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Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces

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Abstract In this paper we introduce definitions of generalized neutrosophic sets. After given the fundamental definitions of generalized neutrosophic set operations, we obtain several properties, and discussed the relationship between generalized neutrosophic sets and others. Finally, we extend the concepts of neutrosophic topological space [9], intuitionistic fuzzy topological space [5, 6], and fuzzy topological space [4] to the case of generalized neutrosophic sets. Possible application to GIS topology rules are touched upon.

Keywords Neutrosophic Set, Generalized Neutrosophic Set, Neutrosophic Topology

1. Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory. The fuzzy set was introduced by Zadeh [10] in 1965, where each element had a degree of membership. The intuitionstic fuzzy set (Ifs for short) on a universe X was introduced by K. Atanassov [1, 2, 3] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. After the introduction of the neutrosophic set concept [7, 8, 9]. In this paper we introduce definitions of generalized neutrosophic sets. After given the fundamental definitions of generalized neutrosophic set operations, we obtain several properties, and discussed the relationship between generalized neutrosophic sets and others. Finally, we extend the concepts of neutrosophic topological space [9].

2. Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [7, 8], Atanassov in[1, 2, 3] and Salama [9]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $\left]0^-,1^+\right[$ is nonstandard unit interval.

Definition.[7, 8]

Let T, I,F be real standard or nonstandard subsets of $\left]0^-,1^+\right[$, with

Sup_T=t_sup, inf_T=t_inf
Sup_I=i_sup, inf_I=i_inf
Sup_F=f_sup, inf_F=f_inf
n-sup=t_sup+i_sup+f_sup
n-inf=t_inf+i_inf+f_inf,
T, I, F are called neutrosophic components

Definition [9]

Let X be a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form $A=\left\{\left\langle x,\mu_A(x),\sigma_A(x),\gamma_A(x)\right\rangle : x\in X\right\}$ Where $\mu_A(x),\sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\gamma_A(x)$) respectively of each element $x\in X$ to the set A.

Definition [9].

The NSS 0_N and 1_N in X as follows:

0_N may be defined as:

(0_1) $0_N=\left\{\left\langle x,0,0,1\right\rangle : x\in X\right\}$

(0_2) $0_N=\left\{\left\langle x,0,1,1\right\rangle : x\in X\right\}$

(0_3) $0_N=\left\{\left\langle x,0,1,0\right\rangle : x\in X\right\}$

(0_4) $0_N=\left\{\left\langle x,0,0,0\right\rangle : x\in X\right\}$

1_N may be defined as:

(1_1) $1_N=\left\{\left\langle x,1,0,0\right\rangle : x\in X\right\}$

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(1_2) $1_N = \{\langle x, 1, 0, 1\rangle : x \in X\}$

(1_3) $1_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$

(1_4) $1_N = \{\langle x, 1, 1, 1\rangle : x \in X\}$

3. Generalized Neutrosophic Sets

We shall now consider some possible definitions for basic concepts of the generalized neutrosophic set.

Definition

Let X be a non-empty fixed set. A generalized neutrosophic set (GNS for short) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ Where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A where the functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \le 0.5$.

Remark

A generalized neutrosophic $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$ can be identified to an ordered triple $< \mu_A, \sigma_A, \gamma_A >$ in $]^-0, 1^+[$ on X, where the triple functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \le 0.5$

Remark

For the sake of simplicity, we shall use the symbol $A = < x, \mu_A, \sigma_A, \gamma_A >$ for the G NS $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$

Example

Every GIFS A a non-empty set X is obviously on GNS having the form

$$A = \{< x, \mu_A(x), 1 - (\mu_A(x) + \gamma_A(x)), \gamma_A(x) >: x \in X\}$$

Definition

Let $A = \langle \mu_A, \sigma_A, \gamma_A \rangle$ a GNSS on X , then the complement of the set A ($C(A)$, for short) maybe defined as three kinds of complements

(C_1) $C(A) = \{\langle x, 1 - \mu_A(x), \sigma_A(x), 1 - \nu_A(x)\rangle : x \in X\}$,

(C_2) $C(A) = \{\langle x, \gamma_A, \sigma_A(x), \mu_A(x)\rangle : x \in X\}$

(C_3) $C(A) = \{\langle x, \gamma_A, 1 - \sigma_A(x), \mu_A(x)\rangle : x \in X\}$

One can define several relations and operations between GNSS as follows:

Definition

Let X be a non-empty set, and GNSS A and B in the form $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$, $B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle$, then we may consider two possible definitions for subsets $(A \subseteq B)$

$(A \subseteq B)$ may be defined as

$A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \gamma_A(x) \ge \gamma$ and $\sigma_A(x) \le \sigma_B(x)$ $\forall x \in X$

$A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \gamma_A(x) \ge \gamma_B(x)$ and $\sigma_A(x) \ge \sigma_B(x)$

Proposition

For any generalized neutrosophic set A the following are holds

$0_N \subseteq A$, $0_N \subseteq 0_N$

$A \subseteq 1_N$, $1_N \subseteq 1_N$

Definition

Let X be a non-empty set, and $A = < x, \mu_A(x), \gamma_A(x), \sigma_A(x) >$, $B = < x, \mu_B(x), \sigma_B(x), \gamma_B(x) >$ are GNSS Then $A \cap B$ maybe defined as:

(I_1) $A \cap B = < x, \mu_A(x).\mu_B(x), \sigma_A(x).\sigma_B(x), \gamma_A(x).\gamma_B(x) >$

(I_2) $A \cap B = < x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) >$

(I_3) $A \cap B = < x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) >$

$A \cup B$ may be defined as:

(U_1) $A \cup B = < x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) >$

(U_2) $A \cup B = < x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) >$

$[\ ]A = < x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x) >$

$< > A = < x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x) >$

Example.3.2. Let $X = \{a, b, c, d, e\}$ and $A = \langle x, \mu_A, \sigma_A, \nu_A\rangle$ given by:

X	$\mu_A(x)$	$\nu_A(x)$	$\sigma_A(x)$	$\mu_A(x) \wedge \nu_A(x) \wedge \sigma_A(x)$
a	0.6	0.3	05	0.3
b	0.5	0.3	0.6	0.3
c	0.4	0.4	0.5	0.4
d	0.3	0.5	0.3	0.3
e	0.3	0.6	0.4	0.3

Then the family $G = \{O_\sim, A\}$ is an GNSS on X.

We can easily generalize the operations of generalized intersection and union in definition 3.4 to arbitrary family of GNSS as follow:

Definition

Let $\{Aj : j \in J\}$ be a arbitrary family of NSS in X , then

$\cap Aj$ maybe defined as:

1) $\cap Aj = \left\langle x, \underset{j \in J}{\wedge} \mu_{Aj}(x), \underset{j \in J}{\wedge} \sigma_{Aj}(x), \vee \gamma_{Aj}(x) \right\rangle$

2) $\bigcap Aj = \left\langle x, \wedge\mu_{Aj}(x), \vee\sigma_{Aj}(x), \vee\gamma_{Aj}(x)\right\rangle$

$\bigcup Aj$ maybe defined as:

1) $\cup A_j = \left\langle x, \underset{j\in J}{\vee}\mu_{A_j}, \wedge\sigma_{A_j}, \wedge\nu_{A_j}\right\rangle$

2) $\cup A_j = \left\langle x, \underset{j\in J}{\vee}\mu_{A_j}, \vee\sigma_{A_j}, \wedge\nu_{A_j}\right\rangle$

Definition

Let A and B are generalized neutrosophic sets then $A|B$ may be defined as

$A|B = \left\langle x, \mu_A \wedge \gamma_B, \sigma_A(x)\sigma_B(x), \gamma_A \vee \mu_B(x)\right\rangle$

Proposition

For all A,B two generalized neutrosophic sets then the following are true

i) $C(A \cap B) = C(A) \cup C(B)$

ii) $C(A \cup B) = C(A) \cap C(B)$

4. Generalized Neutrosophic Topological Spaces

Here we extend the concepts of and intuitionistic fuzzy topological space [5, 7], and neutrosophic topological Space [9] to the case of generalized neutrosophic sets.

Definition

A generalized neutrosophic topology (GNT for short) an a non empty set X is a family τ of generalized neutrosophic subsets in X satisfying the following axioms

$(GNT_1)\ O_N, 1_N \in \tau$,

$(GNT_2)\ G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

$(GNT_3)\ \cup G_i \in \tau \quad \forall\{G_i : i \in J\} \subseteq \tau$

In this case the pair (X,τ) is called a generalized neutrosophic topological space (GNTS for short) and any neutrosophic set in τ is known as neuterosophic open set (NOS for short) in X. The elements of τ are called open generalized neutrosophic sets, A generalized neutrosophic set F is closed if and only if it C (F) is generalized neutrosophic open.

Remark A generalized neutrosophic topological spaces are very natural generalizations of intuitionistic fuzzy topological spaces allow more general functions to be members of intuitionistic fuzzy topology.

Example

Let $X = \{x\}$ and

$A = \{\langle x, 0.5, 0.5, 0.4\rangle : x \in X\}$

$B = \{\langle x, 0.4, 0.6, 0.8\rangle : x \in X\}$

$D = \{\langle x, 0.5, 0.6, 0.4\rangle : x \in X\}$

$C = \{\langle x, 0.4, 0.5, 0.8\rangle : x \in X\}$

Then the family $\tau = \{O_n, 1_n, A, B, C, D\}$ of GNSs in X is generalized neutrosophic topology on X

Example

Let (X, τ_o) be a fuzzy topological space in Changes [4] sense such that τ_0 is not indiscrete suppose now that $\tau_0 = \{0_N, 1_N\} \cup \{V_j : j \in J\}$ then we can construct two G$NTSS$ on X as follows

$\tau_0 = \{0_N, 1_N\} \cup \{< x, V_j, \sigma(x), 0 >: j \in J\}$

$\tau_0 = \{0_N, 1_N\} \cup \{< x, V_j, 0, \sigma(x), 1 - V_j >: j \in J\}$

Proposition

Let (X,τ) be a GNT on X, then we can also construct several GNTSS on X in the following way:

a) $\tau_{o,1} = \{[\ ]G : G \in \tau\}$,

b) $\tau_{o,2} = \{<> G : G \in \tau\}$,

Proof a)

(GNT_1) and (GNT_2) are easy.

(GNT_3)Let $\{[\ ]G_j : j \in J, G_j \in \tau\} \subseteq \tau_{0,1}$.Since

$\cup G_j = \left\{\left\langle x, \vee\mu_{G_j}, \vee\sigma_{G_j}, \wedge\gamma_{G_j}\right\rangle or \left\langle x, \vee\mu_{G_j}, \wedge\sigma_{G_j}, \wedge\gamma_{G_j}\right\rangle or \left\langle x, \vee\mu_{G_j}, \wedge\sigma_{G_j}, \vee\gamma_{G_j}\right\rangle\right\} \in \tau$,

we have

$\cup([\ ]G_j) = \{x, \vee\mu_{G_j}, \vee\sigma_{G_j}, \wedge(1-\mu_{G_j})\} or \{x, \vee\mu_{G_j}, \vee\sigma_{G_j}, (1 - \vee\mu_{G_j})\} \in \tau_{0,1}$

This similar to (a)

Definition

Let $(X,\tau_1), (X,\tau_2)$ be two generalized neutrosophic topological spaces on X. Then τ_1 is said be contained in τ_2 (in symbols $\tau_1 \subseteq \tau_2$) if $G \in \tau_2$ for each $G \in \tau_1$. In this case, we also say that τ_1 is coarser than τ_2.

Proposition

Let $\{\tau_j : j \in J\}$ be a family of $NTSS$ on X. Then $\cap\tau_j$ is A generalized neutrosophic topology on X .Furthermore, $\cap\tau_j$ is the coarsest NT on X containing all. τ_j, s

Proof. Obvious

Definition

The complement of A (C (A) for short) of NOS. A is called a generalized neutrosophic closed set (GNCS for short) in X.

Now, we define generalized neutrosophic closure and interior operations in generalized neutrosophic topological spaces:

Definition

Let (X,τ) be G NTS and $A = < x, \mu_A(x), \gamma_A(x), \sigma_A(x) >$ be a GNS in X.

Then the generalized neutrosophic closer and generalized neutrosophic interior of Aare defined by

G $NCl(A) = \cap\{K : K$ is an NCS in X and $A \subseteq K\}$

G $NInt(A) = \cup\{G : G$ is an NOS in X and $G \subseteq A\}$.It can be also shown that

It can be also shown that $NCl(A)$ is NCS and $NInt(A)$ is a GNOS in X

A is in X if and only if G $NCl(A)$.

A is G $\mathbf{N}$ in X if and only if G $NInt(A) = A$.

Proposition

For any generalized neutrosophic set A in (x,τ) we have

(a) G $NCl(C(A) = C(GNInt(A)$,

(b) G $NInt(C(A)) = C(GNCl(A))$.

Proof.

Let $A = \{< x, \mu_A, \sigma_A, \upsilon_A >: x \in X\}$ and suppose that the family of generalized neutrosophic subsets contained in A are indexed by the family if G NSS contained in A are indexed by the family $A = \{< x, \mu_{G_i}, \sigma_{G_i}, \upsilon_{G_i} >: i \in J\}$. Then we see that $GNInt(A) = \{< x, \vee\mu_{G_i}, \vee\sigma_{G_i}, \wedge\upsilon_{G_i} >\}$ and hence $C(GNInt(A)) = \{< x, \wedge\mu_{G_i}, \vee\sigma_{G_i}, \vee\upsilon_{G_i} >\}$. Since $C(A)$ and $\mu_{G_i} \leq \mu_A$ and $\nu_{G_i} \geq \nu_A$ for each $i \in J$, we obtaining $C(A)$. i.e $GNCl(C(A)) = \{< x, \wedge\upsilon_{G_i}, \vee\sigma_{G_i}, \vee\mu_{G_i} >\}$. Hence $GNCl(C(A) = C(GNInt(A)$, follows immediately

This is analogous to (a).

Proposition

Let (x,τ) be a GNTS and A,B be two neutrosophic sets in X. Then the following properties hold:

$GNInt(A) \subseteq A$,

$A \subseteq GNCl(A)$,

$A \subseteq B \Rightarrow GNInt(A) \subseteq GNInt(B)$,

$A \subseteq B \Rightarrow GNCl(A) \subseteq GNCl(B)$,

$GNInt(GNInt(A)) = GNInt(A) \wedge GNInt(B)$,

$GNCl(A \cup B) = GNCl(A) \vee GNCl(B)$,

$GNInt(1_N) = 1_N$,

$GNCl(O_N) = O_N$,

Proof (a), (b) and (e) are obvious (c) follows from (a) and Definitions.

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