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ACHIEVABLE SINGLE–VALUED NEUTROSOPHIC GRAPHS IN WIRELESS SENSOR NETWORKS

M. HAMIDI AND A. BORUMAND SAEID

Abstract. This paper considers wireless sensor (hyper)networks by single–valued neutrosophic (hyper)graphs. It tries to extend the notion of single–valued neutrosophic graphs to single–valued neutrosophic hypergraphs and it is derived single–valued neutrosophic graphs from single–valued neutrosophic hypergraphs via positive equivalence relation. We use single–valued neutrosophic hypergraphs and positive equivalence relation to create the sensor clusters and access to cluster heads. Finally, the concept of (extended) derivable single–valued neutrosophic graph is considered as the energy clustering of wireless sensor networks and is applied this concept as a tool in wireless sensor (hyper)networks.

Mathematics Subject Classification (2010): 68R10, 05C15, 81Q30, 05C65.
Key words and phrases: Single–valued neutrosophic (graphs) hypergraphs, positive equivalence relation, extendable single–valued neutrosophic graph, WSN.

1. Introduction

Neutrosophy, as a newly"born science, is a branch of philosophy that studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. It can be defined as the incidence of the application of a law, an axiom, an idea, a conceptual accredited construction on an unclear, indeterminate phenomenon, contradictory to the purpose of making it intelligible. Neutrosophic set and neutrosophic logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic) are tools for publications on advanced studies in neutrosophy. In neutrosophic logic, a proposition has a degree of truth (T), indeterminacy (I) and falsity (F), where T, I, F are standard or non–standard subsets of $]^{-}0, 1^{+}[$. In 1995, Smarandache talked for the first time about neutrosophy and in 1999 and 2005 [11, 12] he initiated the theory of neutrosophic set as a new mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data. Alkhazaleh et al. generalized the concept of fuzzy soft set to neutrosophic soft set and they gave some applications of this concept in decision making and medical diagnosis [4]. Smarandache [13, 14] have defined four main categories of neutrosophic graphs, two based on literal indeterminacy (I), whose name were; I–edge neutrosophic graph and I–vertex neutrosophic graph, these concepts have been deeply studied and have gained popularity among the researchers due to their applications in real world problems [7, 15]. The two others graphs were based on (t, i, f) components whose name was; The (t, i, f)–Edge neutrosophic graph and the (t, i, f)–vertex neutrosophic graph, these concepts are not developed at all. Later on, Broumi et al.[5] introduced a third neutrosophic graph model. This model allows the attachment of truth–membership (t), indeterminacy–membership (i) and falsity–membership degrees (f) both to vertices and edges, and investigated some of their properties. M. Akram et. al defined the concepts of single-valued neutrosophic hypergraph, line graph of single-valued neutrosophic hypergraph, dual single-valued neutrosophic hypergraph and transversal single-valued neutrosophic hypergraph [3]. Wireless sensor networks (WSNs) have gained worldwide attention in recent years, particularly with the proliferation of micro–electro–mechanical systems (MEMS) technology, which has facilitated the development of smart sensors. WSNs are used in numerous applications, such as environmental monitoring, habitat monitoring, prediction and detection of natural calamities, medical monitoring, and structural health monitoring. WSNs consist of tiny sensing devices that are spread over a large geographic area and can be used to collect and process environmental data such as temperature, humidity, light conditions, seismic activities, images of the environment, and so on.

Regarding these points, this paper aims to generalize the notion of single-valued neutrosophic graphs by considering the notion of positive equivalence relation and trying to define the concept of derivable single-valued neutrosophic graphs. The relationships between derivable single–valued neutrosophic graphs and single-valued neutrosophic hypergraphs are considered as a natural question. The quotient of single-valued neutrosophic hypergraphs via equivalence relations is the main motivation of this research. Moreover, by using positive equivalence relations, we define a well–defined operation on single-valued neutrosophic hypergraphs that the quotient of any single–valued neutrosophic hypergraphs via this relation is a single–valued neutrosophic graph. We use single–valued neutrosophic hypergraphs to represent wireless sensor hypernetworks. By considering the concept of the wireless sensor networks, the use of wireless sensor hypernetworks appears to be a necessity for exploring these systems and representation their relationships. We have introduced several valuable measures as truth–membership, indeterminacy and falsity–membership values for studying wireless sensor hypernetworks, such as node and hypergraph centralities as well as clustering coefficients for both hypernetworks and networks. Clustering is one of the basic approaches for designing energy–efficient, robust and highly scalable distributed sensor networks. A sensor network reduces the communication overhead by clustering, and decreases the energy consumption and the interference among the sensor nodes, so we via the concept of single-valued neutrosophic (hyper)graphs and equivalence relations considered the wireless sensor hypernetworks.

2. Preliminaries

In this section, we recall some definitions and results are indispensable to our research paper.

Definition 2.1. [6] *Let $G = \{x_1, x_2, \ldots, x_n\}$ be a finite set. A hypergraph on G is a family $H = (E_1, E_2, \ldots, E_m) = (G, \{E_i\}_{i=1}^{m})$ of subsets of G such that*

- (i) *for all $1 \leq i \leq m, E_i \neq \emptyset$;*
- (ii) $\bigcup_{i=1}^{m} E_i = G$.

A simple hypergraph (Sperner family) is a hypergraph $H = (E_1, E_2, \ldots, E_m)$ such that

- (iii) $E_i \subset E_j \Longrightarrow i = j$.

The elements $x_1, x_2, \ldots, x_n$ of G are called vertices, and the sets $E_1, E_2, \ldots, E_m$ are the edges (hyperedges) of the hypergraph. For any $1 \leq k \leq m$ if $|E_k| \geq 2$, then E_k is represented by a solid line surrounding its vertices, if $|E_k| = 1$ by a cycle on the element (loop). If for all $1 \leq k \leq m$ $|E_k| = 2$, the hypergraph becomes an ordinary (undirected) graph.

Definition 2.2. [9] *Let $H = (G, \{E_x\}_{x\in G})$ be a hypergraph. Then $H = (G, \{E_x\}_{x\in G})$ is called a complete hypergraph, if for any $x, y \in G$ there is a hyperedge E such that $\{x, y\} \subseteq E$ and is shown a complete hypergraph with n elements by K_n^*. Let $H = (G, \{E_i\}_{i=1}^{n+1})$ be a complete hypergraph.*

- (i) *$H = (G, \{E_i\}_{i=1}^{n+1})$ is called a joint complete hypergraph, if for any $1 \leq i \leq n$, $|E_i| = i, E_i \subseteq E_{i+1}$ and $|E_{n+1}| = n$;*
- (ii) *$H = (G, \{E_i\}_{i=1}^{n+1})$ is called a discrete complete hypergraph, if for any $1 \leq i \neq j \leq n, |E_i| = |E_j|, E_i \cap E_j = \emptyset$ and $|E_{n+1}| = n$;*

Definition 2.3. [10] *Let $H = (G, \{E_i\}_{i=1}^{n})$ be a hypergraph. Define a binary relation η on G as follows: $\eta_1 = \{(x, x) \mid x \in G\}$ and for every integer $k > 1$,*

$$x \ \eta_k \ y \Longleftrightarrow \exists \ \ E_i^s, \text{ such that } \{x, y\} \subseteq E_i^s, \text{ where } k = |E_i^s| = \min\{|E_t|; \ x, y \in E_t\}$$

and for all $1 \leq i, j \leq n$, there is no $E_i \neq E_i^s$, or $E_j \neq E_i^s$, such that $x \in E_i, y \in E_j$ and $|E_i| < k, |E_j| < k$. Obviously $\eta = \bigcup_{k\geq 1} \eta_k$ is a reflexive and symmetric relation on G. Let η^* be the transitive closure of η (the smallest transitive relation such that contains η).

Theorem 2.4. [10] *Let $H = (G, \{E_x\}_{x\in G})$ be a hypergraph, $\mathbb{N}^* = \mathbb{N} \cup \{0\}$ and $\eta = \eta^*$. Then for any $i \in \mathbb{N}^*$ there exists an operation $*_i$ on G/η such that $H/\eta = (G/\eta, *_i)$ is a graph.*

Definition 2.5. [16] *Let X be a set. A single valued neutrosophic set A in X (SVN–S A) is a function $A : X \longrightarrow [0,1] \times [0,1] \times [0,1]$ with the form $A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$ where the functions T_A, I_A, F_A define respectively the truth–membership function, an indeterminacy–membership function, and a falsity–membership function of the element $x \in X$ to the set A such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. Moreover, $Supp(A) = \{x \mid T_A(x) \neq 0, I_A(x) \neq 0, F_A(x) \neq 0\}$ is a crisp set.*

Definition 2.6. [5] *A single valued neutrosophic graph (SVN–G) is defined to be a form $G = (V, E, A, B)$ where*

- (i) *$V = \{v_1, v_2, \ldots, v_n\}$, $T_A, I_A, F_A : V \longrightarrow [0,1]$ denote the degree of membership, degree of indeterminacy and non–membership of the element $v_i \in V$; respectively, and for every $1 \leq i \leq n$, we have $0 \leq T_A(v_i)+I_A(v_i)+F_A(v_i) \leq 3$.*
- (ii) *$E \subseteq V \times V$, $T_B, I_B, F_B : E \longrightarrow [0,1]$ are called degree of truth–membership, indeterminacy–membership and falsity–membership of the edge $(v_i, v_j) \in E$ respectively, such that for any $1 \leq i, j \leq n$, we have $T_B(v_i, v_j) \leq \min\{T_A(v_i), T_A(v_j)\}, I_B(v_i, v_j) \geq \max\{I_A(v_i), I_A(v_j)\}$, $F_B(v_i, v_j) \geq \max\{F_A(v_i), F_A(v_j)\}$ and $0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3$. Also A is called the single valued neutrosophic vertex set of V and B is called the single valued neutrosophic edge set of E.*

Definition 2.7. [3]

- (i) *A single valued neutrosophic hypergraph (SVN–HG) is defined to be a pair $H = (V, \{E_i\}_{i=1}^m)$, where $V = \{v_1, v_2, \ldots, v_n\}$ is a finite set of vertices and $\{E_i = \{(v_j, T_{E_i}(v_j), I_{E_i}(v_j), F_{E_i}(v_j))\}\}_{i=1}^m$ is a finite family of non–trivial neutrosophic subsets of the vertex V such that $V = \bigcup\limits_{i=1}^{m} supp(E_i)$. Also $\{E_i\}_{i=1}^m$ is called the family of single valued neutrosophic hyperedges of H and V is the crisp vertex set of H.*
- (ii) *Let $1 \leq \alpha, \beta, \gamma \leq 1$, then $A^{(\alpha,\beta,\gamma)} = \{x \in X \mid T_A(x) \geq \alpha, I_A(x) \geq \beta, F_A(x) \leq \gamma\}$ is called (α, β, γ)–level subset of A.*

3. (regular) Single–valued neutrosophic hypergraphs(graphs) (SVN–HG)

In this section, we introduce a concept of regular single–valued neutrosophic graph and construct quotient single–valued neutrosophic hypergraphs, via equivalence relations.

Let $(H, \{E_i\}_{i=1}^n)$ be a hypergraph, $1 \leq i, j \leq n$ and $k \in \mathbb{N}$. Then H is called a partitioned hypergraph, if $\mathcal{P} = \{E_1, E_2, \ldots, E_n\}$ is a partition set of H. We will denote the set of partitioned hypergraphs with $|\mathcal{P}| = k$ on H that $|E_i| = |E_j|$, by $\mathcal{P}_h^{(k)}(H)$ and the set of all partitioned hypergraphs on H, by $\mathcal{P}_h(H)$.

Definition 3.1. *Let $G = (V, E, A, B)$ be a single–valued neutrosophic graph. Then $G = (V, E, A, B)$ is called*

(i) *a weak single–valued neutrosophic graph, if supp$(A) = V$;*

(ii) *a regular single–valued neutrosophic graph, if is weak and for any $v_i, v_j \in V$ have $T_B(v_i, v_j) = \min\{T_A(v_i), T_A(v_j)\}, I_B(v_i, v_j) = \max\{I_A(v_i), I_A(v_j)\}$ and $F_B(v_i, v_j) = \max\{F_A(v_i), F_A(v_j)\}$.*

Proposition 3.2. *Let $V = \{a_1, a_2, \ldots, a_n\}$. Consider the complete graph K_n and define $A : V \to [0, 1]$ by $T_A(a_i) = 1/i, I_A(a_i) = 1/(i + 1), F_A(a_i) = 1/(i + 2)$ and*

(i) *$B : V\times V \to [0, 1]$ by $T_B(a_i, a_j) = T_A(a_i)\times T_A(a_j), I_B(a_i, a_j) = F_B(a_i, a_j) = I_A(a_i) + T_A(a_j)$. It is clear that $G = (V, E, A, B)$ is a single–valued neutrosophic complete graph and since supp$(A) = V$, we get that it is a weak single–valued neutrosophic complete graph.*

(ii) *$B : V \times V \to [0, 1]$ by $T_B(a_i, a_j) = |T_A(a_i) + T_A(a_j)|/2 - |T_A(a_i) - T_A(a_j)|/2, I_B(a_i, a_j) = |I_A(a_i)+I_A(a_j)|/2+|I_A(a_i)-I_A(a_j)|/2$ and $F_B(a_i, a_j) = |F_A(a_i)+F_A(a_j)|/2+|F_A(a_i)-F_A(a_j)|/2$. It is clear that $G = (V, E, A, B)$ is a regular single–valued neutrosophic complete graph.*

Corollary 3.3. *Any finite set can be a (regular)weak single–valued neutrosophic complete graph.*

Proof. Let G be a finite set and R be an equivalence relation on G. Then consider, $H = \big(G, \{R(x) \times R(y)\}_{x,y\in G}\big)$, whence it is a complete graph. By Proposition 3.2, is obtained. □

Lemma 3.4. *Let X be a finite set and $A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$ be a single–valued neutrosophic set in X. If R is an equivalence relation on X, then $A/R = \{(R(x), T_{R(A)}(R(x)), I_{R(A)}(R(x)), F_{R(A)}(R(x)) \mid x \in X\}$ is a single–valued neutrosophic set, where $T_{R(A)}(R(x)) = \bigwedge\limits_{t\ R\ x} T_A(t), I_{R(A)}(R(x)) = \bigvee\limits_{t\ R\ x} I_A(t)$ and $F_{R(A)}(R(x)) = \bigvee\limits_{t\ R\ x} F_A(t)$.*

Proof. Let $X = \{x_1, x_2, \ldots, x_n\}$ and $\mathcal{P} = \{R(x_1), R(x_2), \ldots, R(x_k)\}$ be a partition of X, where $k \leq n$. Since for any $x_i \in X, T_A(x_i) \leq 1, I_A(x_i) \leq 1$ and $F_A(x_i) \leq 1$, we get that $\bigwedge\limits_{t\ R\ x_i} T_A(t) \leq 1$, $\bigvee\limits_{t\ R\ x_i} I_A(t) \leq 1$ and $\bigvee\limits_{t\ R\ x_i} F_A(t) \leq 1$. Hence for any $1 \leq i \leq k$, $0 \leq \bigwedge\limits_{t\ R\ x_i} T_A(t) + \bigvee\limits_{t\ R\ x_i} I_A(t) + \bigvee\limits_{t\ R\ x_i} F_A(t) \leq 3$ and so $R(A) = \{(R(x_i), \bigwedge\limits_{t\ R\ x_i} T_A(t), \bigvee\limits_{t\ R\ x_i} I_A(t), \bigvee\limits_{t\ R\ x_i} F_A(t))\}_{i=1}^{k}$ is a single–valued neutrosophic set in X/R. □

Theorem 3.5. *Let $\overline{V} = \{v_1, v_2, \ldots, v_n\}$ and $H = (\overline{V}, \{(v_j, T_{E_i}(v_j), I_{E_i}(v_j), F_{E_i}(v_j))\}_{i=1}^{m})$ be a single–valued neutrosophic hypergraph. If R is an equivalence relation on*

H, then $H/R = \big(R(\overline{V}), \{R(v_j), T_{R(E_i)}(R(v_j)), I_{R(E_i)}(R(v_j)), F_{R(E_i)}(R(v_j))\}_{i=1}^{m}\big)$ is a partitioned single–valued neutrosophic hypergraph.

Proof. By Lemma 3.4, $\{R(v_j), T_{R(E_i)}(R(v_j)), I_{R(E_i)}(R(v_j)), F_{R(E_i)}(R(v_j))\}_{i=1}^{m}$ is a finite family of single–valued neutrosophic subsets of $\overline{V}/R$. Since $\overline{V} = \bigcup\limits_{i=1}^{m} supp(E_i)$, we get that $\bigcup\limits_{i=1}^{m} supp(R(E_i)) = R(\bigcup\limits_{i=1}^{m} supp(E_i)) = R(\overline{V})$. It follows that $H/R = (R(\overline{V}), \{R(v_j), T_{R(E_i)}(R(v_j)), I_{R(E_i)}(R(v_j)), F_{R(E_i)}(R(v_j))\}_{i=1}^{m})$ is a single–valued neutrosophic hypergraph. Since R is an equivalence relation on $\overline{V}$, for any $x \neq y \in \overline{V}$ we get that $R(x) \cap R(y) = \emptyset$ and so it is a partitioned single–valued neutrosophic hypergraph. $\square$

Example 3.6. *Consider a joint complete single–valued neutrosophic hypergraph $H = (\overline{V}, \{E_i\}_{i=1}^{n})$, where $\overline{V} = \{a_1, a_2, \ldots, a_n\}$ and for any $1 \leq i \leq n, E_i = \{(a_i, i/10^n, (i+1)/10^n), (i+2)/10^n)\}$. Clearly $R = \{(a_i, a_i), (a_r, a_s) \mid r + s = n+1, 1 \leq i \leq n\}$ is an equivalence relation on $\overline{V}$ and so we obtain $\overline{V}/R = \{R(a_1), R(a_2), R(a_3), \ldots, R(a_{(n/2)-1}), R(a_{n/2})\}$. It follows that*

$$\begin{aligned} H/R = \ & \Big(\big\{R(a_1), R(a_2), R(a_3), \ldots, R(a_{(n/2)-1}), R(a_{n/2})\big\}, \\ & \big\{\big(R(a_i), i/10^n, (i+1)/10^n, (i+2)/10^n\big), \big(R(a_{n-i+1}), \\ & (n-i+1)/10^n, (n-i+2)/10^n, (n-i+3)/10^n\big)\big\}_{i=1}^{n/2}\Big). \end{aligned}$$

Computation shows that H/R is a partitioned single–valued neutrosophic hypergraph.

4. Derivable regular single–valued neutrosophic graphs

In this section, we introduce the concept of derivable single–valued neutrosophic graphs via the equivalence relation η^* on single–valued neutrosophic hypergraphs. It is shown that any single–valued neutrosophic graph is not necessarily a derivable single–valued neutrosophic graph and it is proved under some conditions. Furthermore, it can show that some regular trees and all regular single–valued neutrosophic complete bigraphs are derivable single–valued neutrosophic graph and regular single–valued neutrosophic complete graph are self derivable single–valued neutrosophic graphs.

Definition 4.1. *A single–valued neutrosophic graph $G = (V, E, A, B)$ is said to be:*

(i) a derivable single–valued neutrosophic graph if there exists a nontrivial single–valued neutrosophic hypergraph $(H, \{E_k\}_{k=1}^{n})$ such that $(H, \{E_k\}_{k=1}^{n})/\eta^* \cong G = (V, E, A, B)$ and H is called an associated single–valued neutrosophic hypergraph with single–valued neutrosophic graph G. In other words, it is equal to the quotient of nontrivial single–valued neutrosophic hypergraph on η^* up to isomorphic;

(ii) a self derivable single–valued neutrosophic graph, if it is a derivable single–valued neutrosophic graph by itself.

Theorem 4.2. *Let $H = (\overline{V}, \{E_i\}_{i=1}^m)$ be a single–valued neutrosophic hypergraph, $j \in \mathbb{N}^* = \mathbb{N} \cup \{0\}$ and $\eta = \eta^*$. Then there exists an operation "$*_j$" on H/η such that $(H/\eta, *_j)$ is a regular single–valued neutrosophic graph.*

Proof. By Theorem 3.5, $H/\eta = \big(\eta(\overline{V}), \{\eta(v_j), T_{\eta(E_i)}(\eta(v_j)), I_{\eta(E_i)}(\eta(v_j)), F_{\eta(E_i)}(\eta(v_j))\}_{i=1}^m\big)$ is a partitioned single–valued neutrosophic hypergraph, where

$$T_{\eta(E_i)}(\eta(x)) = \bigwedge_{x\ \eta\ t \in X} T_{E_i}(t), I_{\eta(E_i)}(\eta(x)) = \bigvee_{x\ \eta\ t \in X} I_{E_i}(t) \text{ and}$$

$$F_{\eta(E_i)}(\eta(x)) = \bigvee_{x\ \eta\ t \in X} F_{E_i}(t).$$

For any $\eta(x) = \eta((x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x)))$ and $\eta(y) = \eta((y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y)) \in H/\eta$, define an operation "$*_j$" on H/η by

$$\eta(x) *_j \eta(y) = \begin{cases} \widehat{\eta(x), \eta(y)} & \text{if } \big||\eta(x)| - |\eta(y)|\big| = j, \\ \widehat{\emptyset} & \text{otherwise,} \end{cases}$$

where for any $x, y \in G$, $(\widehat{\eta(x), \eta(y)})$ is represented as an ordinary (simple) edge and $\widehat{\emptyset} = \widehat{\eta(x)}$ means that there is not edge. It is easy to see that

$$H/\eta = \big(\eta(\overline{V}), \{\eta(v_j), T_{\eta(E_i)}(\eta(v_j)), I_{\eta(E_i)}(\eta(v_j)), F_{\eta(E_i)}(\eta(v_j))\}_{i=1}^m, *_j\big)$$

is a graph. Now, define $\overline{T}_{\eta(E_i)}, \overline{I}_{\eta(E_i)}, \overline{F}_{\eta(E_i)} : \eta(\overline{V}) \times \eta(\overline{V}) \longrightarrow [0,1]$ by $\overline{T}_{\eta(E_i)}(\eta(x), \eta(y)) = \bigwedge_{a\eta x,\ b\eta y} (T_{\eta(E_i)}(a) \wedge T_{\eta(E_i)}(b)), \overline{I}_{\eta(E_i)}(\eta(x), \eta(y)) = \bigvee_{a\eta x,\ b\eta y} (I_{\eta(E_i)}(a) \vee I_{\eta(E_i)}(b))$ and $\overline{F}_{\eta(E_i)}(\eta(x), \eta(y)) = \bigvee_{a\eta x,\ b\eta y} (F_{\eta(E_i)}(a) \vee F_{\eta(E_i)}(b))$. It is clear to see that $\overline{T}_{\eta(E_i)}(\eta(x), \eta(y)) \leq (T_{\eta(E_i)}(\eta(x)) \wedge T_{\eta(E_i)}(\eta(y))), \overline{I}_{\eta(E_i)}(\eta(x), \eta(y)) \geq (I_{\eta(E_i)}(\eta(x)) \vee I_{\eta(E_i)}(\eta(y)))$ and $\overline{F}_{\eta(E_i)}(\eta(x), \eta(y)) \geq (F_{\eta(E_i)}(\eta(x)) \vee F_{\eta(E_i)}(\eta(y)))$. Hence $H/\eta = \big(\eta(\overline{V}), \{\eta(v_j), T_{\eta(E_i)}(\eta(v_j)), I_{\eta(E_i)}(\eta(v_j)), F_{\eta(E_i)}(\eta(v_j))\}_{i=1}^m, *\big)$ is a single–valued neutrosophic graph. □

Example 4.3. *Let $H = (\{a, b, c, d, e, f, g\}, \{E_1, E_2, E_3, E_4\})$ be a single–valued neutrosophic hypergraph in Figure 1. Since*

$$E_1^s = \{(a, 0.1, 0.2, 0.3), (b, 0.3, 0.2, 0.1)\}, E_2^s = \{(c, 0.4, 0.5, 0.6), (d, 0.6, 0.5, 0.4)\},$$
$$E_3^s = \{(e, 0.7, 0.8, 0.9), (f, 0.9, 0.8, 0.7)\} \text{ and } E_4^s = \{(g, 0.1, 0.3, 0.5)\},$$

by Theorem 4.2, we get that $\eta^* = \eta$ and

$$\eta((a, 0.1, 0.2, 0.3)) = \eta((b, 0.3, 0.2, 0.1)) = \{(a, 0.1, 0.2, 0.3), (b, 0.1, 0.2, 0.3)\},$$
$$\eta((c, 0.4, 0.5, 0.6)) = \eta((d, 0.6, 0.5, 0.4)) = \{(c, 0.4, 0.5, 0.6), (d, 0.4, 0.5, 0.6)\},$$
$$\eta((e, 0.7, 0.8, 0.9)) = \eta((f, 0.9, 0.8, 0.7)) = \{(e, 0.7, 0.8, 0.9), (f, 0.7, 0.8, 0.9)\},$$
$$\text{and } \eta((g, 0.1, 0.3, 0.5)) = \{(g, 0.1, 0.3, 0.5)\}.$$

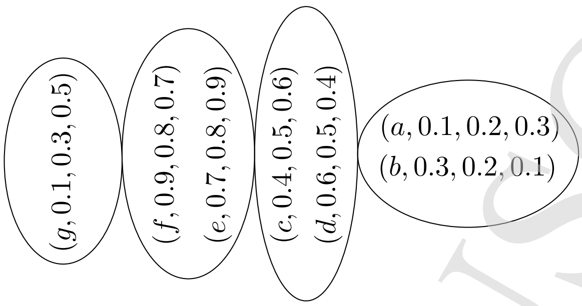

Figure 1. SVN–HG

Now, for $i = 0$, we obtain the regular single–valued neutrosophic graph in Figure 2.

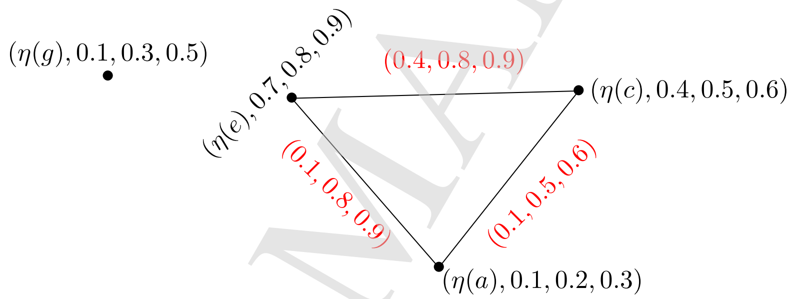

Figure 2. Derivable SVN–G G/η for $i = 0$

G/η is a regular un–connected single–valued neutrosophic graph with 4 vertices and 3 edges. For $i = 1$, we obtain $G/\eta \cong K_{1,3}$ as Figure 3. G/η is a connected regular single–valued neutrosophic graph with 4 vertices and 3 edges. Moreover, for any $i \geq 2$ graph G/η is isomorphic to null single–valued neutrosophic graph $\overline{K}_4$(Figure 4).

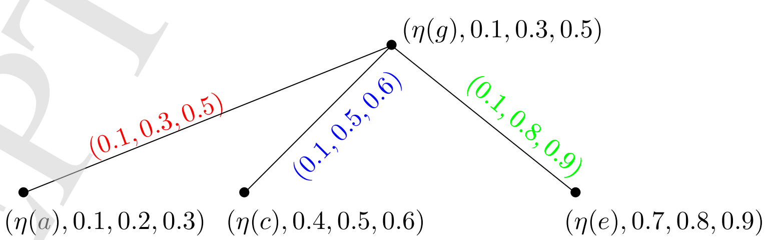

Figure 3. Derivable SVN–G G/η for $i = 1$

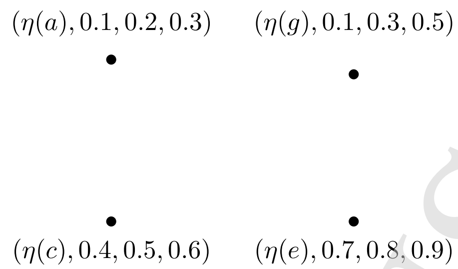

FIGURE 4. Derivable SVN–G G/η for $i \geq 2$

Example 4.4. *Let* $\overline{V} = \{a_1, a_2, a_3, a_4, a_5\}$. *Then consider the single–valued neutrosophic hypergraph* $H = (\overline{V}, E_1, E_2, E_3, E_4, E_5)$ *in Figure 5. Clearly* $\eta^* = \eta$,

$$E^s_{a_1} = \{(a_1, 0.2, 0.4, 0.6)\}, E^s_{a_2} = \{(a_2, 0.4, 0.6, 0.6)\}, E^s_{a_3} = \{(a_3, 0.3, 0.2, 0.4)\},$$
$$E^s_{a_4} = \{(a_4, 0.8, 0.9, 0.1)\} \text{ and } E^s_{a_5} = \{(a_5, 0.1, 0.9, 0.9)\},$$

thus by Theorem 4.2, we obtain

$$\eta((a_1, 0.2, 0.4, 0.6)) = \{(a_1, 0.2, 0.4, 0.6)\}, \eta((a_2, 0.4, 0.6, 0.6)) = \{(a_2, 0.4, 0.6, 0.6)\},$$
$$\eta((a_3, 0.3, 0.2, 0.4)) = \{(a_3, 0.3, 0.2, 0.4)\}, \eta((a_4, 0.8, 0.9, 0.1)) = \{(a_4, 0.8, 0.9, 0.1)\}$$
$$\text{and } \eta((a_5, 0.1, 0.9, 0.9)) = \{(a_5, 0.1, 0.9, 0.9)\}.$$

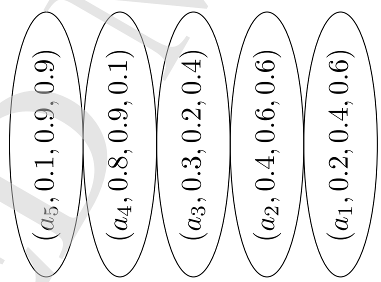

FIGURE 5. SVN–HG

Now, for $i = 0$, $H/\eta = (\eta(\overline{V}), \{(\eta(a_i), T_{\eta(E_i)}(\eta(a_i)), I_{\eta(E_i)}(\eta(a_i)), F_{\eta(E_i)}(\eta(a_i)))\}_{i=1}^{5}$ *and we obtain the regular single–valued neutrosophic graph in Figure 6.*

Theorem 4.5. *Let* $G = (V, E, A, B)$ *be a derivable single–valued neutrosophic graph by a single–valued neutrosophic hypergraph* $H = (\overline{V}, \overline{E} = \{v_j, T_{E_k}(v_j), I_{E_k}(v_j), F_{E_k}(v_j))\}_{k=1}^{m})$. *Then*

(*i*) $|\overline{V}/\eta| = |V|$ *and* $|\overline{E}| = |E|$;
(*ii*) $|\overline{V}| \geq |V|$;
(*iii*) *if* $G = (V, E, A, B)$ *is a connected single–valued neutrosophic graph and* $|\overline{V}| = |V|$, *then* $*_i = *_0$;

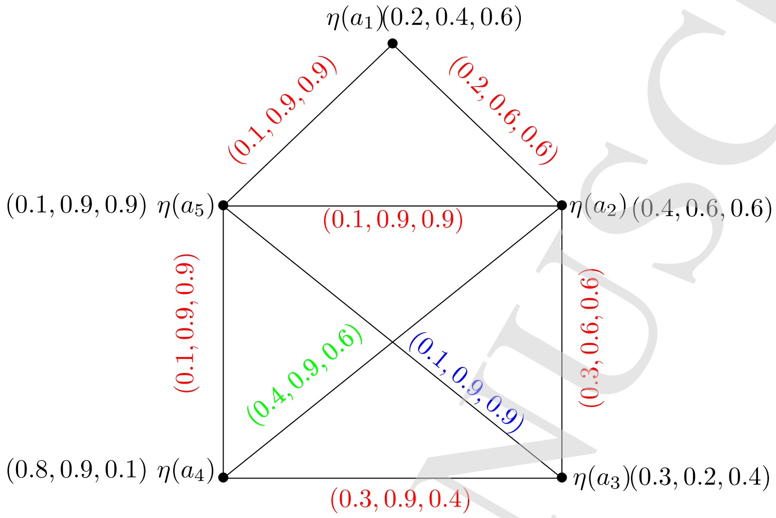

FIGURE 6. Derived cycle SVN–G K_5

(iv) *for any* $1 \leq j \leq m$, $\overline{T}_{\eta(E_i)}(v_j) \leq T_{E_i}(v_j)$, $\overline{I}_{\eta(E_i)}(v_j) \geq I_{E_i}(v_j)$ *and* $\overline{F}_{\eta(E_i)}(v_j) \geq F_{E_i}(v_j)$.

Proof. (iii) Let $|\overline{V}| = |V|$. Then for any $x, y \in \overline{V}, \eta((x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x))) \neq \eta((y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y)))$. Since $G = (V, E, A, B)$ is a connected single–valued neutrosophic graph, we get that for any $x, y \in \overline{V}, |\eta((x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x)))| = |\eta((y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y)))|$ and so $*_i = *_0$. □

Example 4.6. *Let* $\overline{V} = \{a_1, a_2, \ldots, a_n\}$ *and* $i = 0$. *Consider the discrete complete hypergraph in Figure 7. Then it is easy to see that* $K_n^* = (\overline{V}, \{E_k = \{(v_k, T_{E_k}(v_j), I_{E_k}$

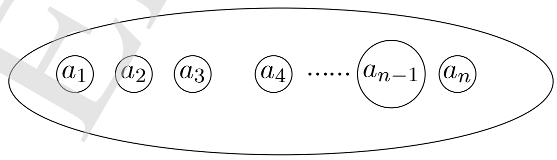

FIGURE 7. Joint complete SVN–HG K_n^*

$(v_j), F_{E_k}(v_j))\}\}_{k=1}^{n+1})$ *is a discrete complete single–valued neutrosophic hypergraph, where for any* $1 \leq k \leq n$, $E_k = \{(a_k, 1/k, 1/k^2, 1/k^3)\}$ *and* $E_{n+1} = \bigcup_{k=1}^{n} E_k$. *Clearly for any* $1 \leq k \leq n$, $\eta(a_k) = E^s_{a_k} = \{(a_k, 1/k, 1/k^2, 1/k^3)\}$. *Hence* $K_n^*/\eta^* = \{\eta(a_k) \mid 1 \leq k \leq n\}$ *and so* $K_n^*/\eta^* \cong K_n$. *For any* $1 \leq k \leq n$, *we get* $\overline{T}_{\eta(E_i)}(\eta(a_k), \eta(a_{k+1})) = (1/k) \wedge 1/(k+1) = 1/(k+1) \leq \overline{T}_{\eta(E_i)}(\eta(a_k)) \wedge \overline{T}_{\eta(E_i)}(\eta(a_{k+1}))$. *Therefore for any* $n \in \mathbb{N}, K_n$ *is a derivable single–valued neutrosophic graph.*

Theorem 4.7. *Let $G = (V, E, A, B)$ be a connected single–valued neutrosophic graph. G is a self derivable single–valued neutrosophic graph if and only if G is a single–valued neutrosophic complete graph.*

Proof. Let $V = \{a_1, a_2, \ldots, a_n\}$. If G is a single–valued neutrosophic complete graph, then by Example 4.6, G is a self derivable single–valued neutrosophic graph.

Conversely, let G be a self derivable single–valued neutrosophic graph. Then by Theorem 4.5, we get $*_i = *_0$ and so for any $x, y \in G, \eta(x) *_i \eta(y) = \widehat{\eta(x), \eta(y)}$. Thus $G = (V, E, A, B)$ is a derivable single–valued neutrosophic complete graph. $\square$

4.1. Derivable cycle single–valued neutrosophic graph.

In this section, we consider derivable cycle single–valued neutrosophic graph and show that C_n is a derivable cycle single–valued neutrosophic graph if and only if $n \in \{3, 4\}$.

Example 4.8. *Consider the cycle single–valued neutrosophic graph C_4 in Figure 8.*

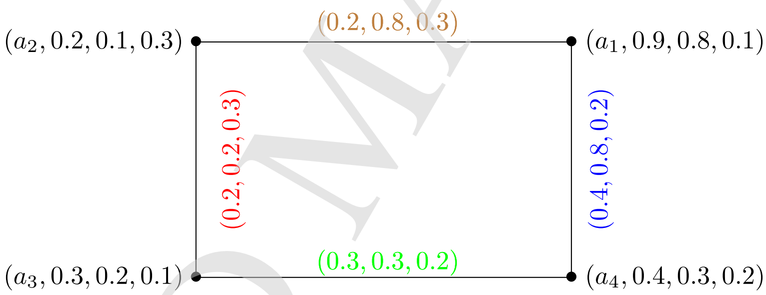

FIGURE 8. Derived cycle SVN–G C_4

Now introduce the single–valued neutrosophic hypergraph H in Figure 9. Clearly

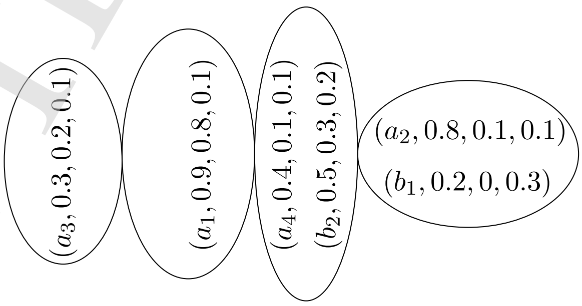

FIGURE 9. SVN–HG

$\eta = \eta^*$,

$$\eta((a_1, 0.9, 0.8, 0.1)) = \{(a_1, 0.9, 0.8, 0.1)\}, \eta((a_2, 0.8, 0.1, 0.1)) =$$
$$\{(a_2, 0.2, 0.1, 0.3), (b_1, 0.2, 0.1, 0.3)\}, \eta((a_3, 0.3, 0.2, 0.1)) = \{(a_3, 0.3, 0.2, 0.1)\}$$
$$\text{and } \eta((a_4, 0.4, 0.1, 0.1)) = \{(a_4, 0.4, 0.3, 0.2), (b_2, 0.4, 0.3, 0.2)\}.$$

By Theorem 4.2 and for $i = 1$, we obtain that $H/\eta \cong C_4$.

Lemma 4.9. *(C_5, A, B) is not a derivable single–valued neutrosophic graph.*

Proof. Consider the cycle single–valued neutrosophic graph C_5 in Figure 10, where for any $1 \leq j \leq 5$ we have $0 \leq T_j + I_j + F_j \leq 3$, $\lambda_j = (T_{jj+1}, I_{jj+1}, F_{jj+1})$ and $0 \leq T_{jj+1} + I_{jj+1} + F_{jj+1} \leq 3$. Let C_5 be a derivable single–valued neutrosophic graph

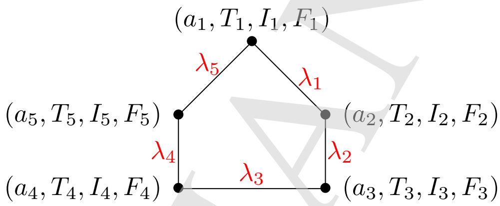

FIGURE 10. Derived SVN–G C_5

and $H = (\overline{V}, \{E_j\}_{j=1})$ be an associated single–valued neutrosophic hypergraph with single–valued neutrosophic graph C_5. Since $\eta(a_1) *_i \eta(a_2) = \widehat{\eta(a_1), \eta(a_2)}$ and $\eta(a_1) *_i \eta(a_5) = \widehat{\eta(a_1), \eta(a_5)}$, we get $k \in \mathbb{N}$, $E_1, E_2, E_3 \subseteq H$ so that $a_1 \in E_1, |E_1| = k, a_2 \in E_2, |E_2| = k + i, a_5 \in E_3$ and $|E_3| = k + i$. On the other hand, $\eta(a_3) *_i \eta(a_2) = \widehat{\eta(a_3), \eta(a_2)}$, $\eta(a_3) *_i \eta(a_4) = \widehat{\eta(a_3), \eta(a_4)}$ and $\eta(a_3) *_i \eta(a_1) = \widehat{\emptyset}$, implies that there exist $E_4, E_5 \subseteq H$ so that $a_3 \in E_4, |E_4| = |E_1|$, $a_4 \in E_5$ and $|E_2| = |E_3| = |E_5| = k + i$. Since $\big||\eta(a_4)| - |\eta(a_5)|\big| \neq i$, $\big||\eta(a_3)| - |\eta(a_5)|\big| = i$ and $\big||\eta(a_4)| - |\eta(a_1)|\big| = i$, we get that $\eta(a_4) *_i \eta(a_5) = \widehat{\emptyset}$, $\eta(a_3) *_i \eta(a_5) = \widehat{\eta(a_3), \eta(a_5)}$ and $\eta(a_4) *_i \eta(a_1) = \widehat{\eta(a_4), \eta(a_1)}$. But $H = (V, \overline{E})$, where $\overline{E} = \{E_1, E_2, E_3, E_4, E_5\}$, $H/\eta^* \cong C_5$ and $|\overline{E}/\eta^*| = 6$, which is a contradiction. Therefore C_5 can not be a derivable single–valued neutrosophic graph. □

Proposition 4.10. *Let $6 \leq n \in \mathbb{N}$. Then (C_n, A, B) is not a derivable single–valued neutrosophic graph.*

Proof. Since for any $6 \leq n \in \mathbb{N}$, C_n is homeomorphic to (C_5, A, B), by Lemma 4.9, (C_n, A, B) is not a derivable single–valued neutrosophic graph. □

Theorem 4.11. *Let $C_n = (V, E, A, B)$ be a cycle single–valued neutrosophic graph. Then C_n is a derivable single–valued neutrosophic graph if and only if $n = 3$ and $n = 4$.*

Proof. If $n = 3$, then $C_3 \cong K_3$ and so by Theorem 4.7, C_3 is a derivable single–valued neutrosophic graph. If $n = 4$, then by Example 4.8, C_4 is a derivable single–valued neutrosophic graph. By Proposition 4.10, the converse, is obtained. □

Corollary 4.12. *Let $G = (V, E, A, B)$ be a single–valued neutrosophic non–complete graph in which has cycle. Then $G = (V, E, A, B)$ is a non–derivable single–valued neutrosophic graph if and only if*

(i) *$G \ncong C_3$ and $G \ncong C_4$;*
(ii) *it contains a single–valued neutrosophic subgraph that is homeomorphic to C_n, where $3, 4 \neq n$.*

Example 4.13. *Consider the cycle single–valued neutrosophic graph C_6 in Figure 11, where $\lambda_1 = (0.1, 0.3, 0.4), \lambda_2 = (0.2, 0.4, 0.5), \lambda_3 = (0.3, 0.5, 0.6), \lambda_4 = (0.4, 0.6, 0.7), \lambda_5 = (0.5, 0.7, 0.8)$ and $\lambda_6 = (0.1, 0.7, 0.8)$. By Proposition 4.10, C_6*

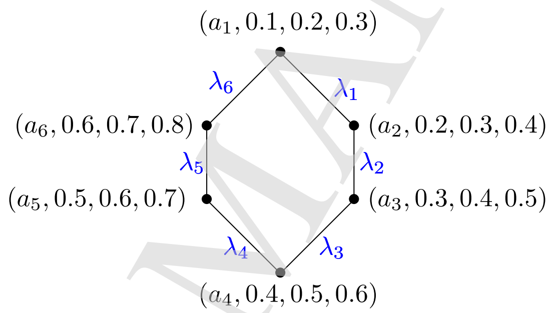

FIGURE 11. Derived cycle SVN–G C_6

is not a derivable single–valued neutrosophic graph. Now we add some edges to C_6 as Figure 12, where $\lambda_7 = (0.1, 0.5, 0.6), \lambda_8 = (0.2, 0.6, 0.7)$ and $\lambda_9 = (0.3, 0.7, 0.8)$. Hence now we consider the single–valued neutrosophic hypergraph $\overline{G} = (\overline{V}, \{\overline{E_j}\}_{j=1}^{n})$

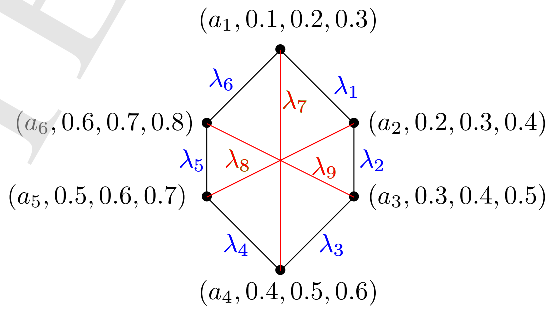

FIGURE 12. SVN–G $C_6^{\nearrow}$

in Figure 13. Clearly $\eta^*(a_1) = \{a_1\}, \eta^*(a_2) = \{a_2, b_1\}, \eta^*(a_3) = \{a_3\}, \eta^*(a_4) =$

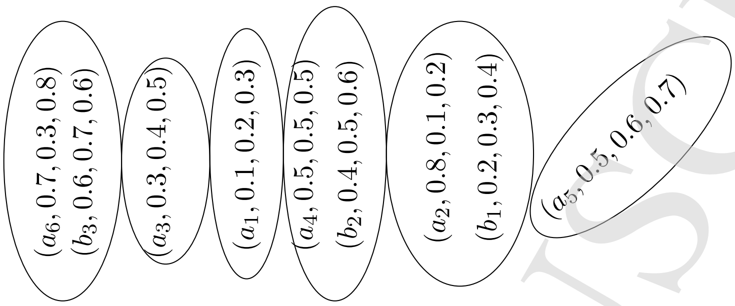

FIGURE 13. SVN–HG

$\{a_4, b_2\}, \eta^*(a_5) = \{a_5\}, \eta^*(a_6) = \{a_6, b_3\}$ *and it is easy to compute that* $H/\eta^* \cong C_6^{\nearrow}$.

In Example 4.13, we saw that C_6 is not a derivable single–valued neutrosophic graph, while we added some edges to this single–valued neutrosophic graph and converted to a derivable single–valued neutrosophic graph. Duo to this problem we will have the following definition.

Definition 4.14. *Let $G = (V, E, A, B)$ be a non derivable single–valued neutrosophic graph and $i \neq 0$. We will call non single–valued neutrosophic complete graph $G^{\nearrow} = (V, E^{\nearrow}, A^{\nearrow}, B^{\nearrow})$ is an extended derivable single–valued neutrosophic graph of G, if $E^{\nearrow}$ is obtained by adding the least number of edges to E such that $G^{\nearrow}$ be a derivable single–valued neutrosophic graph. Also we will say $G = (V, E, A, B)$ is an extended derivable single–valued neutrosophic graph.*

Example 4.15. *By Example 4.13, cycle single–valued neutrosophic graph C_6 is an extended derivable single–valued neutrosophic graph.*

Theorem 4.16. *(C_5, A, B) is not an extended derivable single–valued neutrosophic graph.*

Proof. Since $i \geq 1$, by Lemma 4.9, for any single–valued neutrosophic hypergraph $H = (\overline{V}, \overline{E})$, where $\overline{E} = \{E_1, E_2, E_3, E_4, E_5\}$, we get that H/η^* has a cycle of maximum length 4. By adding any edges to C_5, since $|\overline{V}|$ is odd again, duo to Lemma 4.9, we get that $|\overline{E}/\eta^*| = 6$, else all edges in H/η^* be connected that implies $i = 0$, which is a contradiction. $\square$

Corollary 4.17. *Let $k \in \mathbb{N}$. Then C_{2k+1} is not an extended derivable single–valued neutrosophic graph.*

Theorem 4.18. *Let $k \in \mathbb{N}$. Then*

(*i*) *$C_{2k} = (V, E^{\nearrow}, A^{\nearrow}, B^{\nearrow})$ is an extended derivable single–valued neutrosophic graph;*

(*ii*) $|E^{\nearrow}| = k^2$.

Proof. (i, ii) Let $V = V_1 \cup V_2$, where $V_1 = \{a_1, a_3, a_5, \ldots, a_{2k-1}\}$, $V_2 = \{a_2, a_4, a_6, \ldots, a_{2k}\}$ and for any $1 \leq j \leq 2k, e_j = \widehat{a_j, a_{j+1}}$. Now for any $j \in \{1, 3, 5, \ldots, a_{2k-1}\}$ consider E_j so that $a_j \in E_j$, $|E_j| = k_j$, $a_{j+1} \in E_{j+1}$ and $|E_{j+i}| = k_j + i$, where $k_j \in \mathbb{N}$. A simple computation shows that $H = (\overline{V}, \{E_s = \{(v_j, T_{E_s}(v_j), I_{E_s}(v_j), F_{E_s}(v_j))\}\}_{s=1})$ is a single–valued neutrosophic hypergraph, where $\overline{V} = V \cup W$ and W is any set so that $|W| = i(n/2)$. Moreover, by definition of η^*, we can see that $\eta^*(a_j) = E_j$ and $\eta^*(a_j) *_i \eta^*(a_{j+1}) = \eta^*\widehat{(a_j), \eta^*}(a_{j+1})$, whence $1 \leq j \leq 2k$. Since $|V_1| = |V_2| = n/2$, for any $j \neq 1, \eta^*(a_j) *_i \eta^*(a_{j+1}) = \eta^*\widehat{(a_j), \eta^*}(a_{j+1}), \eta^*(a_j) *_i \eta^*(a_{j-1}) = \eta^*\widehat{(a_j), \eta^*}(a_{j-1})$, $\eta^*(a_1) *_i \eta^*(a_2) = \eta^*\widehat{(a_1), \eta^*}(a_2)$ and $\eta^*(a_1) *_i \eta^*(a_{2k}) = \eta^*\widehat{(a_1), \eta^*}(a_{2k})$, we get that $|E^{\nearrow}| = (2k/2)((2k)/2 - 2) + 2k = k^2$. □

4.2. Derivable single–valued neutrosophic trees.

In this section, we study derivable regular single–valued neutrosophic trees and introduce Y–single–valued neutrosophic tree T_6 which is not a derivable single–valued neutrosophic tree.

Lemma 4.19. *Y–single–valued neutrosophic tree T_6 is not a derivable single–valued neutrosophic graph.*

Proof. Let $V = \{a_1, a_2, a_3, a_4, a_5, a_6\}$. Consider regular single–valued neutrosophic tree $T = (V, E, A, B)$ in Figure 14, where for any $1 \leq j \leq 6$ we have $0 \leq T_j + I_j + F_j \leq 3$, $\lambda_j = (T_{jj+1}, I_{jj+1}, F_{jj+1})$ and $0 \leq T_{jj+1} + I_{jj+1} + F_{jj+1} \leq 3$.

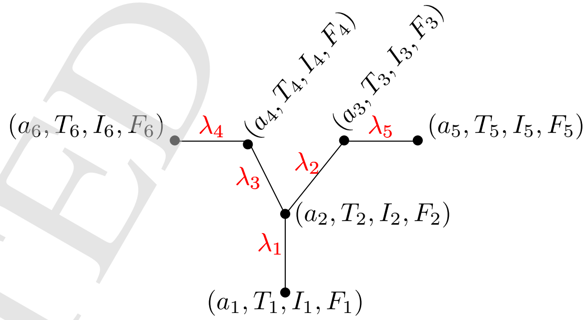

FIGURE 14. Y–SVN–T T_6

Let Y–single–valued neutrosophic tree T_6(Figure 14), be a derivable regular single–valued neutrosophic graph and $H = (\overline{V}, \{E_s = \{(v_j, T_{E_s}(v_j), I_{E_s}(v_j), F_{E_s}(v_j))\}\}_{s=1})$ be an associated single–valued neutrosophic hypergraph with regular single–valued neutrosophic graph Y–single–valued neutrosophic tree T_6. Since for any $1 \leq j \leq 4$, $\eta(a_2) *_i \eta(a_j) = \eta\widehat{(a_2), \eta}(a_j)$, $\eta(a_3) *_i \eta(a_5) = \eta\widehat{(a_3), \eta}(a_5)$ and $\eta(a_4) *_i \eta(a_6) = \eta\widehat{(a_4), \eta}(a_6)$, we get $k \in \mathbb{N}$, $E_1, E_2, E_3, E_4 \subseteq H$ so that $a_1 \in E_1, |E_1| = k, a_2 \in$

$E_2, |E_2| = k + i, a_3, a_4 \in E_3$, $|E_3| = k + 2i$ $a_5, a_6 \in E_4$ and $|E_4| = k + 3i$. It follows that, $\eta(a_3) *_i \eta(a_6) = \widehat{\eta(a_3), \eta(a_6)}$ and $\eta(a_4) *_i \eta(a_5) = \widehat{\eta(a_4), \eta(a_5)}$, which is a contradiction. Therefore Y–single–valued neutrosophic tree T_5 can not be a derivable single–valued neutrosophic graph. □

Theorem 4.20. *Let $T = (V, E, A, B)$ be a regular single–valued neutrosophic tree and for any $v \in V, deg(v) \leq 2$. Then T is a derivable single–valued neutrosophic graph.*

Proof. Let $i \in \mathbb{N}, G = (V, E, A, B, *')$ be a regular single–valued neutrosophic graph with no cycle, in such a way $V = \{a_1, a_2, \ldots, a_n\}$ and $E = \{e_1, e_2, \ldots, e_m\}$, where $m \leq n$. Suppose that for any $1 \leq j \neq j' \leq m$, $e_{jj'} = \{a_j, a_{j'}\} = a_j *' a_{j'}$. Define a single–valued neutrosophic hypergraph $\overline{G} = (\overline{V}, \{\overline{E_s}\}_{s=1}^{n})$ as follows:

$$\overline{E_s} = \{(a_j, T_{E_s}(a_j), I_{E_s}(a_j), F_{E_s}(a_j))\} \cup C_j = \{(a'_j, T_{C_s}(a'_j), I_{C_s}(a'_j), F_{C_s}(a'_j))\}$$

such that $|C_1| = 1$, for any $1 < l \leq n$, $|C_{l+1}| - |C_l| = i$, for any $1 \leq j, j' \leq n$ we have $C_j \cap C_{j+1} = \emptyset$ and $T_A(v) = \bigwedge(T_{E_s}(v) \wedge T_{C_s}(v)), I_A(v) = \bigvee(I_{E_s}(v) \vee I_{C_s}(v))$ and $F_A(v) = \bigvee(F_{E_s}(v) \vee F_{C_s}(v))$. It is easy to see that for any $1 < j, j' \leq n$, $|\overline{E_{j+1}}| - |\overline{E_j}| = i$ and $\overline{E_j} \cap \overline{E_{j'}} = \emptyset$. A simple computation shows that $\overline{V} = \bigcup_{j=1}^{n}(C_j \cup V)$ and $(\overline{V}, \{\overline{E_j}\}_{j=1}^{n})$ is a single–valued neutrosophic hypergraph. Clearly for any $1 \leq j \leq n, \eta((a_j, T_{E_s}(a_j), I_{E_s}(a_j), F_{E_s}(a_j))) = \overline{E_j}$ and since $\overline{E_j} \cap \overline{E_{j'}} = \emptyset$, we get that $G/\eta = \{\eta(a_j) = \eta((a_j, T_{E_s}(a_j), I_{E_s}(a_j), F_{E_s}(a_j))) \mid 1 \leq j \leq n\}$ and so for any $i \in \mathbb{N}^*$ obtain

$$\eta(a_r) *_i \eta(a_s) = \begin{cases} \widehat{\eta(a_r), \eta(a_s)} & \text{if } |r - s| = i, \\ \widehat{\emptyset} & \text{if } |r - s| \neq i. \end{cases}$$

Now, define map $\varphi : (\overline{G}/\eta, *i) \longrightarrow G = (V, E)$ by $\varphi(\eta(a_j)) = a_j$ and $\varphi((\widehat{\eta(a_j), \eta(a_{j'})}))$ $= e_{jj'}$. Let $a_j, a_{j'} \in \overline{V}$. If $\eta(a_j) = \eta(a_{j'})$, then $|\overline{E_j}| = |\overline{E_{j'}}|$ and so $\overline{E_j} = \overline{E_{j'}}$. Thus $\varphi(\eta(a_j)) = \varphi(\eta(a_{j'}))$. Since for any $1 \leq j \neq j' \leq n$,

$$\varphi(\eta(a_j) *_i \eta(a_{j'})) = \varphi((\widehat{\eta(a_j), \eta(a_{j'})})) = e_{jj'} = a_j *' a_{j'} = \varphi(\eta(a_j)) *' \varphi(\eta(a_{j'})),$$

in other words, if $\eta(a_j)$ and $\eta(a_{j'})$ in G/η are adjacent, then $\varphi(\eta(a_j))$ and $\varphi(\eta(a_{j'}))$ in G are adjacent. So φ is a homomorphism. It is easy to see that φ is bijection and so is an isomorphism. It follows that any single–valued neutrosophic graph is a derivable single–valued neutrosophic graph. □

Corollary 4.21. *Let $n \leq 5$. Then T_n is a derivable single–valued neutrosophic graph.*

Example 4.22. *Let $V = \{a_1, a_2, a_3, a_4\}$, $E = \{e_1, e_2, e_3\}$. Consider single–valued neutrosophic tree T_4 in Figure 15:*

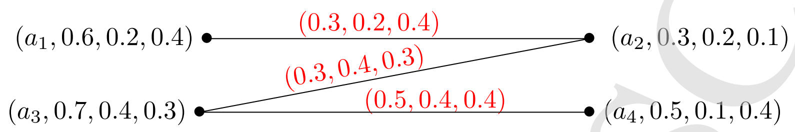

FIGURE 15. SVN–T T_4

For any arbitrary set $B = \bigcup_{i=1}^{10} b_i$ *where for any* j, j' *have* $a_j \neq b'_j$, *define a single–valued neutrosophic hypergraph* $\overline{G} = (\overline{V}, \{\overline{E_j}\})$ *in a way that*

$\overline{E_1} = \{(a_1, 0.9, 0.1, 0.2)\} \cup \{(b_1, 0.6, 0.2, 0.4)\}, \overline{E_2} = \{(a_2, 0.8, 0.1, 0.1)\} \cup$
$\{(b_2, 0.3, 0.1, 0.1), (b_3, 0.9, 0.2, 0.1)\}, \overline{E_3} = \{(a_3, 0.7, 0.2, 0.1)\} \cup$
$\{(b_4, 0.8, 0.1, 0.1), (b_5, 0.7, 0.1, 0.3), (b_6, 0.9, 0.4, 0.2)\}$ *and* $\overline{E_4} = \{(a_4, 0.5, 0.1, 0.2)\}$
$\cup\{(b_7, 0.8, 0.1, 0.3), (b_8, 0.9, 0.1, 0.1), (b_9, 1, 0.1, 0.2), (b_{10}, 0.9, 0.1, 0.4)\}$.

Hence consider the single–valued neutrosophic hypergraph $\overline{G} = (\overline{V}, \{\overline{E_i}\}_{i=1}^{n})$ *in Figure 16. By Theorem 4.20, for* $*_i = *_1$ *and any* $1 \leq j \leq 4, \eta(a_j) = \overline{E_j}$, *so*

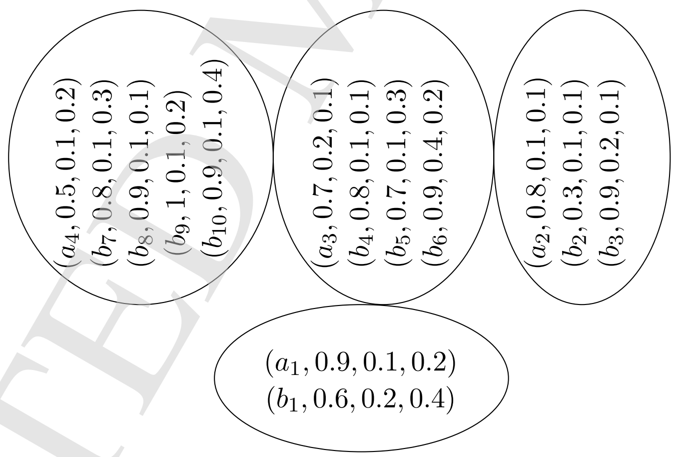

FIGURE 16. SVN–HG

$$\overline{G}/\eta = (\{\eta(a_1), \eta(a_2), \eta(a_3), \eta(a_4)\}, \{\{\eta(a_1), \eta(a_2)\}, \{\eta(a_2), \eta(a_3)\}, \{\eta(a_3), \eta(a_4)\}\})$$

is a single–valued neutrosophic tree in Figure 17. It is easy to see that $T_4 \cong \overline{G}/\eta$ *and so* $T_4 = (V, E, A, B)$ *is a derivable single–valued neutrosophic graph. Moreover, for any* $2 \leq i \in \mathbb{N}$, *we can construct another associated single–valued neutrosophic hypergraphs of single–valued neutrosophic graph* G.

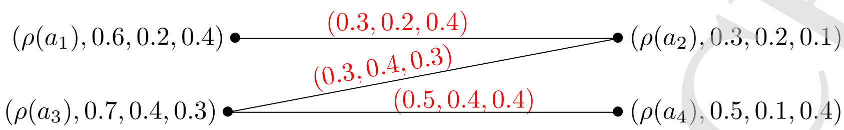

FIGURE 17. Derived SVN–G for $i = 1$

Theorem 4.23. *Let $T_n = (V, E, A, B)$ be a regular single–valued neutrosophic tree which is not contain a single–valued neutrosophic subtree, that is homeomorphic to Y–single–valued neutrosophic tree T_6 and $n \geq 6$. Then T_n is a derivable single–valued neutrosophic tree.*

Proof. Let $V = \{a_1, a_2, \ldots, a_n\}$. We rearrange the regular single–valued neutrosophic tree T by $n - 1 \geq d_n \geq \ldots \geq d_3 \geq d_2 \geq d_1$, where $d_i = deg(a_i)$. Since $deg(a_1) = d_1$, we get that $a_{j1}, a_{j2}, \ldots, a_{jd_1-1}, E_1, E_2$ so that $a_1 \in E_1, a_{j1}, a_{j2}, \ldots, a_{jd_1-1} \in E_2$ and $\big||E_1| - |E_2|\big| = i$. By Lemma 4.19, for any $1 \leq j' \leq d_1 - 1$, $deg(a_{jj'}) = 1$ and by rearrangement $a_{jd_1} = a_2$. If $deg(a_{jd_1}) = 1$ the proof is obtained. If $deg(a_{jd_1}) > 1$ in a similar way, there exist $a_{r1}, a_{r2}, \ldots, a_{jd_2-1}, E_3, E_4$ so that $a_2 \in E_3, a_{r1}, a_{r2}, \ldots, a_{rd_1-1} \in E_4$ and $\big||E_3| - |E_4|\big| = i$. By Lemma 4.19, for any $1 \leq r' \leq d_2 - 1$, $deg(a_{rr'}) = 1$ and by rearrangement $a_{rd_2} = a_3$. If $deg(a_{rd_2}) = 1$ the proof is obtained. If $deg(a_{rd_2}) > 1$ we can continue. Since $|V| < \infty$, then this process stops. A simple computation shows that $H = (V \cup V', \{E_s = \{(a_j, T_{E_s}(a_j), I_{E_s}(a_j), F_{E_s}(a_j))\}\}_{s=1})$ is a single–valued neutrosophic hypergraph, where for any $1 \leq j \leq n$, $(\eta^*(a_j), T_{\eta^*(a_j)}, I_{\eta^*(a_j)}, F_{\eta^*(a_j)})$ and $(a_j, T_{a_j}, I_{a_j}, F_{a_j})$ $T_{\eta^*(a_j)} = T_{a_j}, I_{\eta^*(a_j)} = I_{a_j}$ and $F_{\eta^*(a_j)} = F_{a_j}$. By Theorem 4.20, $H/\eta^* \cong (T_n, A, B)$. □

Corollary 4.24. *Let $T = (V, E, A, B)$ be a regular single–valued neutrosophic tree. Then $T = (V, E, A, B)$ is a non–derivable single–valued neutrosophic graph if and only if it contains a single–valued neutrosophic subtree that is homeomorphic to Y–single–valued neutrosophic tree (T_6, A, B).*

Example 4.25. *Consider the single–valued neutrosophic tree $T5 = (V, E, A, B)$ in Figure 18. Now construct the single–valued neutrosophic hypergraph H in figure 19. By Theorem 4.20, for $*_i = *_1$, we have*

$$\eta^*(a, 0.4, 0.5, 0.6) = \{(a, 0.4, 0.5, 0.6)\}, \eta^*(b, 0.1, 0.2, 0.3) = \{(b, 0.1, 0.2, 0.3), (b_1, 0.1, 0.2, 0.3)\}, \eta^*(c, 0.7, 0.8, 0.9) = \{(c, 0.7, 0.8, 0.9), (b_2, 0.7, 0.8, 0.9)\}, \eta^*(d, 0.2, 0.4, 0.6) = \{(d, 0.2, 0.1, 0.3), (b_3, 0.2, 0.1, 0.3)\} \text{ and } \eta^*(e, 0.3, 0.5, 0.7) = \{(e, 0.4, 0.5, 0.6), (b_4, 0.4, 0.5, 0.6)\}.$$

So $H/\eta^* \cong (T_5, V, E, A, B)$ and so (T_5, V, E, A, B) is a derivable single–valued neutrosophic tree.

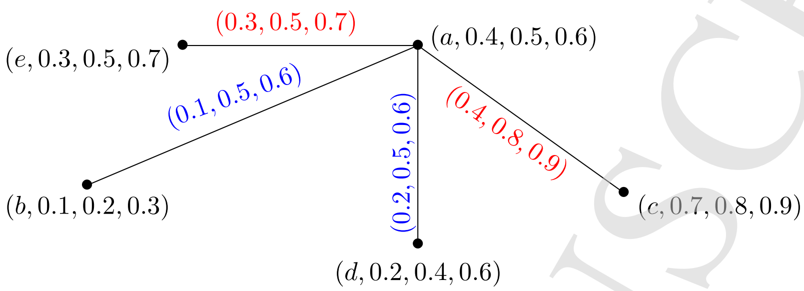

FIGURE 18. SVN–T T_5

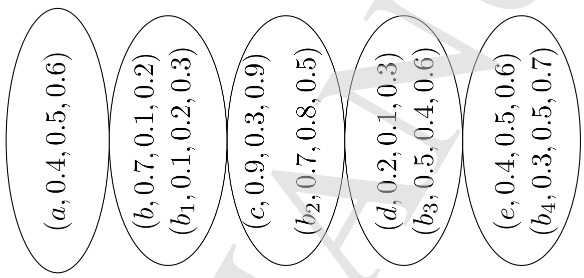

FIGURE 19. SVN–HG

Theorem 4.26. *Let $T = (V, E, A, B)$ be a single–valued neutrosophic tree and $i \in \mathbb{N}^*$. If $H = (V, \{E_j\}_j)$ is a single–valued neutrosophic hypergraph of single–valued neutrosophic tree T, then $\Delta(V/\eta^*) = |E| - i$.*

Proof. Let $V = \{a_1, a_2, \ldots, a_n\}$, for any $1 \leq j \leq n, deg(a_j) = d_j$ and $\Delta(G) = d_p$, where $1 \leq p \leq n$. Set $E_1 = \{a_p\}, E_2 = \{a_j \mid j \neq p$ and $\widehat{a_j, a_p} \neq \widehat{\emptyset}\}$ so that $|E_2| = i + 1, E_k = \{a_k \mid a_k \notin E_1 \cup E_2\}$ and $|E_k| = 1$. A simple calculation shows that $H = (V, \{E_i\}_i)$ is a single–valued neutrosophic hypergraph. Since $|V| = n$, we get $\big|\{E_j;\ |E_j| = 1\}\big| = n - (i + 1)$ and so single–valued neutrosophic hypergraph $H = (V, \{E_i\}_i)$ has $(n - i)$ hyperedges. It is easy to see that

$$V/\eta^* = \{\eta^*(a_p), \eta^*(a_k), \eta^*(a_{k'}) \mid a_k \in E_k, a_{k'} \notin E_1 \cup E_2\},$$

$|\eta^*(a_p)| = 1$, for any $a_k \in E_k$ we have $|\eta^*(a_k)| = i + 1$ and for any $a_{k'} \notin E_1 \cup E_2, |\eta^*(a_{k'})| = 1$. So for any $r \neq r'$,

$$\eta(a_r) *_i \eta(a_{r'}) = \begin{cases} \widehat{\emptyset} & \text{if } \{a_r, a_{r'}\} \subseteq E_1 \cup E_k \cup E_{k'}, \\ \widehat{\eta(a_r), \eta(a_{r'})} & \text{otherwise.} \end{cases}$$

Since $|V| = n$ and T is a single–valued neutrosophic tree, we get that $|E| = n - 1$ and so $\Delta(V/\eta^*) = n - i - 1 = |E| - i$. $\square$

Example 4.27. *Let $V = \{1, 2, 3, 4, 5, 6, 7\}$. Consider single–valued neutrosophic tree $T = (V, E, A, B)$ in Figure 20. Since $\Delta(T) = 4$, we obtain $*_i = *_3$ and the*

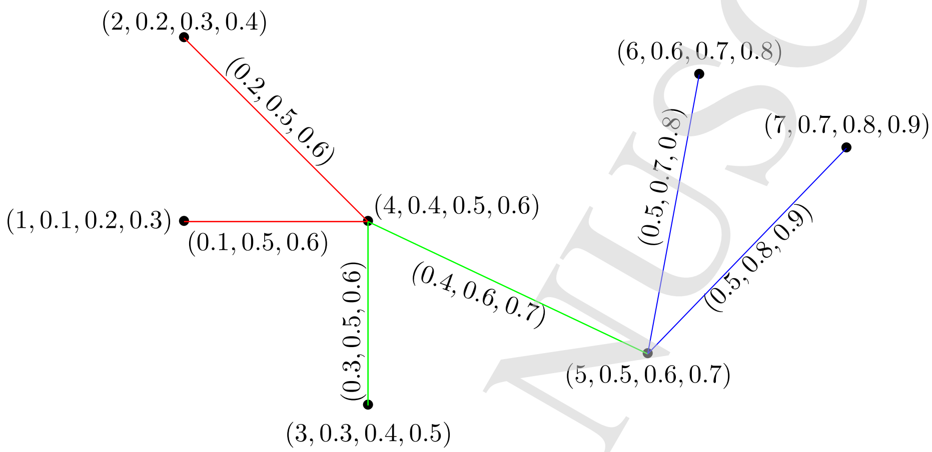

FIGURE 20. SVN–T T

single–valued neutrosophic hypergraph in Figure 21. Hence V/η^* is a single–valued

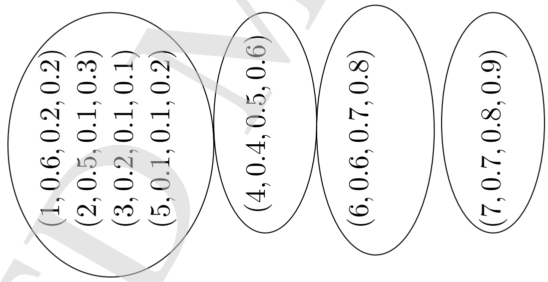

FIGURE 21. SVN–HG

neutrosophic tree with 4 vertices and 3 edges such that $\Delta(V/\eta^*) = 3$ (Figure 22).

Corollary 4.28. *Let $n \in \mathbb{N}$.*

(*i*) *If $n \geq 2$, then any single–valued neutrosophic tree T_n is not a self derivable single–valued neutrosophic graph;*

(*ii*) *C_n is not a self derivable single–valued neutrosophic graph.*

Proof. (*i*) Let $|V| = n$, $T_n = (V, E)$ and T_n be a self derivable single–valued neutrosophic graph. By Theorem 4.26, T_n is a self derivable single–valued neutrosophic

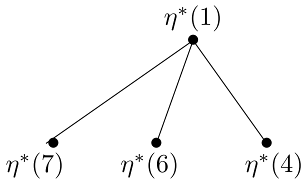

FIGURE 22. Derived SVN–G G/η^* for $i = 3$

graph if and only if $|E| - i = |E|$ if and only if $i = 0$. By Theorem 4.7, must $T_n \cong K_n$ where is a contradiction.

(ii) Since C_n has cycle, by Theorem 4.7, is clear. □

4.3. Derivable single–valued neutrosophic complete graphs.

In this section, we consider regular single–valued neutrosophic complete graphs, regular single–valued neutrosophic complete bigraphs and we investigate their associated single–valued neutrosophic hypergraphs.

Theorem 4.29. *Let $G = (V, E, A, B)$ be a derivable single–valued neutrosophic complete graph via associated single–valued neutrosophic hypergraph H and $|V| = n$. Then*

- (i) *if H is a discrete complete single–valued neutrosophic hypergraph, where their hyperedges are $2 \leq k$-hyperedge, then $*_i = *_0$;*
- (ii) *if H is a discrete complete single–valued neutrosophic hypergraph, where their hyperedges are $2 \leq k$-hyperedge, then $|H| = kn$.*

Proof. Clearly $H/\eta \cong G$.

(i) Since H/η is a single–valued neutrosophic complete graph, we get that for any $\eta(x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x)), \eta(y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y)) \in H/\eta$,

$$\begin{aligned}&\eta(x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x)) *_i \eta(y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y))\\ &= \eta(x, T_{E_i}(x), I_{E_i}(x), F_{E_i}\widehat{(x)), \eta(}y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y))\end{aligned}$$

and so $\big||\eta(x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x))| - |\eta(y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y))|\big| = i$. On the other hand, H is a discrete complete single–valued neutrosophic hypergraph, where $2 \leq k$-hyperedge, then for any $\eta(x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x)), \eta(y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y)) \in H/\eta$, $|\eta(x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x))| = |\eta(y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y))| = k$. It follows that $i = \big||\eta(x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x))| - |\eta(y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y))|\big| = |k - k| = 0$.

(ii) Since for any $\eta(x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x)), \eta(y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y)) \in H/\eta$, $|\eta(x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x))| = |\eta(y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y))| = k$ and $n = |H/\eta| = |H|/|\eta|$, we get that $|H| = n|\eta| = nk$. □

Theorem 4.30. *Let $H = (\overline{V}, \{E_i = \{(a_j, T_{E_i}(a_j), I_{E_i}(a_j), F_{E_i}(a_j))\}\}_{i=1}^m)$ be a discrete complete single–valued neutrosophic hypergraph, where $2 \leq r$-hyperedge and $|H| = rn$. Then*

(i) *if $*_i = *_0$, then its derivable single–valued neutrosophic graph is isomorphic to regular single–valued neutrosophic complete graph $K_{n/r}$;*

(ii) *if for $n \in \mathbb{N}$ $*_i = *_n$, then its derivable single–valued neutrosophic graph is isomorphic to single–valued neutrosophic complete graph $\overline{K}_{n/r}$.*

Proof. Since H is a discrete complete single–valued neutrosophic hypergraph, where $2 \leq r$-hyperedge, then for all $\eta(x) = \eta(x, T_{E_i}(x), I_{E_i}(x), F_{E_i}(x)), \eta(y) = \eta(y, T_{E_i}(y), I_{E_i}(y), F_{E_i}(y)) \in H/\eta, |\eta(x)| = |\eta(y)| = r$.

(i) If $*_i = *_0$, then for any $\eta(x), \eta(y) \in H/\eta, \eta(x) *_0 \eta(y) = \eta\widehat{(x), \eta}(y)$. Since $|H| = n$ we get that $|H/\eta| = n/r$.

(ii) If for any $n \in \mathbb{N}$, $*_i = *_n$, then for any $\eta(x), \eta(y) \in H/\eta, \eta(x) *_n \eta(y) = \widehat{\emptyset}$. Hence H/η is null single–valued neutrosophic graph and $|H| = n$ implies that $|H/\eta| = n/r$. □

Theorem 4.31. *Let $G = (V, E, A, B)$ be a derivable regular single–valued neutrosophic graph by $*_i = *_0$ and $|V| = n$. Then*

(i) *if $G = (V, E, A, B)$ is a connected single–valued neutrosophic graph, then there exists $n \in \mathbb{N}^*$ so that $G \cong K_n$,*

(ii) *if $G = (V, E, A, B)$ is a non–connected single–valued neutrosophic graph, then all connected components of G as G' are isomorphic to K_k or $\overline{K}_k$ whence, $1 \leq k \leq n \in \mathbb{N}^*$.*

Proof. Since $G = (V, E, A, B)$ is a derivable regular single–valued neutrosophic graph, then we have a single–valued neutrosophic hypergraph

$$H = (\overline{V}, \{E_i = \{(a_j, T_{E_i}(a_j), I_{E_i}(a_j), F_{E_i}(a_j))\}\}_{i=1}^m)$$

as associated single–valued neutrosophic hypergraph to G, where $m \geq n$, $T_{\eta(E_i)}(\eta(x)) = T_A(x)$, $I_{\eta(E_i)}(\eta(x)) = I_A(x)$ and $F_{\eta(E_i)}(\eta(x)) = F_A(x)$.

(i) Since $G = (V, E, A, B)$ is a connected derivable regular single–valued neutrosophic graph, $|V| = n$ and $*_i = *_0$, we get that there exist $a_1, a_2, \ldots, a_n \in H$ so that for any $1 \leq j, j' \leq n, \eta(a_j) *_i \eta(a_{j'}) = \eta\widehat{(a_j), \eta}(a_{j'})$.

(ii) Since $G = (V, E, A, B)$ is a non–connected derivable single–valued neutrosophic graph, then there exist $a_1, \ldots, a_n \in H$ such that for any $1 \leq j, j' \leq n, |\eta(a_j)| \neq |\eta(a_{j'})|$ or $|\eta(a_j)| = |\eta(a_{j'})|$. Let $1 \leq l \leq m$ and $\mathcal{P}_l = \{\eta(a_j) \mid |\eta(a_j)| = l\}$, then for any $1 \leq l \leq m$, it is easy to see that $\sum\limits_{l=1}^{m}(l.|\mathcal{P}_l|) = m$. If G' be a connected components of G, then there exists $1 \leq l \leq m$ so that $G' \cong K_{|\mathcal{P}_l|}$.

□

Theorem 4.32. *Let $m, n \in \mathbb{N}$. Then for any $1 \leq i$ there exists a partitioned single–valued neutrosophic hypergraph H such that $H/\eta \cong K_{m,n}$, where $K_{m,n}$ is a regular single–valued neutrosophic complete bigraph.*

Proof. Let $1 \leq i$. Then we consider $t \in \mathbb{N}$ and a partitioned single–valued neutrosophic hypergraph $H = (\overline{V}, \{E_j\}_j)$, where has hyperedges $E_1, E_2, \ldots, E_m$ that for any $1 \leq j \leq m, |E_j| = t$ and hyperedges $E_{m+1}, E_{m+2}, \ldots, E_{n+m}$ that for any $m+1 \leq j \leq n, |E_j| = t+i$. Let for any $1 \leq j \leq m$,

$$E_j = \{(a_j^1, T_{E_i}(a_j^1), I_{E_i}(a_j^1), F_{E_i}(a_j^1)), (a_j^2, T_{E_i}(a_j^2), I_{E_i}(a_j^2), F_{E_i}(a_j^1)),$$
$$\ldots, (a_j^t, T_{E_i}(a_j^t), I_{E_i}(a_j^t), F_{E_i}(a_j^t))\}$$

and for any $m+1 \leq j \leq n+m$,

$$E_j = \{(a_j^1, T_{E_i}(a_j^1), I_{E_i}(a_j^1), F_{E_i}(a_j^1)), (a_j^2, T_{E_i}(a_j^2), I_{E_i}(a_j^2), F_{E_i}(a_j^2)),$$
$$\ldots, (a_j^1, T_{E_i}(a_j^{t+i}), I_{E_i}(a_j^{t+i}), F_{E_i}(a_j^{t+i}))\}.$$

A simple calculation shows that for any $1 \leq j \leq m$, for any $1 \leq j' \leq t$,

$$\eta((a_j^{j'}, T_{E_i}(a_j^{j'}), I_{E_i}(a_j^{j'}), F_{E_i}(a_j^{j'}))) = \{(a_j^1, T_{E_i}(a_j^1), I_{E_i}(a_j^1), F_{E_i}(a_j^1)),$$
$$(a_j^2, T_{E_i}(a_j^2), I_{E_i}(a_j^2), F_{E_i}(a_j^2)), \ldots, (a_j^t, T_{E_i}(a_j^t), I_{E_i}(a_j^t), F_{E_i}(a_j^t))\}$$

and for any $m+1 \leq l \leq n+m$, for any $1 \leq l' \leq t+i$,

$$\eta((a_l^{l'}, T_{E_i}(a_l^{l'}), I_{E_i}(a_l^{l'}), F_{E_i}(a_l^{l'}))) = \{(a_l^1, T_{E_i}(a_l^1), I_{E_i}(a_l^1), F_{E_i}(a_l^1)),$$
$$(a_l^2, T_{E_i}(a_l^2), I_{E_i}(a_l^2), F_{E_i}(a_l^2)), \ldots, (a_l^{t+i}, T_{E_i}(a_l^{t+i}), I_{E_i}(a_l^{t+i}), F_{E_i}(a_l^{t+i}))\}.$$

So

$$H/\eta = \{\eta((a_r^1, T_{E_i}(a_r^1), I_{E_i}(a_r^1), F_{E_i}(a_r^1))), \eta((a_s^1, T_{E_i}(a_s^1),$$
$$I_{E_i}(a_s^1), F_{E_i}(a_s^1))) \mid 1 \leq r \leq m \text{ and } m+1 \leq s \leq n+m\},$$

whence for any $1 \leq r \leq m, |\eta((a_r^1, T_{E_i}(a_r^1), I_{E_i}(a_r^1), F_{E_i}(a_r^1)))| = t$ and for any $m+1 \leq s \leq n+m, |\eta((a_s^1, T_{E_i}(a_s^1), I_{E_i}(a_s^1), F_{E_i}(a_s^1)))| = t+i$. Thus for any $1 \leq r \leq m$, and for any $m+1 \leq s \leq n+m$ we have

$$\eta((a_r^1, T_{E_i}(a_r^1), I_{E_i}(a_r^1), F_{E_i}(a_r^1))) *_i \eta((a_s^1, T_{E_i}(a_s^1), I_{E_i}(a_s^1), F_{E_i}(a_s^1)))$$
$$= \eta((a_r^1, T_{E_i}(a_r^1), I_{E_i}(a_r^1), F_{E_i}(a_r^1\widehat{))), \eta(}(a_s^1, T_{E_i}(a_s^1), I_{E_i}(a_s^1), F_{E_i}(a_s^1))),$$

for any $1 \leq r \leq m$,

$$\eta((a_r^1, T_{E_i}(a_r^1), I_{E_i}(a_r^1), F_{E_i}(a_r^1))) *_i \eta((a_s^1, T_{E_i}(a_s^1), I_{E_i}(a_s^1), F_{E_i}(a_s^1))) = \widehat{\emptyset}$$

and for any $m+1 \leq s \leq n+m$,

$$\eta((a_s^1, T_{E_i}(a_s^1), I_{E_i}(a_s^1), F_{E_i}(a_s^1))) *_i \eta((a_s^1, T_{E_i}(a_s^1), I_{E_i}(a_s^1), F_{E_i}(a_s^1))) = \widehat{\emptyset}.$$

Therefore $H/\eta \cong (K_{m,n}, A, B)$. □

5. Wireless sensor (hyper)networks and achievable single–valued neutrosophic (hyper)graphs

In this section, we describe some applications of achievable single–valued neutrosophic graphs.

The study of complex networks plays a main role in the important area of multidisciplinary research involving physics, chemistry, biology, social sciences, and information sciences. These systems are commonly represented by means of simple or directed graphs that consist of sets of nodes representing the objects under investigation, e.g., people or groups of people, molecular entities, computers, etc., joined together in pairs by links if the corresponding nodes are related by some kind of relationship. These networks include the internet, the world wide web, social networks, information networks, neural networks, food webs, and protein–protein interaction networks. In some cases, the use of simple or directed graphs to represent complex networks do not provide a complete description of the real–world systems under investigation. For instance, WSN is undergoing intensive research to overcome its complexity and constraint challenges in terms of storage resources, computational capabilities, communication bandwidth, and more importantly, power supply [8]. The main components of a sensor node and its associated energy consumption are discussed. Typically, sensor nodes are grouped hierarchically in clusters (sections), and each cluster has some nodes that acts as the cluster head (CH). All the nodes forward their sensor data to the CH, which in turn aggregates data reports and routes them to a specialized node called the sink node or base station (BS). A natural way of representing these systems is to use hypergraphs. Hyper–edges in hypergraph can relate groups of more than two nodes. Thus, we can represent the collaboration network as a hypergraph in which nodes represent authors and hyper–edges represent the groups of authors who have published papers together. Despite the fact that complex weighted networks have been covered in some detail in the physical literature, there are no reports on the use of hypergraphs to represent complex systems. Consequently, we will formally introduce the hypergraph concept as a generalization for representing complex networks and will call them complex hyper–networks . The hypergraph concept includes, as particular cases, a wide variety of other mathematical structures that are appropriate for the study of complex networks. Since these representations still are unsuccessful to deal with all the competitions of world, for that purpose SVN–HG are introduced. Now, we discuss applications of SVN–HG for studying the competition along with algorithms. The SVN–G has many utilizations in different areas, where we by using the especial equivalence relations connect SVN–G and SVN–HG. We will show some examples of complex systems for which hypergraph representation is necessary.

Example 5.1. *Let* $X = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o\}$ *be a WSN and* a, b, c, d, e $, f, g, h, i, j, k, l, m, n, o$ *be nodes of its. These nodes create some groups as* $E_1 =$

TABLE 1. DRE, DSRA and DR of WS–HN

Node	*Truth–membership*	*Indeterminacy–membership*	*Falsity–membership*
a	0.6	0.2	0.3
b	0.9	0.2	0.4
c	1	0.2	0.2
d	1	0.2	0.3
e	1	0.2	0.5
f	0.8	0.3	0.2
g	0.9	0.2	0.2
h	0.8	0.2	0.4
i	1	0.5	0.3
j	0.9	0.2	0.2
k	0.4	0.2	0.2
l	1	0.3	0.2
m	1	0.2	0.3
n	0.7	0.3	0.5
o	0.4	1	1

$\{a, b, c, d, e\}, E_2 = \{f, g, h, i\}, E_3 = \{j, k, l\}, E_4 = \{m, n\}$ *and* $E_5 = \{o\}$. *Let, the degree of remaining energy* (DRE) *in the WSN of* a *is* $60/100$, *degree of sensor remaining alive*($DSRA$) *is* $20/100$ *and degree of reliability* (DR) *is* $30/100$, *i.e. the truth–membership, indeterminacy–membership and falsity–membership values of the vertex human is* $(0.6, 0.2, 0.3)$. *The degree of remaining energy, degree of sensor remaining alive and degree of reliability of WSN is shown in the Table 1.*

Consider the clustering for wireless sensor hypernetwork $H = (\{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o\}, \{E_1, E_2, E_3, E_4, E_5\})$ *in Figure 23. Clearly*

$$\eta^*((a, 0.6, 0.2, 0.3)) = \{(a, 0.6, 0.2, 0.5), (b, 0.6, 0.2, 0.5), (c, 0.6, 0.2, 0.5),$$
$$(d, 0.6, 0.2, 0.5), (e, 0.6, 0.2, 0.5)\}, \eta^*((f, 0.8, 0.3, 0.2)) = \{(f, 0.8, 0.5, 0.4),$$
$$(g, 0.8, 0.5, 0.4), (h, 0.8, 0.5, 0.4), (i, 0.8, 0.5, 0.4)\}$$
$$\eta^*((j, 0.9, 0.2, 0.2)) = \{(j, 0.4, 0.3, 0.2), (k, 0.4, 0.3, 0.2), (l, 0.4, 0.3, 0.2)\},$$
$$\eta^*((m, 1, 0.2, 0.3)) = \{(m, 0.7, 0.3, 0.5), (n, 0.7, 0.3, 0.5)\} \text{ and}$$
$$\eta^*((o, 0.4, 1, 1)) = \{(o, 0.4, 1, 1)\}.$$

So we obtained the single–valued neutrosophic graph in Figure 24, where $\lambda_1 = (0.6, 0.5, 0.5), \lambda_2 = (0.4, 0.5, 0.4), \lambda_3 = (0.4, 0.3, 0.5)$ *and* $\lambda_4 = (0.4, 1, 1)$. *By Figure 24, for society* X, *we have* 5 *representatives* $\eta^*(a), \eta^*(f), \eta^*(j), \eta^*(m)$ *and* $\eta^*(o)$ *where the likeness, indeterminacy and dislikeness of contribution in the business relationships of group of this society is equal share and is shown in the Table 2.*

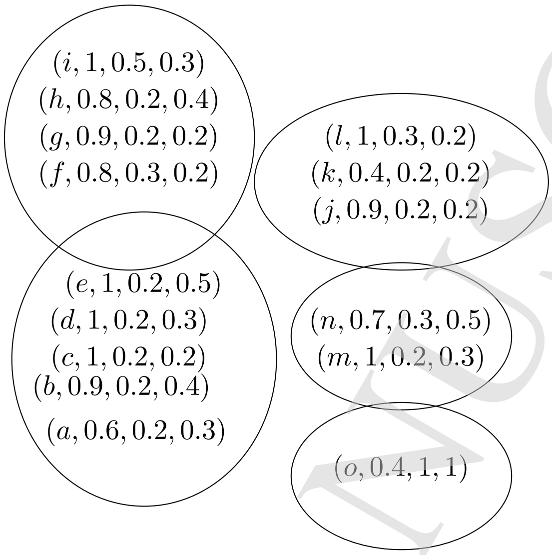

FIGURE 23. Clustering for wireless sensor hypernetwork (WS–HN)

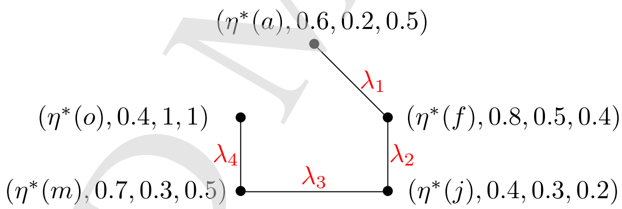

FIGURE 24. Clustering of wireless sensor network(WSN)

TABLE 2. DRE, DSRA and DR of WSN

Cluster head	*Cluster head*	*Truth*	*Indeterminacy*	*Falsity*
$\eta^*(a)$	$\eta^*(f)$	0.6	0.5	0.4
$\eta^*(f)$	$\eta^*(j)$	0.4	0.5	0.4
$\eta^*(j)$	$\eta^*(m)$	0.4	0.3	0.5
$\eta^*(m)$	$\eta^*(o)$	0.4	1	1

Example 5.2. *Let $X = \{a, b, c, d, e, f, g, h\}$ be a WSN and a, b, c, d, e, f, g, h be nodes. These nodes create some groups as $E_1 = \{a, b\}, E_2 = \{c, d\}, E_3 = \{e\}, E_4 = \{f\}$ and $E_5 = \{g, h\}$. Let, the degree of stability period and network lifetime*

TABLE 3. DSPNL, DECT and DCT of WS–HN

Node	*Truth–membership*	*Indeterminacy–membership*	*Falsity–membership*
a	*0.7*	*0*	*0.1*
b	*0.8*	*0.2*	*0.3*
c	*0.4*	*0.4*	*0.4*
d	*0.3*	*0.4*	*0.5*
e	*0*	*0.1*	*0.2*
f	*0.2*	*0.3*	*0.4*
g	*0.5*	*0.5*	*0.5*
h	*0.6*	*0.7*	*0.7*

(DSPNL) of d is $30/100$, *degree of energy consumption and throughput (DECT) is* $40/100$ *and degree of computational time (DCT) is* $50/100$, *i.e. the truth–membership, indeterminacy–membership and falsity–membership values of this node is* $(0.3, 0.4, 0.5)$. *The degree of stability period and network lifetime, degree of energy consumption and throughput and computational time this WSN is shown in the Table 3.*

Consider the energy of wireless sensor hyper–network is illustrated in Figure 25. Since

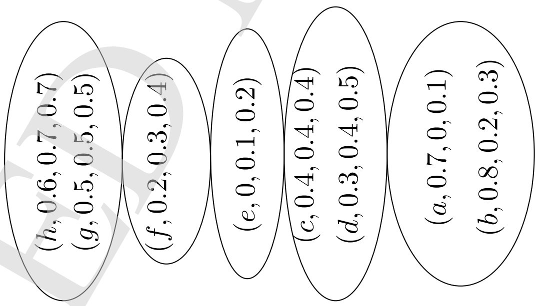

FIGURE 25. Energy of wireless sensor hypernetwork(WS–HN)

$$\eta^*((a, 0.7, 0, 0.1)) = \{(a, 0.7, 0.2, 0.3), (b, 0.7, 0.2, 0.3)\},$$
$$\eta^*((c, 0.4, 0.4, 0.4)) = \{(c, 0.3, 0.4, 0.5), (d, 0.3, 0.4, 0.5)\}$$
$$\eta^*((e, 0, 0.1, 0.2)) = \{(e, 0, 0.1, 0.2)\}, \eta^*((f, 0.2, 0.3, 0.4)) = \{(f, 0.2, 0.3, 0.4)\}$$
$$\text{and } \eta^*((g, 0.5, 0.5, 0.5)) = \{(g, 0.5, 0.7, 0.7), (h, 0.5, 0.7, 0.7)\}.$$

So we obtained the single–valued neutrosophic graph in Figure 26. By Figure 26, for society X, we have 5 representatives $\eta^*(a), \eta^*(c), \eta^*(e), \eta^*(f)$ and $\eta^*(g)$ where

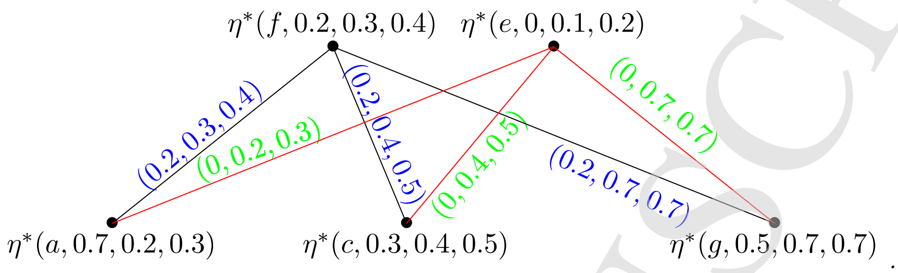

FIGURE 26. Energy of wireless sensor network(WSN)

TABLE 4. DSPNL, DECT and DCT of WSN

Cluster head	*Cluster head*	*Truth*	*Indeterminacy*	*Falsity*
$\eta^*(f)$	$\eta^*(a)$	*0.2*	*0.3*	*0.4*
$\eta^*(f)$	$\eta^*(c)$	*0.2*	*0.4*	*0.5*
$\eta^*(f)$	$\eta^*(g)$	*0.2*	*0.7*	*0.7*
$\eta^*(e)$	$\eta^*(a)$	*0*	*0.2*	*0.3*
$\eta^*(e)$	$\eta^*(c)$	*0*	*0.4*	*0.5*
$\eta^*(e)$	$\eta^*(g)$	*0*	*0.7*	*0.7*

the likeness, indeterminacy and dislikeness of trophic relations between species of group of this society is shown in the Table 4.

6. Conclusion

The current paper considered the concept of single–valued neutrosophic hypergraphs as a generalization of single–valued neutrosophic hypergraphs via η, as positive relation on single–valued neutrosophic hypergraphs. Moreover

(i) It is considered single–valued neutrosophic hypergraphs corresponding to wireless sensor hypernetworks such that vertices (V) represent the sensors and the set of links (E) represents the connections between vertices.

(ii) Using the relation η^*, is constructed G/η^* as sensor clusters and via this quotient single–valued neutrosophic hypergraph the concept of (self) derivable single–valued neutrosophic graph and extended derivable single–valued neutrosophic graphs is introduced.

(iii) It is obtained an equivalent condition that a single–valued neutrosophic graph is a non–derivable single–valued neutrosophic graph.

(*iv*) The concept of intuitionistic neutrosophic sets provides an additional possibility to represent imprecise, uncertain, inconsistent and incomplete information which exist in real situations. In this research paper, we have described the concept of single–valued neutrosophic graphs. We have also presented applications of single–valued neutrosophic hypergraphs and single–valued neutrosophic graphs in wireless sensor network.

We hope that these results are helpful for further studies in single–valued neutrosophic graph theory. In our future studies, we hope to obtain more results regarding single–valued neutrosophic graphs, single–valued neutrosophic hypergraphs and their applications.

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Mohammad Hamidi
Dept. of Math., Faculty of Mathematics, Payame Noor University, Tehran, Iran
m.hamidi@pnu.ac.ir,
Arsham Borumand Saeid
Dept. of Pure Math., Faculty of Mathematics and Computer, Shahid Bahonar University of Kerman, Kerman, Iran
arsham@uk.ac.ir