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Certain Properties of Bipolar Neutrosophic Graphs

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Abstract.

In this paper, we discuss spacial types of bipolar neutrosophic graphs, including edge irregular bipolar neutrosophic graphs and totally edge irregular bipolar neutrosophic graphs. We illustrate these types by several examples and investigate some of their interesting properties.

Keywords: Bipolar neutrosophic graphs, Edge irregular bipolar neutrosophic graphs, Totally edge irregular bipolar neutrosophic graphs.

1. Introduction

Fuzzy set theory plays a vital role in complex phenomena which is not easily characterized by classical set theory. The concept of bipolar fuzzy sets [20] is a new mathematical theory for dealing with uncertainty. Bipolar fuzzy sets are the extension of fuzzy sets whose membership degree lies in the interval $[-1, 1]$. The membership degree $(0, 1]$ shows that the element satisfies a certain property. The membership degree $[-1, 0)$ shows that the object satisfies the implicit counter property. Positive information indicates what is considered to be possible and negative information represent what is granted to be impossible. In fact, a number of decision making problems are depend on two-sided bipolar judgements on a positive side and a negative side. Smarandache [16, 17] proposed the idea of neutrosophic sets by combining the non-standard analysis. In neutrosophic set, the membership value is associated with three components: truth-membership, indeterminacy-membership and falsity-membership, in which each membership value is a real standard or non-standard subset of the non-standard unit interval $]0^-, 1^+[$ and there is no restriction on their sum. Neutrosophic set is a mathematical tool for dealing real life problems having imprecise, indeterminate and inconsistent data. Neutrosophic set theory is considered as a generalization of classical set theory, fuzzy set theory and intuitionistic fuzzy set theory. Wang et al. [18] presented the notion of single-valued neutrosophic sets to apply neutrosophic sets in real life problems more conveniently. In single-valued neutrosophic sets, three components are independent and their values are taken from the standard unit interval $[0, 1]$. Recently, as a generalized form of bipolar fuzzy sets and single-valued neutrosophic sets, bipolar neutrosophic sets are defined by Deli et al. [9].

Graph theory has become a powerful conceptual framework for modeling and solution of combinatorial problems that arise in various fields, including mathematics, engineering and computer science. However, in some cases, some aspects of graph theoretic concepts may be uncertain. In such cases, it is important to deal with uncertainties using the methods of fuzzy sets and logics. Kaufmann [11] gave the definition of a fuzzy graph on the basis of Zadeh's fuzzy relations [19]. Fuzzy graphs were narrated by Rosenfeld [14]. After that, some remarks on fuzzy graphs were represented by Bhattacharya [8]. He showed that all the concepts on crisp graph theory do not have similarities in fuzzy graphs. Santhimaheswari and Sekar [15] studied strongly edge irregular fuzzy graphs and strongly edge totally irregular fuzzy graphs. In [1-6], Akram with his different co-authors has discussed several concepts, including bipolar neutrosophic graphs, bipolar neutrosophic planar graphs, vague graphs, bipolar neutrosophic graph structures and m-polar fuzzy graphs. In this paper, we discuss spacial types of bipolar neutrosophic graphs, including edge irregular bipolar neutrosophic graphs and totally edge irregular bipolar neutrosophic graphs. We illustrate these types by several examples and investigate some of their interesting properties.

2. Bipolar Neutrosophic Graphs

Definition 2.1. *[18] A bipolar fuzzy set S on a non-empty set M is an object having the form*

$$S = \{(m, \mu^+(m), \mu^-(m)) : m \in M\},$$

where the mappings $\mu^+ : M \to [0, 1]$ and $\mu^- : M \to [-1, 0]$ denote the positive membership function and the negative membership function, respectively. The positive membership value $\mu^+(m)$ and the negative membership value $\mu^-(m)$ of an element m indicate the strength of satisfaction of element m to a certain property and the strength of satisfaction of element m to some counter property of bipolar fuzzy set S.

- *If $\mu^+(m) \neq 0$ and $\mu^-(m) = 0$, then it will show that element m has only* *truth satisfaction value for the property of S.*

- *If $\mu^+(m) = 0$ and $\mu^-(m) \neq 0$, then it will show that element m is not satisfying the property of S but satisfying the counter property of S.*
- *If $\mu^+(m) \neq 0$ and $\mu^-(m) \neq 0$, then it will show that element m is satisfying the property of S as well as the counter property of S.*

Definition 2.2. *[9] A bipolar neutrosophic set S on a non-empty set M is an object having the form*

$$S = \{(m, t^+(m), i^+(m), f^+(m), t^-(m), i^-(m), f^-(m)) : m \in M\},$$

where the mappings $t^+, i^+, f^+ : M \to [0,1]$ and $t^-, i^-, f^- : M \to [-1,0]$ denote the positive membership functions and the negative membership functions, respectively. The positive membership values $t^+(m), i^+(m), f^+(m)$ and the negative membership values $t^-(m), i^-(m), f^-(m)$ of an element m indicate the strength of truth, indeterminacy, falsity of element m to a certain property of bipolar neutrosophic set S and the strength of truth, indeterminacy, falsity of element m to some counter property of bipolar neutrosophic set S.

Definition 2.3. *[9] Let M be a non-empty set. A bipolar neutrosophic relation T on M is a mapping $T : M \times M \to [0,1] \times [-1,0]$ having the form $T = \{mn, t_T^+(mn), i_T^+(mn), f_T^+(mn), t_T^-(mn), i_T^-(mn), f_T^-(mn) : mn \in M \times M\}$, where t_T^+, i_T^+, f_T^+ $: M \to [0,1]$ and t_T^-, i_T^-, f_T^- $: M \to [-1,0]$ are the membership functions.*

Definition 2.4. *[9] A bipolar neutrosophic graph $G = (S,T)$ on M is a pair, where $S = (\mu_S^+, \mu_S^-)$ is a bipolar neutrosophic set on M and $T = (\mu_T^+, \mu_T^-)$ is a bipolar neutrosophic relation on M such that*

- $t_T^+(mn) \leq t_S^+(m) \wedge t_S^+(n)$, $\quad i_T^+(mn) \leq i_S^+(m) \wedge i_S^+(n)$, $\quad f_T^+(mn) \leq f_S^+(m) \vee f_S^+(n)$,
- $t_T^-(mn) \geq t_S^-(m) \vee t_S^-(n)$, $\quad i_T^-(mn) \geq i_S^-(m) \vee i_S^-(n)$, $\quad f_T^-(mn) \geq f_S^-(m) \wedge f_S^-(n)$,

for all $m, n \in M$, where $T(mn) = (0,0,0,\ 0,0,0)$ for all $mn \in M \times\ M - L$. Note that S and T are called the bipolar neutrosophic vertex set and the bipolar neutrosophic edge set of G, respectively.

Example 2.5. Consider a crisp graph $G^* = (M, L)$ such that $M = \{m_1, m_2, m_3, m_4, m_5\}$ and $L = \{m_1m_4,\ m_2m_4,\ m_2m_5,\ m_3m_5\}$. Let M and L be a bipolar

Table 2: Bipolar neutrosophic edge set

T	m_1m_4	m_2m_4	m_2m_5	m_3m_5
t_T^+	0.1	0.2	0.1	0.2
i_T^+	0.3	0.3	0.2	0.4
f_T^+	0.3	0.4	0.5	0.5
t_T^-	-0.1	-0.4	-0.3	-0.3
i_T^-	-0.3	-0.3	-0.2	-0.2
f_T^-	-0.3	-0.5	-0.5	-0.5

neutrosophic subset of M and a bipolar neutrosophic subset of $L \subseteq M \times M$, respectively, defined by

Table 1: Bipolar neutrosophic vertex set

S	m_1	m_2	m_3	m_4	m_5
t_S^+	0.1	0.4	0.2	0.2	0.2
i_S^+	0.4	0.3	0.4	0.5	0.4
f_S^+	0.3	0.4	0.3	0.3	0.5
t_S^-	-0.3	-0.5	-0.7	-0.5	-0.4
i_S^-	-0.5	-0.3	-0.2	-0.4	-0.3
f_S^-	-0.2	-0.6	-0.4	-0.3	-0.6

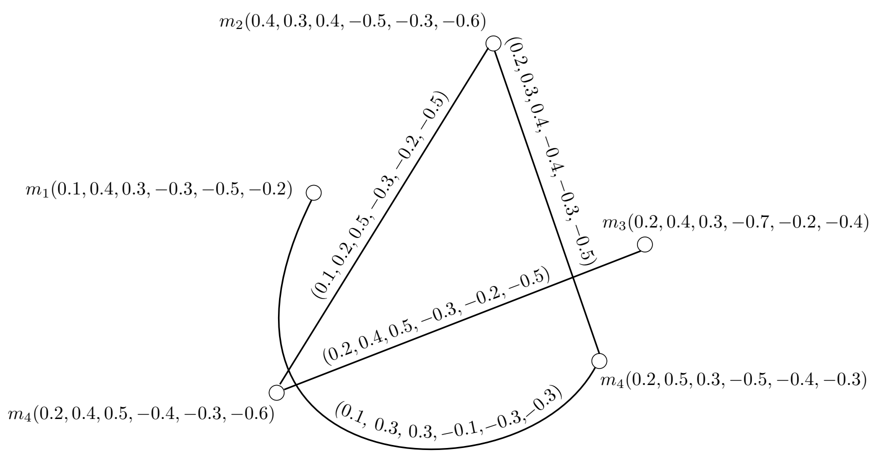

Figure 1: Bipolar neutrosophic graph G

Definition 2.6. *The degree and the total degree of a vertex m of a bipolar neutro-*

*sophic graph G are denoted by $\mathcal{D}_G(m) = (\mathcal{D}_{t^+}(m),\ \mathcal{D}_{i^+}(m),\ \mathcal{D}_{f^+}(m),\ \mathcal{D}_{t^-}(m),\ \mathcal{D}_{i^-}(m),\ \mathcal{D}_{f^-}(m))$ and $\mathcal{TD}_G(m) = (\mathcal{TD}_{t^+}(m),\ \mathcal{TD}_{i^+}(m),$
$\mathcal{TD}_{f^+}(m),\ \mathcal{TD}_{t^-}(m),\ \mathcal{TD}_{i^-}(m),\ \mathcal{TD}_{f^-}(m))$, respectively, and are defined as*

$$\begin{aligned}
\mathcal{D}_G(m) &= (\sum_{m\neq n} t_T^+(mn), \sum_{m\neq n} i_T^+(mn), \sum_{m\neq n} f_T^+(mn), \sum_{m\neq n} t_T^-(mn), \sum_{m\neq n} i_T^-(mn), \\
&\quad \sum_{m\neq n} f_T^-(mn)), \\
\mathcal{TD}_G(m) &= (\sum_{m\neq n} t_T^+(mn) + t_S^+(m), \sum_{m\neq n} i_T^+(mn) + i_S^+(m), \sum_{m\neq n} f_T^+(mn) + f_S^+(m), \\
&\quad \sum_{m\neq n} t_T^-(mn) + t_S^-(m), \sum_{m\neq n} i_T^-(mn) + i_S^-(m), \sum_{m\neq n} f_T^-(mn) + f_S^-(m)),
\end{aligned}$$

for $mn \in L$, where $m \in M$.

Definition 2.7. *A bipolar neutrosophic graph $G = (S, T)$ is called an irregular bipolar neutrosophic graph if there exists a vertex which is adjacent to vertices with distinct degrees. A bipolar neutrosophic graph $G = (S, T)$ is called a totally irregular bipolar neutrosophic graph if there exists a vertex which is adjacent to vertices with distinct total degrees.*

Example 2.8.

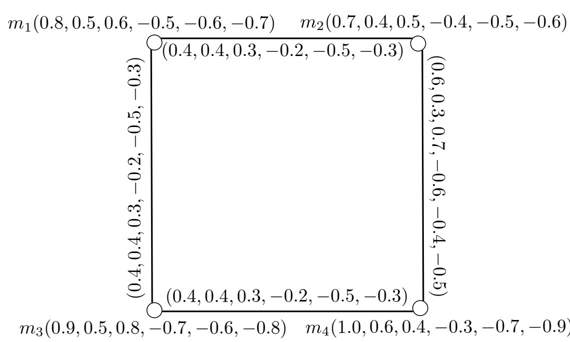

Figure 2: Irregular bipolar neutrosophic graph G

Consider a crisp graph $G^* = (M, L)$ such that $M = \{m_1,\ m_2,\ m_3,\ m_4\}$ and $L = \{m_1m_2,\ m_1m_3,\ m_2$
$m_4,\ m_3m_4\}$. The corresponding bipolar neutrosophic graph $G = (S, T)$ is shown in Fig. 2. By direct calculations, we have $\mathcal{D}_G(m_1) = \mathcal{D}_G(m_3) = (0.8, 0.8, 0.6,$
$-0.4, -1.0,$

$-0.6)$, $\mathcal{D}_G(m_2) = \mathcal{D}_G(m_4) = (1.0, 0.7, 1.0, -0.8, -0.9, -0.8)$, $\mathcal{TD}_G(m_1) = (1.6, 1.3, 1.2, -0.9, -1.6, -1.3)$, $\mathcal{TD}_G(m_2) = (1.7, 1.1, 1.5, -1.2, -1.4, -1.4)$, $\mathcal{TD}_G(m_3) = (1.7, 1.3, 1.4, -1.1, -1.6, -1.4)$ and $\mathcal{TD}_G(m_4) = (2.0, 1.3, 1.4, -1.1, -1.6, -1.7)$. It is easy to see that m_1 is adjacent to vertices of distinct degrees and distinct total degrees. Therefore, G is an irregular bipolar neutrosophic graph as well as totally irregular bipolar neutrosophic graph.

Definition 2.9. *A bipolar neutrosophic graph $G = (S, T)$ is called a strongly irregular bipolar neutrosophic graph if each vertex has distinct degree. A bipolar neutrosophic graph $G = (S, T)$ is called a strongly totally irregular bipolar neutrosophic graph if each vertex has distinct total degree.*

Example 2.10. Consider a crisp graph $G^* = (M, L)$ such that $M = \{m_1, m_2, m_3\}$ and $L = \{m_1m_2, m_2m_3, m_3m_1\}$. The corresponding bipolar neutrosophic graph $G = (S, T)$ is shown in Fig. 3.

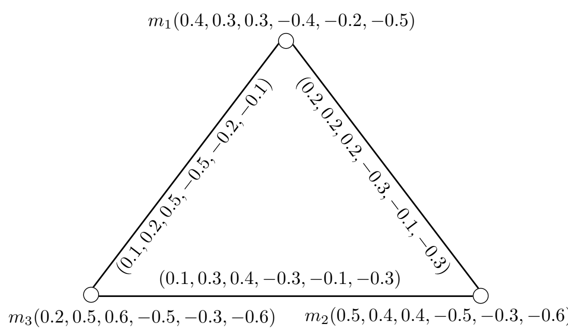

Figure 3: Strongly irregular bipolar neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(m_1) = (0.3, 0.4, 0.7, -0.8, -0.3, -0.4)$, $\mathcal{D}_G(m_2) = (0.3, 0.5, 0.6, -0.6, -0.2, -0.6)$, $\mathcal{D}_G(m_3) = (0.2, 0.5, 0.9, -0.8, -0.3, -0.4)$, $\mathcal{TD}(m_1) = (0.7, 0.7, 1.0, -1.2, -0.5, -0.9)$, $\mathcal{TD}(m_2) = (0.8, 0.9, 1.0, -1.1, -0.5, -1.2)$ and $\mathcal{TD}(m_3) = (0.4, 1.0, 1.5, -1.3, -0.6, -1.0)$. From Fig. 3, it is clear that each vertex has distinct degree and distinct total degree. Therefore, G is strongly irregular bipolar neutrosophic graph as well as strongly totally irregular bipolar neutrosophic graph.

Definition 2.11. *A bipolar neutrosophic graph $G = (S, T)$ is called a highly irregular bipolar neutrosophic graph if each vertex in G is adjacent to vertices having distinct degrees. A bipolar neutrosophic graph $G = (S, T)$ is called a highly totally irregular bipolar neutrosophic graph if each vertex in G is adjacent to vertices having distinct total degrees.*

Example 2.12. Consider the bipolar neutrosophic graph $G = (S, T)$ as shown in Fig. 2. It is easy to see that each vertex is adjacent to vertices of distinct degree therefore G is highly irregular bipolar neutrosophic graph and highly totally irregular bipolar neutrosophic graph.

Definition 2.13. *The degree and the total degree of an edge mn of a bipolar neutrosophic graph G are denoted by $\mathcal{D}_G(mn) = (\mathcal{D}_{t^+}(mn), \mathcal{D}_{i^+}(mn), \mathcal{D}_{f^+}(mn), \mathcal{D}_{t^-}(mn), \mathcal{D}_{i^-}(mn), \mathcal{D}_{f^-}(mn))$ and $\mathcal{TD}_G(mn) = (\mathcal{TD}_{t^+}(mn), \mathcal{TD}_{i^+}(mn), \mathcal{TD}_{f^+}(mn), \mathcal{TD}_{t^-}(mn), \mathcal{TD}_{i^-}(mn), \mathcal{TD}_{f^-}(mn))$, respectively, and are defined as*

$$\begin{aligned}\mathcal{D}_G(mn) &= \mathcal{D}_G(m) + \mathcal{D}_G(n) - 2(t_T^+(mn), i_T^+(mn), f_T^+(mn), t_T^-(mn), i_T^-(mn),\\ &\quad f_T^-(mn)),\end{aligned}$$

$$\mathcal{TD}_G(mn) = \mathcal{D}_G(mn) + (t_T^+(mn), i_T^+(mn), f_T^+(mn), t_T^-(mn), i_T^-(mn), f_T^-(mn)).$$

Definition 2.14. *A connected bipolar neutrosophic graph $G = (S, T)$ is called a neighbourly edge irregular bipolar neutrosophic graph if every two adjacent edges in G have distinct degrees. A connected bipolar neutrosophic graph $G = (S, T)$ is called a neighbourly edge totally irregular bipolar neutrosophic graph if every two adjacent edges in G have distinct total degrees.*

Example 2.15. Consider a crisp graph $G^* = (M, L)$ such that $M = \{m_1, m_2, m_3\}$ and $L = \{m_1m_2, m_2m_3, m_3m_1\}$. The corresponding bipolar neutrosophic graph $G = (S, T)$ is shown in Fig. 4.

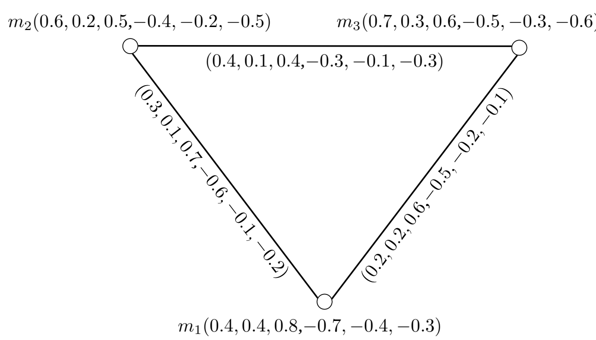

Figure 4: Strongly irregular bipolar neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(m_1) = (0.5, 0.3, 1.3, -1.1, -0.3, -0.3)$, $\mathcal{D}_G(m_2) = (0.7, 0.2, 1.1, -0.9, -0.2, -0.5)$ and $\mathcal{D}_G(m_3) = (0.6, 0.3, 1.0, -0.8, -0.$

$3, -0.4)$. The degree of each edge is $\mathcal{D}_G(m_1m_2) = (0.6, 0.3, 1.0, -0.8, -0.3, -0.4)$, $\mathcal{D}_G(m_1m_3) = (0.7, 0.2, 1.1, -0.9, -0.2, -0.5)$ and $\mathcal{D}_G(m_2m_3) = (0.5, 0.3, 1.3, -1.1, -0.3, -0.3)$. From Fig. 4, it is clear that every two adjacent edges in G have distinct degrees therefore G is neighbourly edge irregular bipolar neutrosophic graph. It is also clear that every two adjacent edges in G have distinct total degrees therefore G is neighbourly edge totally irregular bipolar neutrosophic graph.

Definition 2.16. *Let G^* be a crisp graph. A bipolar neutrosophic graph $G = (S, T)$ of G^* is called a strongly edge irregular bipolar neutrosophic graph if each edge in G has distinct degree, that is, no two edges in G have the same degree.*

Example 2.17. Consider a crisp graph $G^* = (M, L)$ such that $M = \{m_1, m_2, m_3\}$ and $L = \{m_1m_2, m_1m_3, m_2m_3\}$. The corresponding bipolar neutrosophic graph $G = (S, T)$ is shown in Fig. 5.

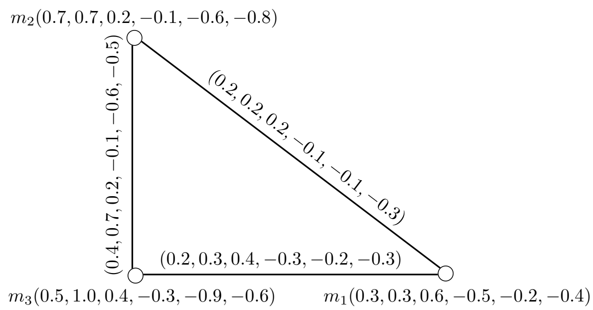

Figure 5: Strongly edge irregular bipolar neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(m_1) = (0.4, 0.5, 0.6, -0.4, -0.3, -0.6)$, $\mathcal{D}_G(m_2) = (0.6, 0.9, 0.4, -0.2, -0.7, -0.8)$, and $\mathcal{D}_G(m_3) = (0.6, 1.0, 0.6, -0.4, -0.8, -0.8)$. The degree of each edge is $\mathcal{D}_G(m_1m_2) = (0.6, 1.0, 0.6, -0.4, -0.8, -0.8)$, $\mathcal{D}_G(m_1m_3) = (0.6, 0.9, 0.4, -0.2, -0.7, -0.8)$ and $\mathcal{D}_G(m_2m_3) = (0.8, 0.5, 0.6, -0.4, -0.3, -0.4)$. Since no two edges in G have the same degree therefore G is a strongly edge irregular bipolar neutrosophic graph.

Definition 2.18. *Let G^* be a crisp graph. A bipolar neutrosophic graph $G = (S, T)$ of G^* is called a strongly edge totally irregular bipolar neutrosophic graph if each edge in G has distinct total degree, that is, no two edges in G have the same total degree.*

Example 2.19. Consider a crisp graph $G^* = (M, L)$ such that $M = \{m_1, m_2,$

m_3, $m_4\}$ and $L = \{m_1m_2,\ m_1m_4,\ m_2m_3,\ m_3m_4\}$. The corresponding bipolar neutrosophic graph $G = (S, T)$ is shown in Fig. 6.

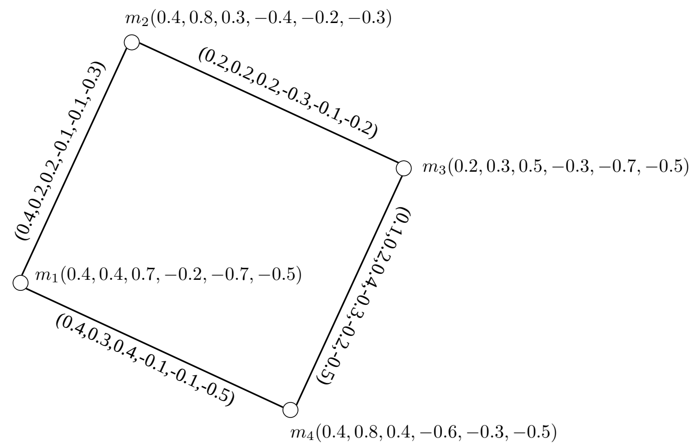

Figure 6: Strongly edge totally irregular bipolar neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(m_1) = (0.8, 0.5, 0.6, -0.2, -0.2, -0.8)$, $\mathcal{D}_G(m_2) = (0.6, 0.4, 0.4, -0.4, -0.2, -0.5)$, $\mathcal{D}_G(m_3) = (0.3, 0.4, 0.6, -0.6, -0.3, -0.7)$ and $\mathcal{D}_G(m_4) = (0.5, 0.5, 0.8, -0.4, -0.3, -1.0)$. The total degree of each edge is $\mathcal{TD}_G(m_1m_2)(1.0, 0.7, 0.8, -0.5, -0.3, -1.0)$, $\mathcal{TD}_G(m_1m_4)(0.9, 0.7, 1.0, -0.5, -0.4, -1.3)$, $\mathcal{TD}_G(m_2m_3)(0.7, 0.6, 0.8, -0.7, -0.4, -1.0)$ and $\mathcal{TD}_G(m_3m_4)(0.6, 0.5, 0.6, -0.4, -0.2, -0.7)$. Since no two edges in G have the same total degree therefore G is a strongly edge totally irregular bipolar neutrosophic graph.

Remark 2.20. A strongly edge irregular bipolar neutrosophic graph G may not be strongly edge totally irregular bipolar neutrosophic graph.

Example 2.21. Consider a crisp graph $G^* = (M, L)$ such that $M = \{m_1, m_2, m_3\}$ and $L = \{m_1m_2, m_1m_3, m_2m_3\}$. The corresponding bipolar neutrosophic graph $G = (S, T)$ is shown in Fig. 7.

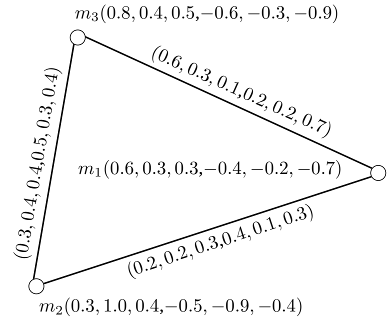

Figure 7: Strongly edge irregular bipolar neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(m_1) = (0.8,\ 0.5, 0.4,\ -0.6, -0.3, -1.0)$, $\mathcal{D}_G(m_2) = (0.5, 0.6, 0.7, -0.9, -0.4, -0.7)$ and $\mathcal{D}_G(m_3) = (0.9, 0.7, 0.5, -0.7, -0.5, -1.1)$. The degree of each edge is $\mathcal{D}_G(m_1m_2) = (0.9, 0.7, 0.5, -0.7, -0.5, -1.1)$, $\mathcal{D}_G(m_2m_3) = (0.8, 0.5, 0.4,\ -0.6, -0.3, -1.0)$ and $\mathcal{D}_G(m_1m_3) = (0.5, 0.6, 0.7, -0.9, -0.4, -0.7)$. Since all the edges have distinct degrees therefore G is a strongly edge irregular bipolar neutrosophic graph. The total degree of each edge is $\mathcal{TD}_G(m_1m_2) = (1.1, 0.9, 0.8, -1.1, -0.6, -1.4) = \mathcal{TD}_G(m_1m_3) = \mathcal{TD}_G(m_2m_3)$. Since each edge of G has the same total degree therefore G is not a strongly edge totally irregular bipolar neutrosophic graph.

Remark 2.22. A strongly edge totally irregular bipolar neutrosophic graph G may not be strongly edge irregular bipolar neutrosophic graph.

Example 2.23. Consider a crisp graph $G^* = (M, L)$ such that $M = \{m_1, m_2, m_3, m_4\}$ and $L = \{m_1m_2, m_1m_4, m_2m_3,\ m_3m_4\}$. The corresponding bipolar neutrosophic graph $G = (S, T)$ is shown in Fig. 8.

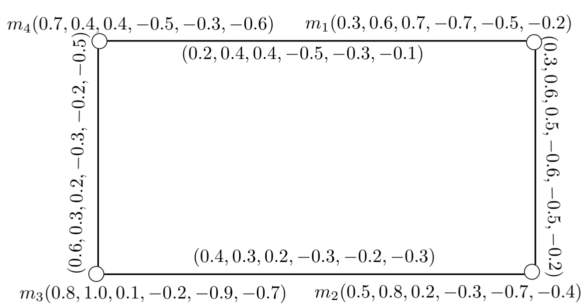

Figure 8: Strongly edge totally irregular bipolar neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(m_1) = (0.5, 1.0, 0.9, -1.1, -0.8, -0.3)$, $\mathcal{D}_G(m_2) = (0.7, 0.9, 0.7, -0.9, -0.7, -0.5)$ $\mathcal{D}_G(m_3) = (0.1, 0.6, 0.4, -0.6, -0.4, -$

0.8) and $\mathcal{D}_G(m_4) = (0.8, 0.7, 0.6, -0.8, -0.5, -0.6)$.
The degree of each edge is $\mathcal{D}_G(m_1m_2) = (0.6, 0.7, 0.6, -0.8, -0.5, -0.4)$, $\mathcal{D}_G(m_1m_4) = (1.1, 1.3, 1.1, -1.4, -1.0, -0.8)$, $\mathcal{D}_G(m_2m_3) = (0.9, 0.9, 0.7, -0.9, -0.7, -0.7)$ and $\mathcal{D}_G(m_3m_4) = (0.6, 0.7, 0.6, -0.8, -0.5, -0.4)$. It is easy to see that $\mathcal{D}_G(m_1m_2) = \mathcal{D}_G(m_3m_4)$ and $\mathcal{D}_G(m_2m_3) = \mathcal{D}_G(m_1m_4)$. Therefore, G is not a strongly edge irregular bipolar neutrosophic graph.
The total degree of each edge is $\mathcal{TD}_G(m_1m_2) = (0.9, 1.3, 1.1, -1.4, -1.0, -0.6)$, $\mathcal{TD}_G(m_1m_4) = (1.1, 1.3, 1.1, -1.4, -1.0, -0.6)$, $\mathcal{TD}_G(m_2m_3) = (1.3, 1.2, 0.9, -1.2, -0.9, -1.0)$ and $\mathcal{TD}_G(m_3m_4) = (1.2, 1.0, 0.8, -1.1, -0.7, -0.9)$. Since all the edges have distinct total degrees therefore G is a strongly edge totally irregular bipolar neutrosophic graph.

Theorem 2.1 *If $G = (S, T)$ is a strongly edge irregular connected bipolar neutrosophic graph, where T is a constant function. Then G is a strongly edge totally irregular bipolar neutrosophic graph.*

Proof. *Let $G = (S, T)$ be a strongly edge irregular connected bipolar neutrosophic graph. Suppose that T is a constant function. Then $t_T^+(mn) = s_1$, $i_T^+(mn) = s_2$, $f_T^+(mn) = s_3$, $t_T^-(mn) = s_4$, $t_T^-(mn) = s_5$ and $t_T^-(mn) = s_6$ for all $mn \in L$, where s_j, $j = 1, 2, \ldots, 6$ are constants. Consider a pair of edges mn and uv in L. Since G is a strongly edge irregular bipolar neutrosophic graph therefore $\mathcal{D}_G(mn) \neq \mathcal{D}_G(uv)$, where mn and uv are a pair of edges in L. This shows that $\mathcal{D}_G(mn)+(s_1, s_2, s_3, s_4, s_5, s_6) \neq \mathcal{D}_G(uv)+(s_1, s_2, s_3, s_4, s_5, s_6)$. This implies that $\mathcal{D}_G(mn) + (t_T^+(mn), i_T^+(mn), f_T^+(mn), t_T^-(mn), i_T^-(mn), f_T^-(mn)) \neq \mathcal{D}_G(uv) + (t_T^+(uv), i_T^+(uv), f_T^+(uv), t_T^-(uv), i_T^-(uv), f_T^-(uv))$. Thus $\mathcal{TD}_G(mn) \neq \mathcal{TD}_G(uv)$, where mn and uv are a pair of edges in L. Since the pair of edges mn and uv were taken to be arbitrary this shows that every pair of edges in G have distinct total degrees. Hence G is a strongly edge totally irregular bipolar neutrosophic graph.* ■

Theorem 2.2 *If $G = (S, T)$ is a strongly edge totally irregular connected bipolar neutrosophic graph, where T is a constant function. Then G is a strongly edge irregular bipolar neutrosophic graph.*

Proof. *Let $G = (S, T)$ be a strongly edge totally irregular connected bipolar neutrosophic graph. Suppose that T is a constant function. Then $t_T^+(mn) = s_1$, $i_T^+(mn) = s_2$, $f_T^+(mn) = s_3$, $t_T^-(mn) = s_4$, $t_T^-(mn) = s_5$ and $t_T^-(mn) = s_6$ for all $mn \in L$, where s_j, $j = 1, 2, \ldots, 6$ are constants. Consider a pair of edges mn and uv in L. Since G is a strongly edge totally irregular bipolar neutrosophic graph therefore $\mathcal{TD}_G(mn) \neq \mathcal{TD}_G(uv)$, where mn and uv are a pair of edges in L. This shows that $\mathcal{D}_G(mn)+(t_T^+(mn), i_T^+(mn), f_T^+(mn), t_T^-(mn), i_T^-(mn), f_T^-(mn)) \neq \mathcal{D}_G(uv) + (t_T^+(uv), i_T^+(uv), f_T^+(uv), t_T^-(uv), i_T^-(uv), f_T^-(uv))$. This implies that $\mathcal{D}_G(mn) + (s_1, s_2, s_3, s_4, s_5, s_6) \neq \mathcal{D}_G(uv) + (s_1, s_2, s_3, s_4, s_5, s_6)$. Thus $\mathcal{D}_G(mn) \neq \mathcal{D}_G(uv)$, where mn and uv are a pair of edges in L. Since the pair of edges mn and uv were taken to be arbitrary this shows that every pair of edges in G have distinct degrees. Hence G is a strongly edge irregular bipolar neutrosophic graph.* ■

Remark 2.24. If $G = (S, T)$ is both strongly edge irregular bipolar neutrosophic graph and strongly edge totally irregular bipolar neutrosophic graph, then it is not necessary that T is a constant function.

Example 2.25. Consider a crisp graph $G^* = (M, L)$ such that $T = \{m_1, m_2, m_3, m_4, m_5\}$ and $L = \{m_1m_2, m_1m_5,\ m_2m_3,\ m_3m_4, m_4m_5\}$. The corresponding bipolar neutrosophic graph $G = (S, T)$ is shown in Fig. 9.

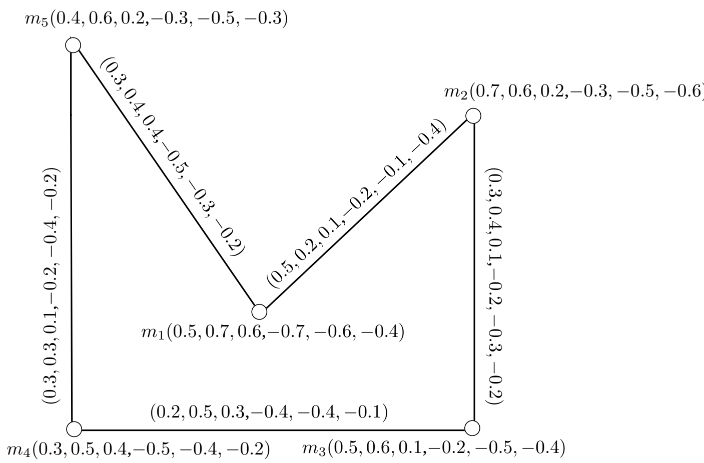

Figure 9: Bipolar neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(m_1) = (0.8,\ 0.6,\ 0.5,\ -0.7,\ -0.4,\ -0.6)$, $\mathcal{D}_G(m_2) = (0.8,\ 0.6,\ 0.2,\ -0.4,\ -0.4,\ -0.6)$, $\mathcal{D}_G(m_3) = (0.5,\ 0.9,\ 0.4,\ -0.6,\ -0.7,\ -0.3)$, $\mathcal{D}_G(m_4) = (0.5,\ 0.8,\ 0.4,\ -0.6,\ -0.8,\ -0.3)$ and $\mathcal{D}_G(m_5) = (0.6,\ 0.7,\ 0.5,\ -0.7, -0.7, -0.4)$. The degree of each edge is $\mathcal{D}_G(m_1m_2) = (0.6, 0.8, 0.5, -0.7, -0.6, -0.4)$, $\mathcal{D}_G(m_2m_3) = (0.7, 0.7, 0.4,\ -0.6, -0.5, -0.5)$, $\mathcal{D}_G(m_3m_4) = (0.6, 0.7, 0.2,\ -0.4, -0.7, -0.4)$, $\mathcal{D}_G(m_4m_5) = (0.5, 0.9, 0.7, -0.9, -0.7, -0.3)$ and $\mathcal{D}_G(m_5m_1) = (0.8, 0.5, 0.2,\ -0.4, -0.5, -0.6)$. It is easy to see that all the edges have distinct degrees. Therefore, G is a strongly edge irregular bipolar neutrosophic graph.
The total degree of each edge is $\mathcal{TD}_G(m_1m_2) = (1.1, 1.0, 0.6,\ -0.9, -0.7, -0.8)$, $\mathcal{TD}_G(m_2m_3) = (1.0, 1.1, 0.5,\ -0.8, -0.8, -0.7)$, $\mathcal{TD}_G(m_3m_4) = (0.8, 1.2, 0.5, -0.8, -1.1, -0.5)$, $\mathcal{TD}_G(m_4m_5) = (0.8, 1.2, 0.8,\ -1.1, -1.1, -0.5)$ and $\mathcal{TD}_G(m_5\, m_1) = (1.1, 0.9, 0.6,\ -0.9, -0.8, -0.8)$. Since all the edges have distinct total degrees therefore G is a strongly edge totally irregular bipolar neutrosophic graph. This shows that G is both strongly edge irregular bipolar neutrosophic graph and strongly edge totally irregular bipolar neutrosophic graph but Y is not a constant function.

Theorem 2.3 *Let $G = (S, T)$ be a strongly edge irregular bipolar neutrosophic graph. Then G is a neighbourly edge irregular bipolar neutrosophic graph.*

Proof. Suppose that G is a strongly edge irregular bipolar neutrosophic graph. Then each edge in G has distinct degree. This shows that every pair of edges in G have distinct degrees. Therefore, G is a neighbourly edge irregular bipolar neutrosophic graph. ■

Theorem 2.4 *Let $G = (S, T)$ be a strongly edge totally irregular bipolar neutrosophic graph. Then G is a neighbourly edge totally irregular bipolar neutrosophic graph.*

Proof. Suppose that G is a strongly edge totally irregular bipolar neutrosophic graph. Then each edge in G has distinct total degree. This shows that every pair of edges in G have distinct total degrees. Therefore, G is a neighbourly edge totally irregular bipolar neutrosophic graph. ■

Remark 2.26. If G is a neighbourly edge irregular bipolar neutrosophic graph then it is not necessary that G is a strongly edge irregular bipolar neutrosophic graph.

Example 2.27. Consider a crisp graph $G^* = (M, L)$ such that $M = \{m_1, m_2, m_3, m_4\}$ and $L = \{m_1m_2, m_2m_3, m_3m_4\}$. The corresponding bipolar neutrosophic graph$G = (S, T)$ is shown in Fig. 10.

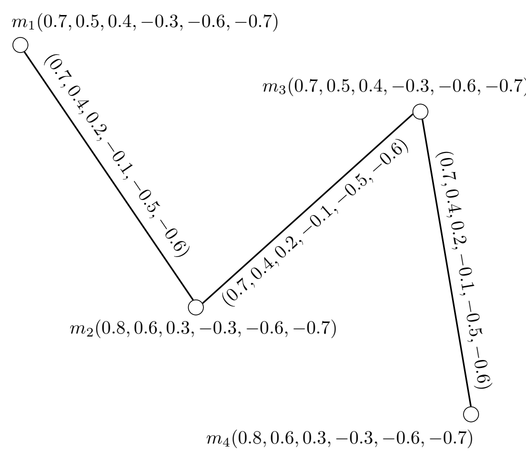

Figure 10: Bipolar neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(m_1) = (0.7, 0.4, 0.2, -0.1, -0.5, -0.6)$, $\mathcal{D}_G(m_2) = (1.4, 0.8, 0.4, -0.2, -1.0, -1.2)$, $\mathcal{D}_G(m_3) = (1.4, 0.8, 0.4, -0.2, -1.0, -1.2)$ and $\mathcal{D}_G(m_4) = (0.7, 0.4, 0.2, -0.1, -0.5, -0.6)$. The degree of each edge is $\mathcal{D}_G(m_1m_2) = (0.7, 0.4, 0.2, -0.1, -0.5, -0.6)$, $\mathcal{D}_G(m_2m_3) = (1.4, 0.8, 0.4, -0.2, -$

$1.0, -1.2)$ and $\mathcal{D}_G(m_3m_4) = (0.7, 0.4, 0.2, -0.1, -0.5, -0.6)$. G is neighbourly edge irregular bipolar neutrosophic graph since every two adjacent edges in G have distinct total degrees, that is, $\mathcal{D}_G(m_1m_2) \neq \mathcal{D}_G(m_2m_3)$ and $\mathcal{D}_G(m_2m_3) \neq \mathcal{D}_G(m_3m_4)$. It is easy to see that $\mathcal{D}_G(m_1m_2) = \mathcal{D}_G(m_3m_4)$. Therefore, G is not a strongly edge irregular bipolar neutrosophic graph.

Remark 2.28. If G is a neighbourly edge totally irregular bipolar neutrosophic graph then it is not necessary that G is a strongly edge totally irregular bipolar neutrosophic graph.

Example 2.29. Consider the bipolar neutrosophic graph $G = (S, T)$ as shown in Fig. 10. The total degree of each edge is $\mathcal{TD}_G(m_1m_2) = (1.4, 0.8, 0.4, -0.2, -1.0, -1.2)$, $\mathcal{TD}_G(m_2m_3) = (2.1, 1.2, 0.6, -0.3, -1.5, -1.8)$ and $\mathcal{TD}_G(m_1m_2) = (1.4, 0.8, 0.4, -0.2, -1.0, -1.2)$. It is easy to see that every two adjacent edges in G have distinct total degrees, that is, $\mathcal{TD}_G(m_1m_2) \neq \mathcal{TD}_G(m_2m_3)$ and $\mathcal{TD}_G(m_2m_3) \neq \mathcal{TD}_G(m_3m_4)$. Therefore, G is neighbourly edge totally irregular bipolar neutrosophic graph. It is easy to see that $\mathcal{TD}_G(m_1m_2) = \mathcal{TD}_G(m_3m_4)$. Hence G is not a strongly edge totally irregular bipolar neutrosophic graph.

Theorem 2.5 *Let $G = (S, T)$ be a strongly edge irregular connected bipolar neutrosophic graph, with T as constant function. Then G is an irregular bipolar neutrosophic graph.*

Proof. Let $G = (S, T)$ be a strongly edge irregular connected bipolar neutrosophic graph, with T as constant function. Then $t_T^+(mn) = s_1$, $i_T^+(mn) = s_2$, $f_T^+(mn) = s_3$, $t_T^-(mn) = s_4$, $t_T^-(mn) = s_5$ and $t_T^-(mn) = s_6$, for each edge $mn \in L$, where s_j, $j = 1, 2, \ldots, 6$ are constants. Also, every edge in G has distinct degrees, since G is strongly edge irregular bipolar neutrosophic graph. Let mn and nu be any two adjacent edges in G such that $\mathcal{D}_G(mn) \neq \mathcal{D}_G(nu)$. This implies that $\mathcal{D}_G(m) + \mathcal{D}_G(n) - 2(t_T^+(mn), i_T^+(mn), f_T^+(mn), t_T^-(mn), i_T^-(mn), f_T^-(mn)) \neq \mathcal{D}_G(n) + \mathcal{D}_G(u) - 2(t_T^+(nu), i_T^+(nu), f_T^+(nu), t_T^-(nu), i_T^-(nu), f_T^-(nu))$. This implies that $\mathcal{D}_G(m) + \mathcal{D}_G(n) - 2(s_1, s_2, s_3, s_4, s_5, s_6) \neq \mathcal{D}_G(n) + \mathcal{D}_G(u) - 2(s_1, s_2, s_3, s_4, s_5, s_6)$. This shows that $\mathcal{D}_G(m) \neq \mathcal{D}_G(u)$. Thus there exists a vertex n in G which is adjacent to the vertices with distinct degrees. This shows that G is an irregular bipolar neutrosophic graph. ■

Theorem 2.6 *Let $G = (S, T)$ be a strongly edge totally irregular connected bipolar neutrosophic graph, with T as constant function. Then G is an irregular bipolar neutrosophic graph.*

Proof. Let $G = (S, T)$ be a strongly edge totally irregular connected bipolar neutrosophic graph, with T as constant function. Then $t_T^+(mn) = s_1$, $i_T^+(mn) = s_2$, $f_T^+(mn) = s_3$, $t_T^-(mn) = s_4$, $t_T^-(mn) = s_5$ and $t_T^-(mn) = s_6$ for each edge $xy \in L$, where s_j, $j = 1, 2, \ldots, 6$ are constants. Also, every edge in G has distinct total degrees, since G is strongly edge totally irregular bipolar neutrosophic graph. Let mn and nu be any two adjacent edges in G such that $\mathcal{TD}_G(mn) \neq \mathcal{TD}_G(nu)$. This implies that

$\mathcal{D}_G(mn) + (t_T^+(mn), i_T^+(mn), f_T^+(mn), t_T^-(mn), i_T^-(mn), f_T^-(mn)) \neq \mathcal{D}_G(nu) + (t_T^+(nu), i_T^+(nu), f_T^+(nu), t_T^-(nu), i_T^-(nu), f_T^-(nu))$. This implies that $\mathcal{D}_G(m) + \mathcal{D}_G(n) - (t_T^+(mn), i_T^+(mn), f_T^+(mn), t_T^-(mn), i_T^-(mn), f_T^-(mn)) \neq \mathcal{D}_G(n) + \mathcal{D}_G(u) - (t_T^+(nu), i_T^+(nu), f_T^+(nu), t_T^-(nu), i_T^-(nu), f_T^-(nu))$. This implies that $\mathcal{D}_G(m)+\mathcal{D}_G(n)-2(s_1, s_2, s_3, s_4, s_5, s_6) \neq \mathcal{D}_G(n)+\mathcal{D}_G(u)-2(s_1, s_2, s_3, s_4, s_5, s_6)$. This shows that $\mathcal{D}_G(m) \neq \mathcal{D}_G(u)$. Thus there exists a vertex n in G which is adjacent to the vertices with distinct degrees. This shows that G is an irregular bipolar neutrosophic graph. ■

Remark 2.30. If $G = (S, T)$ is an irregular bipolar neutrosophic graph, with T as a constant function. Then it is not necessary that G is a strongly edge irregular bipolar neutrosophic graph.

Example 2.31. Consider a crisp graph $G^* = (M, L)$ such that $M = \{m_1, m_2, m_3, m_4\}$ and $L = \{m_1m_2, m_1m_4, m_2m_3, m_2m_4, m_3m_4\}$. The corresponding bipolar neutrosophic graph $G = (S, T)$ is shown in Fig. 11.

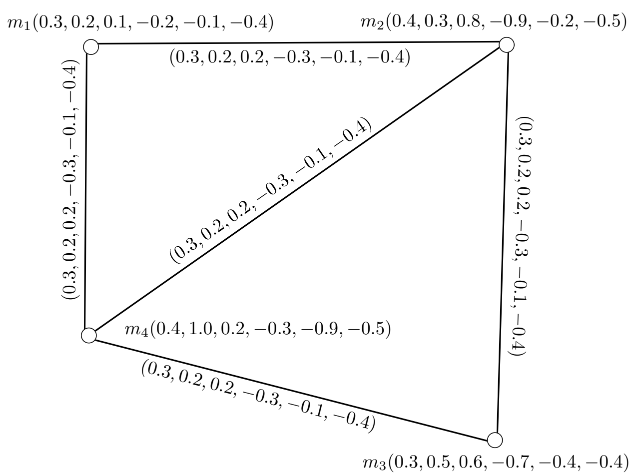

Figure 11: Irregular bipolar neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(m_1) = (0.9, 0.6, 0.6, -0.9, -0.3, -1.2)$, $\mathcal{D}_G(m_2) = (0.9, 0.6, 0.6, -0.9, -0.3, -1.2)$, $\mathcal{D}_G(m_3) = (0.9, 0.6, 0.6, -0.9, -0.3, -1.2)$, and $\mathcal{D}_G(m_4) = (0.9, 0.6, 0.6, -0.9, -0.3, -1.2)$. The degree of each edge is $\mathcal{D}_G(m_1m_2) = (0.9, 0.6, 0.6, -0.9, -0.3, -1.2) = \mathcal{D}_G(m_1m_4) = \mathcal{D}_G(m_2m_3) = \mathcal{D}_G(m_3m_4)$ and $\mathcal{D}_G(m_1m_3) = (1.2, 0.8, 0.8, -1.2, -0.4, -1.6)$. It is easy to see that all the edges have the same degree except the edge m_1m_3. Therefore, G is not a strongly edge irregular bipolar neutrosophic graph.

Remark 2.32. If $G=(S,T)$ is an irregular bipolar neutrosophic graph, with T as a constant function. Then it is not necessary that G is a strongly edge totally irregular bipolar neutrosophic graph.

Example 2.33. Consider the bipolar neutrosophic graph $G=(S,T)$ as shown in Fig. 11. The total degree of each edge is $\mathcal{TD}_G(m_1m_2)=(1.2,0.8,0.8,-1.2,-0.4,-1.6)=\mathcal{D}_G(m_1m_4)=\mathcal{D}_G(m_2m_3)=\mathcal{D}_G(m_3m_4)$ and $\mathcal{TD}_G(m_1m_3)=(1.5,1.0,1.0,-1.5,-0.5,-2.0)$. It is easy to see that all the edges have the same total degree except the edge m_1m_3. Therefore, G is not a strongly edge totally irregular bipolar neutrosophic graph.

Theorem 2.7 *Let $G=(S,T)$ be a strongly edge irregular connected bipolar neutrosophic graph, with T as a constant function. Then G is highly irregular bipolar neutrosophic graph.*

Proof. *Let $G=(S,T)$ be a strongly edge irregular connected bipolar neutrosophic graph, with T as a constant function. Then $t_T^+(mn)=s_1$, $i_T^+(mn)=s_2$, $f_T^+(mn)=s_3$, $t_T^-(mn)=s_4$, $t_T^-(mn)=s_5$ and $t_T^-(mn)=s_6$ for each edge $mn\in L$, where s_j, $j=1,2,\ldots,6$ are constants. Also every pair of adjacent edges in G have distinct degrees. Let n be any vertex in G which is adjacent to vertices n and u. Since G is strongly edge irregular bipolar neutrosophic graph therefore, $\mathcal{D}_G(mn)\neq\mathcal{D}_G(nu)$. This implies that $\mathcal{D}_G(m)+\mathcal{D}_G(n)-2(t_T^+(mn),i_T^+(mn),f_T^+(mn),t_T^-(mn),i_T^-(mn),f_T^-(mn))\neq\mathcal{D}_G(n)+\mathcal{D}_G(u)-2(t_T^+(nu),i_T^+(nu),f_T^+(nu),t_T^-(nu),i_T^-(nu),f_T^-(nu))$. This implies that $\mathcal{D}_G(m)+\mathcal{D}_G(n)-2(s_1,s_2,s_3,s_4,s_5,s_6)\neq\mathcal{D}_G(n)+\mathcal{D}_G(u)-2(s_1,s_2,s_3,s_4,s_5,s_6)$. This shows that $\mathcal{D}_G(m)\neq\mathcal{D}_G(u)$. Thus there exists a vertex n in G which is adjacent to the vertices with distinct degrees. Since n was taken to be an arbitrary vertex in G, therefore all the vertices in G are adjacent to vertices having distinct degrees. Hence G is a highly irregular bipolar neutrosophic graph.* ■

Theorem 2.8 *Let $G=(S,T)$ be a strongly edge totally irregular connected bipolar neutrosophic graph, with T as a constant function. Then G is highly irregular bipolar neutrosophic graph.*

Proof. *Let $G=(S,T)$ be a strongly edge totally irregular connected bipolar neutrosophic graph, with T as a constant function. Then $t_T^+(mn)=s_1$, $i_T^+(mn)=s_2$, $f_T^+(mn)=s_3$, $t_T^-(mn)=s_4$, $t_T^-(mn)=s_5$ and $t_T^-(mn)=s_6$ for each edge $mn\in L$, where s_j, $j=1,2,\ldots,6$ are constants. Also every pair of adjacent edges in G have distinct total degrees. Let n be any vertex in G which is adjacent to vertices m and u. Since G is strongly edge totally irregular bipolar neutrosophic graph therefore, $\mathcal{TD}_G(mn)\neq\mathcal{TD}_G(nu)$. This implies that $\mathcal{D}_G(mn)\neq\mathcal{D}_G(nu)$. This implies that $\mathcal{D}_G(m)+\mathcal{D}_G(n)-2(t_T^+(mn),i_T^+(mn),f_T^+(mn),t_T^-(mn),i_T^-(mn),f_T^-(mn))\neq\mathcal{D}_G(n)+\mathcal{D}_G(u)-2(t_T^+(nu),i_T^+(nu),f_T^+(nu),t_T^-(nu),i_T^-(nu),f_T^-(nu))$. This implies that $\mathcal{D}_G(m)+\mathcal{D}_G(n)-2(s_1,s_2,s_3,s_4,s_5,s_6)\neq\mathcal{D}_G(n)+\mathcal{D}_G(u)-2(s_1,s_2,s_3,s_4,s_5,s_6)$. This shows that $\mathcal{D}_G(m)\neq\mathcal{D}_G(u)$. Thus there exists a vertex n in G which is adjacent to the vertices with distinct degrees. Since n was taken to be an arbitrary ver-*

tex in G, therefore all the vertices in G are adjacent to vertices having distinct degrees. Hence G is a highly irregular bipolar neutrosophic graph. ■

Remark 2.34. If $G = (S, T)$ is a highly irregular bipolar neutrosophic graph, with T as a constant function. Then it is not necessary that G is strongly edge irregular bipolar neutrosophic graph.

Example 2.35. Consider a crisp graph $G^* = (M, L)$ such that $M = \{m_1, m_2, m_3, m_4\}$ and $L = \{m_1m_3, m_1m_4, m_2m_3\}$. The corresponding bipolar neutrosophic graph $G = (S, T)$ is shown in Fig. 12.

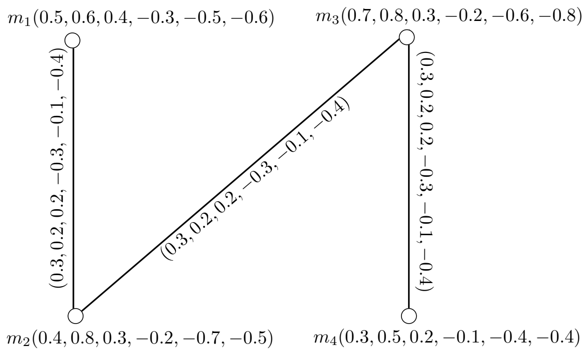

Figure 12: Highly irregular bipolar neutrosophic graph G

By direct calculations, we have $\mathcal{D}_G(m_1) = (0.3, 0.2, 0.2, -0.1, -0.1, -0.4)$, $\mathcal{D}_G(m_2) = (0.6, 0.4, 0.4, -0.2, -0.2, -0.8)$, $\mathcal{D}_G(m_3) = (0.6, 0.4, 0.4, -0.2, -0.2, -0.8)$, and $\mathcal{D}_G(m_4) = (0.3, 0.2, 0.2, -0.1, -0.1, -0.4)$. The degree of each edge is $\mathcal{D}_G(m_1m_2) = (0.3, 0.2, 0.2, -0.1, -0.1, -0.4)$, $\mathcal{D}_G(m_2m_3) = (0.6, 0.4, 0.4, -0.2, -0.2, -0.8)$ and $\mathcal{D}_G(m_3m_4) = (0.3, 0.2, 0.2, -0.1, -0.1, -0.4)$. Since every vertex is adjacent to vertices with distinct degrees, G is a highly irregular bipolar neutrosophic graph. Since the edges m_1m_2 and m_3m_4 in G have the same degree i.e., $\mathcal{D}_G(m_1m_2) = \mathcal{D}_G(m_3m_4)$. Therefore, G is not strongly edge irregular bipolar neutrosophic graph.

Remark 2.36. If $G = (S, T)$ is a highly irregular bipolar neutrosophic graph, with T as a constant function. Then it is not necessary that G is strongly edge totally irregular bipolar neutrosophic graph.

Example 2.37. Consider the bipolar neutrosophic graph $G = (S, T)$ as shown in Fig. 12. The total degree of each edge is $\mathcal{TD}_G(m_1m_2) = (0.6, 0.4, 0.4, -0.2, -0.2, -0.8)$, $\mathcal{TD}_G(m_2m_3) = (0.9, 0.6, 0.6, -0.3, -0.3, -1.2)$ and $\mathcal{TD}_G(m_3m_4) = (0.6, 0.$

$4, 0.4, -0.2, -0.2, -0.8)$. Since the edges m_1m_2 and m_3m_4 in G have the same total degree. Therefore, G is not a strongly edge totally irregular bipolar neutrosophic graph.

Definition 2.38. *A bipolar single-valued neutrosophic path $\mathcal{P}$ is a sequence of distinct vertices and edges such that for all k, $t_T^+(m_km_{k+1}) > 0$, $i_T^+(m_km_{k+1}) > 0$, $f_T^+(m_km_{k+1}) > 0$, $t_T^-(m_km_{k+1}) < 0$, $i_T^-(m_km_{k+1}) < 0$ and $f_T^-(m_km_{k+1}) < 0$. A bipolar single-valued neutrosophic path is said to be a bipolar single-valued neutrosophic cycle if $m = n$.*

Example 2.39. Consider a crisp graph $G^* = (M, L)$ such that $M = \{m_1, m_2, m_3, m_4, m_5\}$ and $L = \{m_1m_2,\ m_2m_3, m_3m_4, m_2m_5, m_3m_5\}$. The corresponding bipolar neutrosophic graph $G = (S, T)$ is shown in Fig. 13.

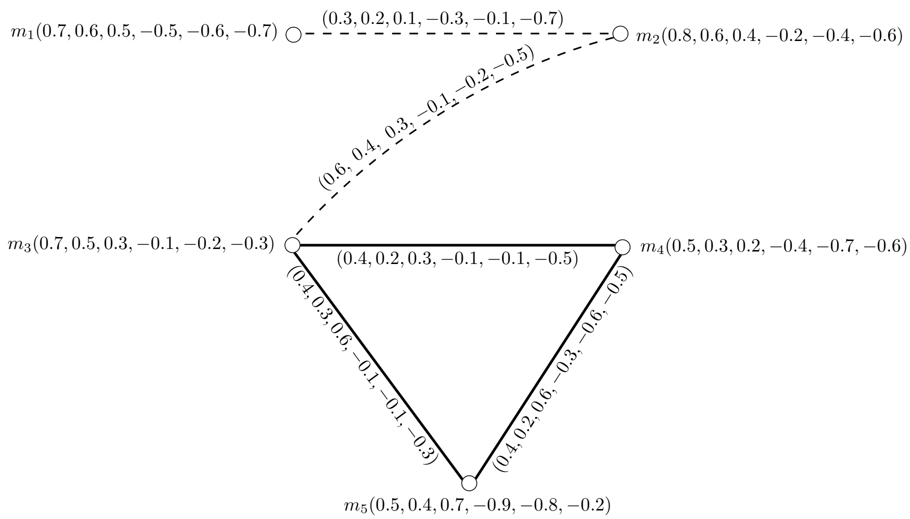

Figure 13: Bipolar neutrosophic graph G

The path $\mathcal{P}$ from m_1 to m_3 is shown with dashed lines and the cycle $\mathcal{C}$ from m_3 to m_3 is shown with bold lines in Fig. 13.

Theorem 2.9 *Let $G^* = (M, L)$ be a path on $2k(k > 1)$ vertices and $G = (S, T)$ be a bipolar neutrosophic graph of G^*. Let $L_1, L_2, L_3, \ldots, L_{2k-1}$ be the edges in G having $b_1, b_2, b_3, \ldots, b_{2k-1}$ as their membership values, respectively. Suppose that $b_1 < b_2 < b_3 < \ldots < b_{2k-1}$, where $b_j = (t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^-)$, $j = 1, 2, 3, \ldots, 2k-1$. Then G is both strongly edge irregular and strongly edge totally irregular bipolar neutrosophic graph.*

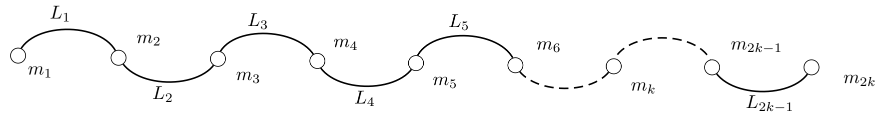

Figure 14: Bipolar neutrosophic path $\mathcal{P}$

Proof. *Let $G = (S, T)$ be a bipolar neutrosophic graph of a crisp graph $G^* = (M, L)$. Suppose that G is a bipolar single-valued neutrosophic path on $2k (k > 1)$ vertices. Suppose that $b_j = (t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^-)$ be the membership values of the edges L_j in G, where $j = 1, 2, 3, \ldots, 2k-1$. We suppose that $b_1 < b_2 < b_3 < \ldots < b_{2k-1}$.*
The degree of each vertex in G is calculated as:

$$\begin{aligned}
\mathcal{D}_G(m_1) &= b_1 = (t_1^+, i_1^+, f_1^+, t_1^-, i_1^-, f_1^-), \quad for \quad j = 1. \\
\mathcal{D}_G(m_j) &= b_{j-1} + b_k = (t_{j-1}^+, i_{j-1}^+, f_{j-1}^+, t_{j-1}^-, i_{j-1}^-, f_{j-1}^-) \\
&\quad +(t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^-), \\
&= (t_{j-1}^+ + t_j^+, i_{j-1}^+ + i_j^+, f_{j-1}^+ + f_j^+, t_{j-1}^- + t_j^-, i_{j-1}^- + i_j^-, f_{j-1}^- + f_j^-), \\
&\quad for \quad k = 2, 3, \ldots, 2k-1. \\
\mathcal{D}_G(m_{2j}) &= b_{2j-1} = (t_{2k-1}^+, i_{2k-1}^+, f_{2k-1}^+, t_{2k-1}^-, i_{2k-1}^-, f_{2k-1}^-), \quad for \quad j = 2k.
\end{aligned}$$

The degree of each edge in G is calculated as:

$$\begin{aligned}
\mathcal{D}_G(L_1) &= b_2 = (t_2^+, i_2^+, f_2^+, t_2^-, i_2^-, f_2^-), \quad for \quad j = 1. \\
\mathcal{D}_G(L_j) &= b_{j-1} + b_{j+1} \\
&= (t_{j-1}^+, i_{j-1}^+, f_{j-1}^+, t_{j-1}^-, i_{j-1}^-, f_{j-1}^-) + (t_{j+1}^+, i_{j+1}^+, f_{j+1}^+, t_{j+1}^-, i_{j+1}^-, f_{j+1}^-), \\
&= (t_{j-1}^+ + t_{j+1}^+, i_{j-1}^+ + i_{j+1}^+, f_{j-1}^+ + f_{j+1}^+, t_{j-1}^- + \\
&\quad t_{j+1}^-, i_{j-1}^- + i_{j+1}^-, f_{j-1}^- + f_{j+1}^-), \quad for \quad j = 2, 3, \ldots, 2k-2. \\
\mathcal{D}_G(L_{2k-1}) &= b_{2k-2} = (t_{2k-2}^+, i_{2k-2}^+, f_{2k-2}^+, t_{2k-2}^-, i_{2k-2}^-, f_{2k-2}^-), \quad for \quad j = 2k-1.
\end{aligned}$$

Since each edge in G has distinct degree therefore G is strongly edge irregular bipolar neutrosophic graph. We now calculate the total degree of each edge in G

as:

$$
\begin{aligned}
\mathcal{TD}_G(L_1) &= b_1+b_2=(t_1^+ + t_2^+, i_1^+ + i_2^+, f_1^+ + f_2^+, t_1^- + t_2^-, i_1^- + i_2^-, f_1^- + f_2^-),\\
&\quad for \quad j=1.\\
\mathcal{TD}_G(L_j) &= b_{j-1}+b_j+b_{j+1}=(t_{j-1}^+, i_{j-1}^+, f_{j-1}^+, t_{j-1}^-, i_{j-1}^-, f_{j-1}^-)+(t_j^+, i_j^+, f_j^+,\\
&\quad t_j^-, i_j^-, f_j^-)+(t_{j+1}^+, i_{j+1}^+, f_{j+1}^+, t_{j+1}^-, i_{j+1}^-, f_{j+1}^-),\\
&= (t_{j-1}^+ + t_j^+ + t_{j+1}^+, i_{j-1}^+ + i_j^+ + i_{j+1}^+, f_{j-1}^+ + f_j^+ + f_{j+1}^+, t_{j-1}^- + t_j^- +\\
&\quad t_{j+1}^-, i_{j-1}^- + i_j^- + i_{j+1}^-, f_{j-1}^- + f_j^- + f_{j+1}^-),\\
&\quad for \quad j=2,3,\ldots,2k-2.\\
\mathcal{TD}_G(L_{2k-1}) &= b_{2k-2}+b_{2k-1}=(t_{2j-2}^+, i_{2j-2}^+, f_{2j-2}^+, t_{2j-2}^-, i_{2j-2}^-, f_{2j-2}^-)+(t_{2j-1}^+,\\
&\quad i_{2j-1}^+, f_{2j-1}^+, t_{2j-1}^-, i_{2j-1}^-, f_{2j-1}^-),\\
&= (t_{2j-2}^+ + t_{2j-1}^+, i_{2j-2}^+ + i_{2j-1}^+, f_{2j-2}^+ + f_{2j-1}^+, t_{2j-2}^- + t_{2j-1}^-, i_{2j-2}^- +\\
&\quad i_{2j-1}^-, f_{2j-2}^- + f_{2j-1}^-), \quad for \quad j=2k-1.
\end{aligned}
$$

Since each edge in G has distinct total degree therefore G is strongly edge totally irregular bipolar neutrosophic graph. Hence G is both strongly edge irregular and strongly edge totally irregular bipolar neutrosophic graph. ■

Definition 2.40. *A complete bipartite graph is a graph whose vertex set can be partitioned into two subsets N_1 and N_2 such that no edge has both endpoints in the same subset, and every possible edge that could connect vertices in different subsets is the part of the graph. A complete bipartite graph with partition of size $|N_1| = k$ and $|N_2| = l$, is denoted by $\mathcal{K}_{(k,l)}$. A complete bipartite graph $\mathcal{K}_{(1,l)}$ or $\mathcal{K}_{(k,1)}$ that is a tree with one internal vertex and l or k leaves is called a star $\mathcal{S}_l$ or $\mathcal{S}_k$.*

Theorem 2.10 *Let $G^* = (M, L)$ be a star $\mathcal{K}_{(k,1)}$ and $G = (S, T)$ be a bipolar neutrosophic graph of G^*. If each edge in G has distinct membership values then G is strongly edge irregular bipolar neutrosophic graph but not strongly edge totally irregular bipolar neutrosophic graph.*

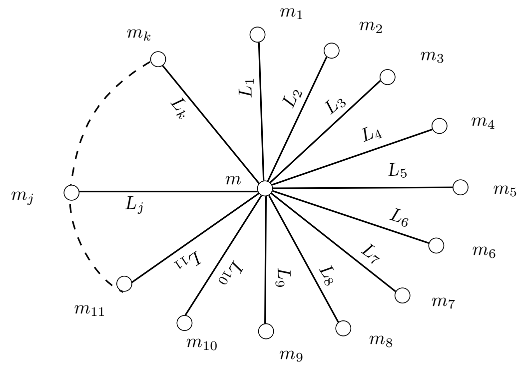

Figure 15: Bipolar neutrosophic graph

Proof. *Let $G = (S,T)$ be a bipolar neutrosophic graph of a crisp graph $G^* = (M,L)$. We assume that G is a star $\mathcal{K}_{(k,1)}$. Let $m, m_1, m_2, \ldots, m_k$ be the vertices of the star $\mathcal{K}_{(k,1)}$, where m is the center vertex and $m_1, m_2, \ldots, m_k$ are the vertices adjacent to vertex m as shown in Fig. 15. Suppose that $b_j = (t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^-)$ be the membership values of the edges L_j in G, where $j = 1, 2, \ldots, k$. We suppose that $b_1 \neq b_2 \neq b_3 \neq \ldots \neq b_k$. The degree of each edge in G is calculated as:*

$$\begin{aligned}
\mathcal{D}_G(L_j) &= \mathcal{D}_G(m) + \mathcal{D}_G(m_j) - 2(t_T^+(mm_j), i_T^+(mm_j), f_T^+(mm_j, t_T^-(mm_j), i_T^-(m \\
&\quad m_j), f_T^-(mm_j)), \\
&= (b_1, b_2, \ldots, b_k) + (t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^-) - 2(t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^-), \\
&= (t_1^+, i_1^+, f_1^+, t_1^-, i_1^-, f_1^-), (t_2^+, i_2^+, f_2^+, t_2^-, i_2^-, f_2^-), \ldots, (t_k^+, i_k^+, f_k^+, t_k^-, \\
&\quad i_k^-, f_k^-) + (t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^-) - 2(t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^-), \\
&= (t_1^+ + t_2^+ + \ldots + t_k^+, i_1^+ + i_2^+ + \ldots + i_k^+, f_1^+ + f_2^+ + \ldots + f_k^+, \\
&\quad t_1^- + t_2^- + \ldots + t_k^-, i_1^- + i_2^- + \ldots + i_k^-, f_1^- + f_2^- + \ldots + f_k^-) - \\
&\quad (t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^-).
\end{aligned}$$

It is easy to see that each edge in G has distinct degree therefore G is strongly edge irregular bipolar neutrosophic graph. We now calculate the total degree of each edge in G as:

$$\begin{aligned}
\mathcal{TD}_G(L_j) &= \mathcal{TD}_G(m) + \mathcal{TD}_G(m_j) - (t_T^+(mm_j), i_T^+(mm_j), f_T^+(mm_j, t_T^-(mm_j), \\
&\quad i_T^-(mm_j), f_T^-(mm_j)), \\
&= (b_1, b_2, \ldots, b_k) + (t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^-)(t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^-), \\
&= (t_1^+, i_1^+, f_1^+, t_1^-, i_1^-, f_1^-), (t_2^+, i_2^+, f_2^+, t_2^-, i_2^-, f_2^-), \ldots, (t_j^+, i_j^+, f_j^+, \\
&\quad t_j^-, i_j^-, f_j^-), \\
&= (t_1^+ + t_2^+ + \ldots + t_k^+, i_1^+ + i_2^+ + \ldots + i_k^+, f_1^+ + f_2^+ + \ldots + f_k^+, \\
&\quad t_1^- + t_2^- + \ldots + t_k^-, i_1^- + i_2^- + \ldots + i_k^-, f_1^- + f_2^- + \ldots + f_k^-).
\end{aligned}$$

since all the edges in G have the same total degree therefore G is not a strongly edge totally irregular bipolar neutrosophic graph ■

Definition 2.41. *The m-barbell graph $\mathcal{B}_{m,m}$ is the simple graph obtained by connecting two copies of a complete graph $\mathcal{K}_m$ by a bridge.*

Theorem 2.11 *Let $G = (S,T)$ be a bipolar neutrosophic graph of $G^* = (M,L)$, the m-barbell graph $\mathcal{B}_{m,m}$. If each edge in G has distinct membership values then G is a strongly edge irregular bipolar neutrosophic graph but not a strongly edge totally irregular bipolar neutrosophic graph.*

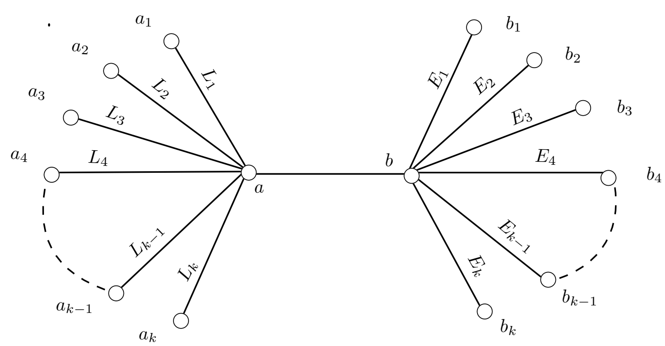

Figure 16: Bipolar neutrosophic graph

Proof. *Let $G = (S,T)$ be a bipolar neutrosophic graph of a crisp graph $G^* = (M,L)$. Suppose that G^* is a m-barbell graph then there exists a bridge, say ab, connecting k new vertices to each of its end vertex a and b. Let $c = (t^+, i^+, f^+, t^-, i^-, f^-)$ be the membership values of the bridge ab. Suppose that $a_1, a_2, \ldots, a_k$ and $b_1, b_2, \ldots, b_k$ are the vertices adjacent to vertices a and b, respectively. Let $c_j = (t_j^+, i_j^+, f_j^+, t_j^-, i_j^-, f_j^-)$ be the membership values of the edges L_j with vertex a, where $j = 1, 2, \ldots, k$ and $c_1 < c_2 < \ldots < c_k$. Let $d_j = (\widetilde{t_j^+}, \widetilde{i_j^+}, \widetilde{f_j^+}, \widetilde{t_j^-}, \widetilde{i_j^-}, \widetilde{f_j^-})$ be the membership values of the edges E_j with vertex b, where $j = 1, 2, \ldots, k$ and $d_1 < d_2 < \ldots < d_k$. Assume that $c_1 < c_2 < \ldots < c_k < d_1 < d_2 < \ldots < d_k < c$. The degree of each edge in G is* *calculated as:*

$$
\begin{aligned}
\mathcal{D}_G(ab) &= \mathcal{D}_G(a)+\mathcal{D}_G(b)-2c,\\
&= c_1+c_2+\ldots+c_k+c+d_1+d_2+\ldots+d_k+c-2c,\\
&= (t_1^+,i_1^+,f_1^+,t_1^-,i_1^-,f_1^-)+(t_2^+,i_2^+,f_2^+,t_2^-,i_2^-,f_2^-)+\ldots+(t_k^+,i_k^+,f_k^+,\\
&\quad t_k^-,i_k^-,f_k^-)+(\widetilde{t}_1^+,\widetilde{i}_1^+,\widetilde{f}_1^+,\widetilde{t}_1^-,\widetilde{i}_1^-,\widetilde{f}_1^-)+(\widetilde{t}_2^+,\widetilde{i}_2^+,\widetilde{f}_2^+,\widetilde{t}_2^-,\widetilde{i}_2^-,\widetilde{f}_2^-)+\ldots\\
&\quad +(\widetilde{t}_k^+,\widetilde{i}_k^+,\widetilde{f}_k^+,\widetilde{t}_k^-,\widetilde{i}_k^-,\widetilde{f}_k^-),\\
&= (t_1^++t_2^++\ldots+t_k^+,i_1^++i_2^++\ldots+i_k^+,f_1^++f_2^++\ldots+f_k^+,\\
&\quad t_1^-+t_2^-+\ldots+t_k^-,i_1^-+i_2^-+\ldots+i_k^-,f_1^-+f_2^-+\ldots+f_k^-)\\
&+ (\widetilde{t}_1^++\widetilde{t}_2^++\ldots+\widetilde{t}_m^+,\widetilde{i}_1^++\widetilde{i}_2^++\ldots+\widetilde{i}_k^+,\widetilde{f}_1^++\widetilde{f}_2^++\ldots+\widetilde{f}_k^+,\\
&\quad \widetilde{t}_1^-+\widetilde{t}_2^-+\ldots+\widetilde{t}_m^-,\widetilde{i}_1^-+\widetilde{i}_2^-+\ldots+\widetilde{i}_k^-,\widetilde{f}_1^-+\widetilde{f}_2^-+\ldots+\widetilde{f}_k^-).
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{D}_G(L_j) &= \mathcal{D}_G(a)+\mathcal{D}_G(a_j)-2c_j,\quad \textit{where}\quad j=1,2,\ldots,k.\\
&= c_1+c_2+\ldots+c_k+c+c_j-2c_j,\\
&= (t_1^+,i_1^+,f_1^+,t_1^-,i_1^-,f_1^-)+(t_2^+,i_2^+,f_2^+,t_2^-,i_2^-,f_2^-)+\ldots\\
&\quad +(t_k^+,i_k^+,f_k^+,t_k^-,i_k^-,f_k^-)+(t^+,i^+,f^+,t^-,i^-,f^-)-c_j,\\
&= (t_1^++t_2^++\ldots+t_k^++t^+,i_1^++i_2^++\ldots+i_k^++i^+,f_1^++f_2^++\ldots\\
&\quad +f_k^++f^+,t_1^-+t_2^-+\ldots+t_k^-+t^-,i_1^-+i_2^-+\ldots+i_k^-+i^-,\\
&\quad f_1^-+f_2^-+\ldots+f_k^-+f^-)-(t_j^+,i_j^+,f_j^+,t_j^-,i_j^-,f_j^-).
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{D}_G(E_j) &= \mathcal{D}_G(b)+\mathcal{D}_G(b_j)-2d_j,\quad \textit{where}\quad j=1,2,\ldots,k.\\
&= d_1+d_2+\ldots+d_k+c+d_j-2d_j,\\
&= (\widetilde{t}_1^+,\widetilde{i}_1^+,\widetilde{f}_1^+,\widetilde{t}_1^-,\widetilde{i}_1^-,\widetilde{f}_1^-)+(\widetilde{t}_2^+,\widetilde{i}_2^+,\widetilde{f}_2^+,\widetilde{t}_2^-,\widetilde{i}_2^-,\widetilde{f}_2^-)+\ldots\\
&\quad +(\widetilde{t}_k^+,\widetilde{i}_k^+,\widetilde{f}_k^+,\widetilde{t}_k^-,\widetilde{i}_k^-,\widetilde{f}_k^-)+(t^+,i^+,f^+,t^-,i^-,f^-)-d_j,\\
&= (\widetilde{t}_1^++\widetilde{t}_2^++\ldots+\widetilde{t}_k^++t^+,\widetilde{i}_1^++\widetilde{i}_2^++\ldots+\widetilde{i}_k^++i^+,\\
&\quad \widetilde{f}_1^++\widetilde{f}_2^++\ldots+\widetilde{f}_k^++f^+,\widetilde{t}_1^-+\widetilde{t}_2^-+\ldots+\widetilde{t}_k^-+t^-,\\
&\quad \widetilde{i}_1^-+\widetilde{i}_2^-+\ldots+\widetilde{i}_k^-+i^-,\widetilde{f}_1^-+\widetilde{f}_2^-+\ldots+\widetilde{f}_k^-+f^-)-\\
&\quad (\widetilde{t}_j^+,\widetilde{i}_j^+,\widetilde{f}_j^+,\widetilde{t}_j^-,\widetilde{i}_j^-,\widetilde{f}_j^-).
\end{aligned}
$$

It is easy to see that all the edges in G have distinct degrees therefore G is strongly edge irregular bipolar neutrosophic graph. The total degree of each edge *in G is calculated as:*

$$\begin{aligned}
\mathcal{TD}_G(ab) &= \mathcal{D}_G(ab) + c,\\
&= c_1 + c_2 + \ldots + c_k + d_1 + d_2 + \ldots + d_k + c,\\
&= (t_1^+, i_1^+, f_1^+, t_1^-, i_1^-, f_1^-) + (t_2^+, i_2^+, f_2^+, t_2^-, i_2^-, f_2^-) + \ldots\\
&\quad +(t_k^+, i_k^+, f_k^+, t_k^-, i_k^-, f_k^-) + (\widetilde{t}_1^+, \widetilde{i}_1^+, \widetilde{f}_1^+, \widetilde{t}_1^-, \widetilde{i}_1^-, \widetilde{f}_1^-)\\
&\quad +(\widetilde{t}_2^+, \widetilde{i}_2^+, \widetilde{f}_2^+, \widetilde{t}_2^-, \widetilde{i}_2^-, \widetilde{f}_2^-) + \ldots + (\widetilde{t}_k^+, \widetilde{i}_k^+, \widetilde{f}_k^+, \widetilde{t}_k^-, \widetilde{i}_k^-, \widetilde{f}_k^-)\\
&\quad +(t^+, i^+, f^+, t^-, i^-, f^-),\\
&= (t_1^+ + t_2^+ + \ldots + t_k^+, i_1^+ + i_2^+ + \ldots + i_k^+, f_1^+ + f_2^+ + \ldots + f_k^+,\\
&\quad t_1^- + t_2^- + \ldots + t_k^-, i_1^- + i_2^- + \ldots + i_k^-, f_1^- + f_2^- + \ldots + f_k^-)\\
&\quad +(\widetilde{t}_1^+ + \widetilde{t}_2^+ + \ldots + \widetilde{t}_m^+, \widetilde{i}_1^+ + \widetilde{i}_2^+ + \ldots + \widetilde{i}_k^+, \widetilde{f}_1^+ + \widetilde{f}_2^+ + \ldots + \widetilde{f}_k^+,\\
&\quad \widetilde{t}_1^- + \widetilde{t}_2^- + \ldots + \widetilde{t}_m^-, \widetilde{i}_1^- + \widetilde{i}_2^- + \ldots + \widetilde{i}_k^-, \widetilde{f}_1^- + \widetilde{f}_2^- + \ldots + \widetilde{f}_k^-)\\
&\quad +(t^+, i^+, f^+, t^-, i^-, f^-).
\end{aligned}$$

$$\begin{aligned}
\mathcal{TD}_G(L_j) &= \mathcal{D}_G(L_j) + c_j, \quad \textit{where} \quad j = 1, 2, \ldots, k.\\
&= c_1 + c_2 + \ldots + c_k + c + c_j - 2c_j + c_j,\\
&= (t_1^+, i_1^+, f_1^+, t_1^-, i_1^-, f_1^-) + (t_2^+, i_2^+, f_2^+, t_2^-, i_2^-, f_2^-) + \ldots\\
&\quad +(t_k^+, i_k^+, f_k^+, t_k^-, i_k^-, f_k^-) + (t^+, i^+, f^+, t^-, i^-, f^-),\\
&= (t_1^+ + t_2^+ + \ldots + t_k^+ + t^+, i_1^+ + i_2^+ + \ldots + i_k^+ + i^+,\\
&\quad f_1^+ + f_2^+ + \ldots + f_k^+ + f^+, t_1^- + t_2^- + \ldots + t_k^- + t^-,\\
&\quad i_1^- + i_2^- + \ldots + i_k^- + i^-, f_1^- + f_2^- + \ldots + f_k^- + f^-).
\end{aligned}$$

$$\begin{aligned}
\mathcal{TD}_G(E_j) &= \mathcal{D}_G(E_j) + d_j, \quad \textit{where} \quad j = 1, 2, \ldots, k.\\
&= d_1 + d_2 + \ldots + d_k + c + d_j - 2d_j + d_j,\\
&= (\widetilde{t}_1^+, \widetilde{i}_1^+, \widetilde{f}_1^+, \widetilde{t}_1^-, \widetilde{i}_1^-, \widetilde{f}_1^-) + (\widetilde{t}_2^+, \widetilde{i}_2^+, \widetilde{f}_2^+, \widetilde{t}_2^-, \widetilde{i}_2^-, \widetilde{f}_2^-) + \ldots\\
&\quad +(\widetilde{t}_k^+, \widetilde{i}_k^+, \widetilde{f}_k^+, \widetilde{t}_k^-, \widetilde{i}_k^-, \widetilde{f}_k^-) + (t^+, i^+, f^+, t^-, i^-, f^-),\\
&= (\widetilde{t}_1^+ + \widetilde{t}_2^+ + \ldots + \widetilde{t}_k^+ + t^+, \widetilde{i}_1^+ + \widetilde{i}_2^+ + \ldots + \widetilde{i}_k^+ + i^+,\\
&\quad \widetilde{f}_1^+ + \widetilde{f}_2^+ + \ldots + \widetilde{f}_k^+ + f^+, \widetilde{t}_1^- + \widetilde{t}_2^- + \ldots + \widetilde{t}_k^- + t^-,\\
&\quad \widetilde{i}_1^- + \widetilde{i}_2^- + \ldots + \widetilde{i}_k^- + i^-, \widetilde{f}_1^- + \widetilde{f}_2^- + \ldots + \widetilde{f}_k^- + f^-).
\end{aligned}$$

Since each edge L_j and E_j in G has the same total degree, where $j = 1, 2, \ldots, k$. Therefore, G is not a strongly edge totally irregular bipolar neutrosophic graph.

■

3. Conclusion

Neutrosophic sets are the generalization of the concept of fuzzy sets and intuitionistic fuzzy sets. Neutrosophic models give more flexibility, precisions and compatibility to the system as compared to the classical, fuzzy and intuitionistic fuzzy models. Bipolar fuzzy graph theory has many applications in technology and science, especially in the fields of operations research, neural networks, decision making and artificial intelligence. Bipolar neutrosophic graph is the extension of bipolar fuzzy graph. In this research paper, we have discussed certain types of edge irregular bipolar neutrosophic graphs. Here, we have established some theorems on bipolar neutrosophic graphs.

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