; \\ G(\varepsilon), & \text{if } \varepsilon \in B - A; \\ H(\varepsilon) \cap G(\varepsilon), & \text{if } \varepsilon \in A \cap B; \end{cases}$$

where $\cap$ denotes the complex neutrosophic intersection.

MAPPING ON COMPLEX NEUTROSOPHIC SOFT EXPERT SETS

In this section, we introduce the notion of a mapping on complex neutrosophic soft expert classes. We will define the complex neutrosophic soft expert images and complex neutrosophic soft expert inverse images of complex neutrosophic soft expert sets, and support them with an illustrative example. We will give also some operations and properties related with this concept.

Now we propose the definition of a complex neutrosophic soft expert class, followed by the definitions of complex neutrosophic soft expert image and inverse image of the complex neutrosophic soft expert set.

Definition 9. Let U be a universe, E a set of parameters, X a set of experts and $O = \{1 = agree, 0 = disagree\}$ a set of opinions. Let $Z = E \times X \times O$. Then the collection of all complex neutrosophic soft expert sets over U with parameters from Z is called a *complex neutrosophic soft expert class* and is denoted as (U, Z).

Definition 10 . Let (U, Z) and (V, Z') be complex neutrosophic soft expert classes. Let $r: U \to V$ and $s: Z \to Z'$ be mappings. Then a mapping $F: (U, Z) \to (V, Z')$ is defined as follows:

For a complex neutrosophic soft expert set (H, A) in (U, Z), $(F(H, A), M)$, where $M = s(Z) \subseteq Z'$ is a complex neutrosophic soft expert set in (V, Z') obtained as follows:

$$F(H,A)(\beta)(v) = \begin{cases} \bigcup\limits_{u \in r^{-1}(v)} \Big(\bigcup\limits_{\alpha \in s^{-1}(\beta) \cap A} H(\alpha) \Big)(u) & \text{if } r^{-1}(v) \text{ and } s^{-1}(\beta) \cap A \neq \varnothing; \\ (0, 1, 1) & \text{otherwise.} \end{cases}$$

For $u \in r^{-1}(v)$, $\beta \in M \subseteq Z'$, $v \in V$ and $\forall \alpha \in s^{-1}(\beta) \cap A$, $(F(H, A), M)$ is called a *complex neutrosophic soft expert image* of the complex neutrosophic soft expert set (H, A). If $M = Z'$, then we shall write $(F(H, A), M)$ as $F(H, A)$.

Definition 11. Let (U, Z) and (V, Z') be two complex neutrosophic soft expert classes. Let $r: U \to V$ and $s: Z \to Z'$ be mappings. Then a mapping $F^{-1}: (V, Z') \to (U, Z)$ is defined as follows:

For a complex neutrosophic soft expert set (G, B) in (V, Z'), $(F^{-1}(G, B), N)$, where $N = s^{-1}(B)$, is a complex neutrosophic soft expert set in (U, Z) obtained as follows:

$$F^{-1}(G,B)(\alpha)(u) = \begin{cases} G(s(\alpha))(r(u)) & \text{if } s(\alpha) \in B, \\ (0, 1, 1) & \text{otherwise,} \end{cases}$$

for $\alpha \in N \subseteq Z$ and $u \in U$. $(F^{-1}(G, B), N)$ is called a *complex neutrosophic soft expert inverse image* of the complex neutrosophic soft expert set (G, B). If $N = Z$, we shall write $(F^{-1}(G, B), N)$ as $F^{-1}(G, B)$.

The Definitions 10 and 11 are illustrated as follows.

Example 1. Let $U = \{u_1, u_2, u_3\}$, $V = \{v_1, v_2\}$ and let $E = \{e_1, e_2\}$, $E' = \{e_1', e_2'\}$, $X = \{p, q\}$, $X' = \{p', q'\}$ and $Z = E \times X \times O = \{(e_1, p, 1), (e_1, q, 1), (e_2, p, 1), (e_2, q, 1), (e_1, p, 0), (e_1, q, 0), (e_2, p, 0), (e_2, q, 0)\}$ and
$Z' = E' \times X' \times O = \{(e_1', p', 1), (e_1', q', 1), (e_2', p', 1), (e_2', q', 1), (e_1', p', 0), (e_1', q', 0), (e_2', p', 0), (e_2', q', 0)\}$.

Suppose that (U,Z) and (V,Z') are complex neutrosophic soft expert classes. Define $r:U\to V$ and $s:Z\to Z'$ as follows: $r(u_1)=v_2$, $r(u_2)=v_1$, $r(u_3)=v_1$, $s(e_1,p,1)=(e_2',p',1), s(e_1,q,1)=(e_1',p',1),$
$s(e_2,p,1)=(e_1',q',0), s(e_2,q,1)=(e_2',p',1), s(e_1,p,0)=(e_1',q',0), s(e_1,q,0)=(e_1',p',1),$
$s(e_2,p,0)=(e_2',p',1), s(e_2,q,0)=(e_1',p',1)$.

Let $A\subseteq Z=\{(e_1,p,1),(e_1,q,1),(e_2,p,0),(e_2,q,0)\}$, $B\subseteq Z'=\{(e_1',p',1),(e_1',q',0),(e_2',p',0),(e_2',q',1)\}$ and (H,A), (G,B) be two complex neutrosophic soft expert sets over U and V respectively such that

$$
\begin{aligned}
(H,A) =\ &\Big\{\{(e_1,p,1),\{\frac{\langle 0.1e^{j2\Pi(0.5)},0.4e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.7)}\rangle}{u_1},\frac{\langle 0.5e^{j2\Pi(0.4)},0.2e^{j2\Pi(0.7)},0.8e^{j2\Pi(0)}\rangle}{u_2},\\
&\frac{\langle 0.1e^{j2\Pi(0.2)},0.3e^{j2\Pi(0.4)},0.5e^{j2\Pi(0.6)}\rangle}{u_3}\}\},\{(e_1,q,1),\{\frac{\langle 0.9e^{j2\Pi(0.4)},0.5e^{j2\Pi(0.6)},0.8e^{j2\Pi(0.3)}\rangle}{u_1},\\
&\frac{\langle 0.3e^{j2\Pi(0.6)},0.3e^{j2\Pi(0.7)},0.7e^{j2\Pi(0.3)}\rangle}{u_2},\frac{\langle 0.7e^{j2\Pi(0.8)},0.9e^{j2\Pi(0)},0.1e^{j2\Pi(0.2)}\rangle}{u_3}\}\},\{(e_2,p,0),\\
&\frac{\langle 0.1e^{j2\Pi(0.8)},0.9e^{j2\Pi(0.7)},0.9e^{j2\Pi(0.8)}\rangle}{u_1},\frac{\langle 0.6e^{j2\Pi(0.4)},0.5e^{j2\Pi(0.5)},0.4e^{j2\Pi(0.1)}\rangle}{u_2},\\
&\frac{\langle 0.3e^{j2\Pi(0.4)},0.5e^{j2\Pi(0.6)},0.7e^{j2\Pi(0.4)}\rangle}{u_3}\}\},\{(e_2,q,0),\{\frac{\langle 0.8e^{j2\Pi(0.5)},0.4e^{j2\Pi(0.1)},0.8e^{j2\Pi(0.9)}\rangle}{u_1},\\
&\frac{\langle 0.3e^{j2\Pi(0.4)},0.6e^{j2\Pi(0.7)},0.1e^{j2\Pi(0.2)}\rangle}{u_2},\frac{\langle 0.1e^{j2\Pi(0.5)},0.9e^{j2\Pi(0.6)},0.4e^{j2\Pi(0.1)}\rangle}{u_3}\}\}\Big\}
\end{aligned}
$$

$$
\begin{aligned}
(G,B) =\ &\Big\{\{(e_1',p',1),\{\frac{\langle 0.5e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.2)}\rangle}{v_1},\frac{\langle 0.5e^{j2\Pi(0.7)},0.2e^{j2\Pi(0.1)},0.8e^{j2\Pi(0.2)}\rangle}{v_2}\}\},\\
&\{(e_1',q',0),\{\frac{\langle 0.3e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.7)},0.8e^{j2\Pi(0.9)}\rangle}{v_1},\frac{\langle 0.4e^{j2\Pi(0.5)},0.2e^{j2\Pi(0.6)},0.1e^{j2\Pi(0.3)}\rangle}{v_2}\}\},\\
&\{(e_2',p',0),\{\frac{\langle 0.6e^{j2\Pi(0.8)},0.3e^{j2\Pi(0.5)},0.4e^{j2\Pi(0.1)}\rangle}{v_1},\frac{\langle 0.2e^{j2\Pi(0.1)},0.2e^{j2\Pi(0.8)},0.9e^{j2\Pi(0.4)}\rangle}{v_2}\}\},\\
&\{(e_2',q',1),\{\frac{\langle 0.4e^{j2\Pi(0.3)},0.5e^{j2\Pi(0.7)},0.9e^{j2\Pi(0.4)}\rangle}{v_1},\frac{\langle 0.5e^{j2\Pi(0.6)},0.7e^{j2\Pi(0.5)},0.4e^{j2\Pi(0.6)}\rangle}{v_2}\}\}\Big\}.
\end{aligned}
$$

Then we define a mapping $F:(U,Z)\to(V,Z')$ as follows.

For a complex neutrosophic soft expert set (H,A) in (U,Z), $(F(H,A),M)$, where $M=s(Z)=\{(e_1',p',0),(e_1',q',0),(e_2',p',1)\}$, is a complex neutrosophic soft expert set in (V,Z') and is obtained as follows:

$F(H,A)(e_1',p',1)(v_1)=$

$$\bigcup_{u\in r^{-1}(v_1)}\Big(\bigcup_{\alpha\in s^{-1}(e_1',p',1)\cap A} H(\alpha)\Big)(u)=\bigcup_{u\in\{u_2,u_3\}}\Big(\bigcup_{\alpha\in\{(e_2,q,0),(e_1,q,1)\}} H(\alpha)\Big)(u)\quad=\bigcup_{u\in\{u_2,u_3\}}\big(H(e_2,q,0)\cup H(e_1,q,1)\big)(u)$$

$$=\bigcup_{u\in\{u_2,u_3\}}\Big(\Big\{\frac{\langle 0.9e^{j2\Pi(0.5)},0.4e^{j2\Pi(0.1)},0.8e^{j2\Pi(0.3)}\rangle}{u_1},\frac{\langle 0.3e^{j2\Pi(0.6)},0.3e^{j2\Pi(0.7)},0.1e^{j2\Pi(0.2)}\rangle}{u_2},$$
$$\frac{\langle 0.7e^{j2\Pi(0.8)},0.9e^{j2\Pi(0)},0.1e^{j2\Pi(0.1)}\rangle}{u_3}\Big\}\Big)(u)$$
$$=\Big(\frac{\langle 0.3e^{j2\Pi(0.6)},0.3e^{j2\Pi(0.7)},0.1e^{j2\Pi(0.2)}\rangle}{u_2}\Big)\bigcup\Big(\frac{\langle 0.7e^{j2\Pi(0.8)},0.9e^{j2\Pi(0)},0.1e^{j2\Pi(0.1)}\rangle}{u_3}\Big)$$
$$=\langle 0.7e^{j2\Pi(0.8)},0.3e^{j2\Pi(0)},0.1e^{j2\Pi(0.1)}\rangle .$$

$$F(H,A)(e_1',p',1)(v_2)=\bigcup_{u\in r^{-1}(v_2)}\Big(\bigcup_{\alpha\in s^{-1}(e_1',p',1)\cap A} H(\alpha)\Big)(u)\;=\bigcup_{u\in\{u_1\}}\Big(\bigcup_{\alpha\in\{(e_2,q,0),(e_1,q,1)\}} H(\alpha)\Big)(u)$$
$$=\langle 0.9e^{j2\Pi(0.5)},0.4e^{j2\Pi(0.1)},0.8e^{j2\Pi(0.3)}\rangle .$$

Thus $F(H,A)(e_1',p',1)=\Big\{\dfrac{\langle 0.7e^{j2\Pi(0.8)},0.3e^{j2\Pi(0)},0.1e^{j2\Pi(0.1)}\rangle}{v_1},\dfrac{\langle 0.9e^{j2\Pi(0.5)},0.4e^{j2\Pi(0.1)},0.8e^{j2\Pi(0.3)}\rangle}{v_2}\Big\}$,

$F(H,A)(e_1',q',0)(v_1)=(0,1,1)$ and $F(H,A)(e_1',q',0)(v_2)=(0,1,1)$, since $s^{-1}(e_1',q',0)\cap A=\phi$.

Thus $F(H,A)(e_1',q',0)=\left\{\dfrac{\langle 0,1,1\rangle}{v1},\dfrac{\langle 0,1,1\rangle}{v2}\right\}$.

By similar calculations, consequently, we get

$$F(H,A)(e_2',p',1)(v_1)=\langle 0.6e^{j2\Pi(0.4)},0.2e^{j2\Pi(0.4)},0.4e^{j2\Pi(0)}\rangle$$
$$F(H,A)(e_2',p',1)(v_2)=\langle 0.1e^{j2\Pi(0.8)},0.4e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.7)}\rangle .$$

Thus $F(H,A)(e_2',p',1)=\Big\{\dfrac{\langle 0.6e^{j2\Pi(0.4)},0.2e^{j2\Pi(0.4)},0.4e^{j2\Pi(0)}\rangle}{v_1},\dfrac{\langle 0.1e^{j2\Pi(0.8)},0.4e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.7)}\rangle}{v_2}\Big\}$.

Hence,

$$(F(H,A),M)=$$
$$\Big\{\Big\{(e_1',p',1),\Big\{\frac{\langle 0.7e^{j2\Pi(0.8)},0.3e^{j2\Pi(0)},0.1e^{j2\Pi(0.1)}\rangle}{v_1},\frac{\langle 0.9e^{j2\Pi(0.5)},0.4e^{j2\Pi(0.1)},0.8e^{j2\Pi(0.3)}\rangle}{v_2}\Big\}\Big\},$$
$$\Big\{(e_1',q',0),\Big\{\frac{\langle 0,1,1\rangle}{v_1},\frac{\langle 0,1,1\rangle}{v_2}\Big\}\Big\},\Big\{(e_2',p',1),\Big\{\frac{\langle 0.6e^{j2\Pi(0.4)},0.2e^{j2\Pi(0.4)},0.4e^{j2\Pi(0)}\rangle}{v_1},$$
$$\frac{\langle 0.1e^{j2\Pi(0.8)},0.4e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.7)}\rangle}{v_2}\Big\}\Big\}\Big\}.$$

Next for the complex neutrosophic soft expert inverse images, we have the following:

For a complex neutrosophic soft expert set (G,B) in (V,Z'), $(F^{-1}(G,B),N)$, where $N=s^{-1}(B)=\{(e_2,q,0),(e_1,q,0),(e_1,p,0),(e_2,p,1),(e_1,q,1)\}$ is a complex neutrosophic soft expert set in (U,Z), where

$$F^{-1}(G,B)(e_2,q,0)(u_1)=G(s(e_2,q,0))r(u_1)=G(e_1',p',1)(v_2)$$
$$=(\{\frac{\langle 0.5e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.2)}\rangle}{v_1},\frac{\langle 0.5e^{j2\Pi(0.7)},0.2e^{j2\Pi(0.1)},0.8e^{j2\Pi(0.2)}\rangle}{v_2}\})(v_2)$$
$$=\langle 0.5e^{j2\Pi(0.7)},0.2e^{j2\Pi(0.1)},0.8e^{j2\Pi(0.2)}\rangle.$$

$$F^{-1}(G,B)(e_2,q,0)(u_2)=G(s(e_2,q,0))r(u_2)=G(e_1',p',1)(v_1)$$
$$=(\{\frac{\langle 0.5e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.2)}\rangle}{v_1},\frac{\langle 0.5e^{j2\Pi(0.7)},0.2e^{j2\Pi(0.1)},0.8e^{j2\Pi(0.2)}\rangle}{v_2}\})(v_1)$$
$$=\langle 0.5e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.2)}\rangle.$$

In the similar manner,

$$F^{-1}(G,B)(e_2,q,0)(u_3)=\langle 0.5e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.2)}\rangle.$$

Thus,

$$F^{-1}(G,B)(e_2,q,0)=\{\frac{\langle 0.5e^{j2\Pi(0.7)},0.2e^{j2\Pi(0.1)},0.8e^{j2\Pi(0.2)}\rangle}{u_1},\frac{\langle 0.5e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.2)}\rangle}{u_2},$$
$$\frac{\langle 0.5e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.2)}\rangle}{u_3}\}.$$

By similar calculations, consequently, we get

$$(F^{-1}(G,B),N)=$$
$$\{\{(e_2,q,0),\{\frac{\langle 0.5e^{j2\Pi(0.7)},0.2e^{j2\Pi(0.1)},0.8e^{j2\Pi(0.2)}\rangle}{u_1},\frac{\langle 0.5e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.2)}\rangle}{u_2},$$
$$\frac{\langle 0.5e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.2)}\rangle}{u_3}\}\},\{(e_1,q,0),\{\frac{\langle 0.5e^{j2\Pi(0.7)},0.2e^{j2\Pi(0.1)},0.8e^{j2\Pi(0.2)}\rangle}{u_1},$$
$$\frac{\langle 0.5e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.2)}\rangle}{u_2},\frac{\langle 0.5e^{j2\Pi(0.4)},0.8e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.2)}\rangle}{u_3}\}\},\{(e_1,p,0),$$
$$\{\frac{\langle 0.4e^{j2\Pi(0.5)},0.2e^{j2\Pi(0.6)},0.1e^{j2\Pi(0.3)}\rangle}{u_1},\frac{\langle 0.3e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.7)},0.8e^{j2\Pi(0.9)}\rangle}{u_2},$$
$$\frac{\langle 0.3e^{j2\Pi(0.5)},0.6e^{j2\Pi(0.7)},0.8e^{j2\Pi(0.9)}\rangle}{u_3}\}\},\{(e_2,p,1),\{\frac{\langle 0.4e^{j2\Pi(0.5)},0.2e^{j2\Pi(0.6)},0.1e^{j2\Pi(0.3)}\rangle}{u_1},$$

$$\frac{\langle 0.3e^{j2\Pi(0.5)}, 0.6e^{j2\Pi(0.7)}, 0.8e^{j2\Pi(0.9)}\rangle}{u_2}, \frac{\langle 0.3e^{j2\Pi(0.5)}, 0.6e^{j2\Pi(0.7)}, 0.8e^{j2\Pi(0.9)}\rangle}{u_3}\}\}, \{(e_1, q, 1),$$
$$\{\frac{\langle 0.5e^{j2\Pi(0.7)}, 0.2e^{j2\Pi(0.1)}, 0.8e^{j2\Pi(0.2)}\rangle}{u_1}, \frac{\langle 0.5e^{j2\Pi(0.4)}, 0.8e^{j2\Pi(0.5)}, 0.6e^{j2\Pi(0.2)}\rangle}{u_2},$$
$$\frac{\langle 0.5e^{j2\Pi(0.4)}, 0.8e^{j2\Pi(0.5)}, 0.6e^{j2\Pi(0.2)}\rangle}{u_3}\}\}\}.$$

Definition 12. Let $F: (U,Z) \rightarrow (V,Z')$ be a mapping and (H,A), (G,B) complex neutrosophic soft expert sets in (U,Z). Then for $\beta \in Z'$, $v \in V$, the union and intersection of the complex neutrosophic soft expert images $F(H,A)$ and $F(G,B)$ are defined as follows.

$$\left(F(H,A)\hat{\cup}F(G,B)\right)(\beta)(v) = F(H,A)(\beta)(v)\tilde{\cup}F(G,B)(\beta)(v),$$
$$\left(F(H,A)\hat{\cap}F(G,B)\right)(\beta)(v) = F(H,A)(\beta)(v)\tilde{\cap}F(G,B)(\beta)(v),$$

where $\tilde{\cup}$ and $\tilde{\cap}$ denote the complex neutrosophic soft expert union and intersection, respectively.

Definition 13. Let $F: (U,Z) \rightarrow (V,Z')$ be a mapping and (H,A), (G,B) complex neutrosophic soft expert sets in (V,Z'). Then for $\alpha \in Z$, $u \in U$, the union and intersection of the complex neutrosophic soft expert inverse images $F^{-1}(H,A)$ and $F^{-1}(G,B)$ are defined as follows.

$$\left(F^{-1}(H,A)\hat{\cup}F^{-1}(G,B)\right)(\alpha)(u) = F^{-1}(H,A)(\alpha)(u)\tilde{\cup}F^{-1}(G,B)(\alpha)(u),$$
$$\left(F^{-1}(H,A)\hat{\cap}F^{-1}(G,B)\right)(\alpha)(u) = F^{-1}(H,A)(\alpha)(u)\tilde{\cap}F^{-1}(G,B)(\alpha)(u),$$

where $\tilde{\cup}$ and $\tilde{\cap}$ denote the complex neutrosophic soft expert union and intersection, respectively.

Proposition 1. Let $F: (U,Z) \rightarrow (V,Z')$ be a mapping. Then for complex neutrosophic soft expert sets (H,A) and (G,B) in the complex neutrosophic soft expert class (U,Z), we have:

1. $F(\tilde{\Phi}) = \tilde{\Phi}$.
2. $F(\tilde{\Psi}) = \tilde{\Psi}$.
3. $F\left((H,A)\tilde{\cup}(G,B)\right) = F(H,A)\tilde{\cup}F(G,B)$.
4. $F\left((H,A)\tilde{\cap}(G,B)\right) \subseteq F(H,A)\tilde{\cap}F(G,B)$.
5. If $(H,A) \subseteq (G,B)$, then $F(H,A) \subseteq F(G,B)$.

Proof: The proof is straightforward by Definitions 10 and 12.

Proposition 2. Let $F: (U,Z) \rightarrow (V,Z')$ be a mapping. Then for complex neutrosophic soft expert sets (H,A) and (G,B) in the complex neutrosophic soft expert class (V,Z'), we have:

1. $F^{-1}(\tilde{\Phi}) = \tilde{\Phi}$.
2. $F^{-1}(\tilde{\Psi}) = \tilde{\Psi}$.
3. $F^{-1}\left((H,A)\tilde{\cup}(G,B)\right) = F^{-1}(H,A)\tilde{\cup}F^{-1}(G,B)$.

4. $F^{-1}\left(\left(H,A\right)\tilde{\cap}\left(G,B\right)\right)=F^{-1}\left(H,A\right)\tilde{\cap}F^{-1}\left(G,B\right)$.
5. If $\left(H,A\right)\subseteq\left(G,B\right)$, then $F^{-1}\left(H,A\right)\subseteq F^{-1}\left(G,B\right)$.

Proof: The proof is straightforward by Definitions 11 and 13.

CONCLUSION

We established the concept of mapping on complex neutrosophic soft expert sets. The basic operations and properties of mapping on complex neutrosophic soft classes have been studied. We hope these fundamental results will help researchers to enhance and promote the research on complex neutrosophic soft expert theory.

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