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Spanning Tree Problem with Neutrosophic Edge Weights

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Abstract

Neutrosophic set and neutrosophic logic theory are renowned theories to deal with complex, not clearly explained and uncertain real life problems, in which classical fuzzy sets/models may fail to model properly. This paper introduces an algorithm for finding minimum spanning tree (MST) of an undirected neutrosophic weighted connected graph (abbr. UNWCG) where the arc/edge lengths are represented by a single valued neutrosophic numbers. To build the MST of UNWCG, a new algorithm based on matrix approach has been introduced. The proposed algorithm is compared to other existing methods and finally a numerical example is provided

Keywords: Single valued neutrosophic sets; neutrosophic matrix; score function; minimum spanning tree problem

1. Introduction

Smarandache [5] has proposed the idea of "Neutrosophic set" (abbr. NS) which can capture the natural phenomenon of the imprecision and uncertainty that exists in the real life scenarios. The idea of NS is direct extensions of the idea

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of the conventional set, type 1 fuzzy set and intuitionistic fuzzy set. The NSs are described by a truth membership function (t), an indeterminate membership function (i) and a false membership function (f) independently. The values of t, i and f are within the nonstandard unit interval $]^-0, 1^+[$. Moreover, for the sake of applying NSs in real-world problems efficiently, Smarandache [5] introduced the idea of single valued neutrosophic set (abbr. SVNS). Then, Wang et al.[6] described some properties of SVNSs. The NS model is an useful method for dealing with real world problems because it can capture the uncertainty (i.e., incomplete, inconsistent and indeterminate information) of the real world problem. The NSs is applied in various fields [24]. To make distinction between two single valued neutrosophic numbers, a series of score functions are presented by some scholars (see table 1). Many algorithms are available to find minimum spanning tree which has a large applications in divers fields of computers science and engineering. In classical graph theory, there are many algorithms for finding the MST [4], two most well know algorithms are Prim's algorithm and Kruskal algorithm. In the literature, several types of spanning tree problems have been developed by many researchers when the weights of the edges are not precise and there is an uncertainty [1, 2, 3, 10, 28]. Recently using the idea of single valued neutrosophic sets on graph theory, a new theory is introduced and it is defined as single valued neutrosophic graph theory (abbr. SVNGT). The concept of SVNGT and their extensions finds its applications in diverse fields [12- 24]. However, to the best of our knowledge, there are only few studies in the literature to deal with the minimum spanning tree problem in neutrosophic environment. Ye [8] presented a method to design the MST of a graph where nodes (samples) are represented in the form of SVNS and distance between two nodes which represents the dissimilarity between the corresponding samples has been derived. Mullai et al. [27] studied the shortest path problem by minimal spanning tree algorithm using bipolar neutrosophic numbers. Kandasamy [7] proposed a double-valued neutrosophic Minimum Spanning Tree (abbr. DVN-MST) clustering algorithm, to cluster the data represented by double-valued neutrosophic information. Mandal and Basu [9] proposed a solution approach of the optimum spanning tree problems considering the inconsistency, incompleteness and indeterminacy of the information. The authors consider a network problem with multiple criteria which are represented by weight of each edge in neutrosophic sets. The approach proposed by the authors is based on similarity measure. It should be noted that the triangular fuzzy numbers and SVNSs are similar in the mathematical notation, but totally different.

Table 1. Different types of score functions of SVNS

Refrences	Score function
27	$S_{RIDVAN}(A) = \frac{(1+T-2I-F)}{2}$
11	$S_{NANCY}(A) = \frac{1+(1+T-2I-F)(2-T-F)}{2}$
25	$S_{ZHANG}(A) = \frac{(2+T-I-F)}{3}$

The main contribution of this manuscript is to extend the matrix approach for finding the cost minimum spanning tree of an undirected neutrosophic graph. Neutrosophic graphs give more precision, and compatibility to model the MST problem in neutrosophic environment when compared to the fuzzy MST.

The manuscript is organized as follows. We briefly introduce the ideas of NSs, SVNS, and the score function of single valued neutrosophic number in Section 2. Section 3 present the formulation problem. Section 4 describes an algorithm for finding the minimum spanning tree of neutrosophic undirected graph. In Section 5, an example is presented to described the proposed method. In Section 6, A comparative study with others existing methods is presented. We present the conclusion of the paper in Section 7.

2. Preliminaries

Some of the important background knowledge for the materials that are presented in this paper is presented in this section. These results can be found in [5, 6, 25].

Definition 2.1 [5] Le ξ be an universal set. The neutrosophic set A on the universal set ξ categorized in to three membership functions called the true $T_A(x)$, indeterminate $I_A(x)$ and false $F_A(x)$ contained in real standard or non-standard subset of $]^-0, 1^+[$ respectively.

$$^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+ \qquad (1)$$

Definition 2.2 [6] Let ξ be a universal set. The single valued neutrosophic sets (SVNs) A on the universal ξ is denoted as following

$$A = \{< x: T_A(x),\ I_A(x),\ F_A(x) > x \in \xi\} \qquad (2)$$

The functions $T_A(x) \in [0.\ 1]$ is the degree of truth membership of x in A, $I_A(x) \in [0.\ 1]$ is the degree of indeterminacy of x in A and $F_A(x) \in [0.\ 1]$ degree of falsity membership of x in A. The $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfy the following condition:

$$0 \le T_A(x) + I_A(x) + T_A(x) \le 3 \qquad (3)$$

To rank the single valued neutrosophic sets, Zhang [25] defined the score function and the relation order between two SVNs as follows.

Definition 2.3 [25] Let A= (T, I, F) be a SVNs. Then a score function S is defined as follow

$$S_{ZHANG}(A) = \frac{(2 + T - I - F)}{3} \qquad (4)$$

Here, *T, I* and *F* represent the degree of truth membership value, indeterminacy membership value and falsity membership values of *A*.

Remark 2.4: In neutrosophic mathematics, the zero sets are represented by the following form 0_N ={<x, (0, 1, 1)> :x$\in$ X}.

3. Problem formulation

A spanning tree of a connected neutrosophic graph G is an acyclic sub-graph which includes every node of neutrosophic graph G and it also is connected. Every neutrosophic spanning tree has exactly $n - 1$ arcs, where n represents the number of nodes of the neutrosophic graph. A neutrosophic minimum spanning tree (MST) problem is to find a neutrosophic spanning tree such that the sum of all its arc costs/ lengths is minimum. In crisp environment, the MST problem uses the exact costs/lengths associated with the edges of the graph. However, in real life scenarios the arc lengths may be imprecise/uncertain in nature. The decision maker takes their decision based on insufficient information due to lack of evidence or incompleteness. The effective way to work with this imprecision information is to consider a neutrosophic graph. In this paper, we have considered an undirected neutrosophic weighted connected graph. The arc weights of the neutrosophic graph are represented as neutrosophic instead of crisp value. To design the MST, we have introduced an algorithm to solve this problem.

4. Minimum spanning tree algorithm of neutrosophic undirected graph

In this section, a new version of minimum spanning tree problem based on matrix approach is presented and discussed on a graph with neutrosophic edge weight.

In the following, we propose a neutrosophic minimum spanning tree algorithm, whose computing steps are described below:

Algorithm:

Input: Adjacency matrix M $=\left[W_{ij}\right]_{n\times n}$ for the undirected weighted neutrosophic graph G with their edge weight.
Output: MST T of graph G

Step 1: Input neutrosophic adjacency matrix A
Step 2: Using the score function (4), convert the neutrosophic matrix into a score matrix $\left[S_{ij}\right]_{n\times n}$.
Step 3: Iterate step 4 and step 5 until all (n-1) elements of matrix of S are either marked to 0 or all the nonzero (≠0) elements of the matrix are marked.
Step 4: Find the M either column wise or row wise to compute the unmarked minimum element S_{ij} ,which is the cost of the corresponding arc e_{ij} in M.
Step 5: If the corresponding arc e_{ij} of chosen S_{ij} produce a cycle with the previous marked entries of the score matrix S then set $S_{ij}=0$ else mark S_{ij} .
Step 6: Design the tree T including only the marked elements from the S which will be computed MST of G.
Step 7: Stop.

5. Practical example

Consider the graph G= (V, E) depicted in figure 1 where V represents the vertices and E represent the edge of the graph. Each arc consists of neutrosophic edge's weight. Here V= 6 and edge =9. The different steps involved in the design of the MST are presented as follows

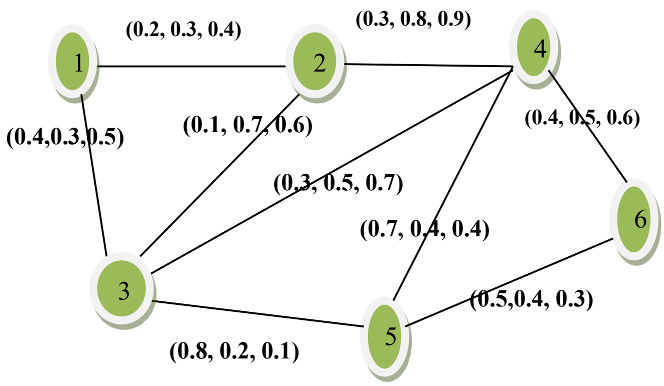

Fig 1. Undirected neutrosophic graphs

The neutrosophic adjacency matrix A of the undirected neutrosophic graph is given below:

$$\begin{bmatrix} 0 & (0.2,0.3,0.4) & (0.4,0.3,0.5) & 0 & 0 & 0 \\ (0.2,0.3,0.4) & 0 & (0.1,0.7,0.6) & (0.3,0.8,0.9) & 0 & 0 \\ (0.4,0.3,0.5) & (0.1,0.7,0.6) & 0 & (0.3,0.5,0.7) & (0.8,0.2,0.1) & 0 \\ 0 & (0.3,0.8,0.9) & (0.3,0.5,0.7) & 0 & (0.7,0.4,0.4) & (0.4,0.5,0.6) \\ 0 & 0 & (0.8,0.2,0.1) & (0.7,0.4,0.4) & 0 & (0.5,0.4,0.2) \\ 0 & 0 & 0 & (0.4,0.5,0.6) & (0.5,0.4,0.2) & 0 \end{bmatrix}$$

Thus, using the score function, we get the score matrix

$$S = \begin{bmatrix} 0 & .5 & 0.533 & 0 & 0 & 0 \\ .5 & 0 & 0.267 & 0.2 & 0 & 0 \\ 0.533 & 0.267 & 0 & 0.367 & 0.833 & 0 \\ 0 & 0.2 & 0.367 & 0 & 0.633 & 0.433 \\ 0 & 0 & 0.833 & 0.633 & 0 & 0.6 \\ 0 & 0 & 0 & 0.433 & 0.6 & 0 \end{bmatrix}$$

Fig. 2. Score matrix

By referring to the figure 2, the minimum entries 0.2 is selected and the corresponding edge (2, 4) is highlighted by red color in figure 3 . Repeat the procedure until the iteration will exist.

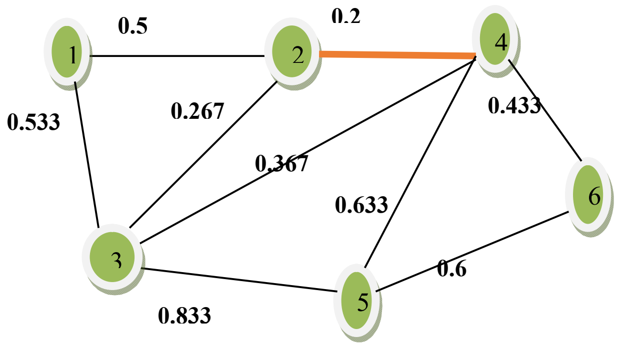

Fig. 3 Undirected neutrosophic graph where the edge (2, 4) is highlighted

By referring to the figure 4, the next non zero minimum entries 0.267 is marked and corresponding edge (2, 3) is highlighted with red color in figure 5.

$$S = \begin{bmatrix} 0 & .5 & 0.533 & 0 & 0 & 0 \\ .5 & 0 & 0.267 & 0.2 & 0 & 0 \\ 0.533 & 0.267 & 0 & 0.367 & 0.833 & 0 \\ 0 & 0.2 & 0.367 & 0 & 0.633 & 0.433 \\ 0 & 0 & 0.833 & 0.633 & 0 & 0.6 \\ 0 & 0 & 0 & 0.433 & 0.6 & 0 \end{bmatrix}$$

Fig. 4

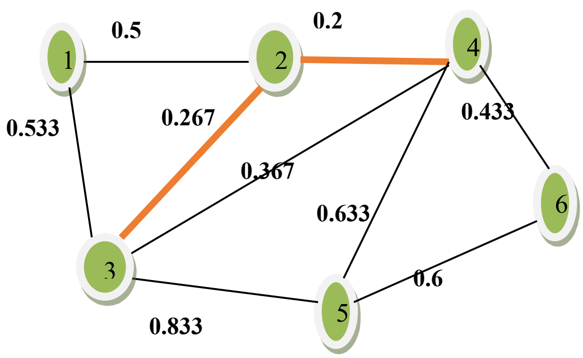

Fig. 5 Undirected neutrosophic graph where the edge (2,3) is highlighted

$$S = \begin{bmatrix} 0 & .5 & 0.533 & 0 & 0 & 0 \\ .5 & 0 & 0.267 & 0.2 & 0 & 0 \\ 0.533 & 0.267 & 0 & \sout{0.367}\ 0 & 0.833 & 0 \\ 0 & 0.2 & 0.367 & 0 & 0.633 & 0.433 \\ 0 & 0 & 0.833 & 0.633 & 0 & 0.6 \\ 0 & 0 & 0 & 0.433 & 0.6 & 0 \end{bmatrix}$$

Fig.6

By referring to the figure 6, the next minimum non zero element 0.367 is marked. But it produces the cycle so we delete and mark it as 0 instead of 0.367. The cycle {2, 3, 4} is shown in figure 7.

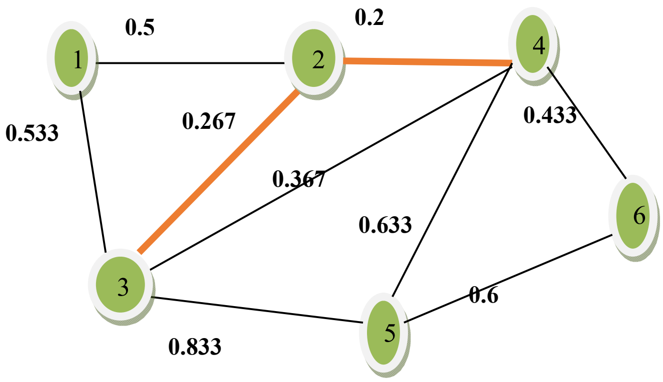

Fig 7. cycle {2, 3, 4}

The next non zero minimum element 0.433 is marked and it is shown in the figure 8. The corresponding marked arc is portrayed in figure 9.

$$S = \begin{bmatrix} 0 & .5 & 0.533 & 0 & 0 & 0 \\ .5 & 0 & 0.267 & 0.2 & 0 & 0 \\ 0.533 & 0.267 & 0 & 0.367 & 0.833 & 0 \\ 0 & 0.2 & 0.367 & 0 & 0.633 & 0.433 \\ 0 & 0 & 0.833 & 0.633 & 0 & 0.6 \\ 0 & 0 & 0 & 0.433 & 0.6 & 0 \end{bmatrix}$$

Fig.8

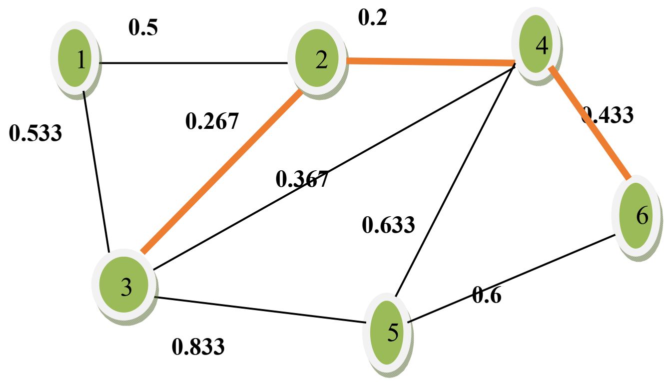

Fig.9. Undirected neutrosophic graph where the edge (4, 6) is highlighted

The next non zero minimum element 0.5 is marked and it is described in the figure 10. The corresponding marked arc is portrayed in figure 11.

$$S = \begin{bmatrix} 0 & .5 & 0.533 & 0 & 0 & 0 \\ .5 & 0 & 0.267 & 0.2 & 0 & 0 \\ 0.533 & 0.267 & 0 & 0.367 & 0.833 & 0 \\ 0 & 0.2 & 0.367 & 0 & 0.633 & 0.433 \\ 0 & 0 & 0.833 & 0.633 & 0 & 0.6 \\ 0 & 0 & 0 & 0.433 & 0.6 & 0 \end{bmatrix}$$

Fig .10

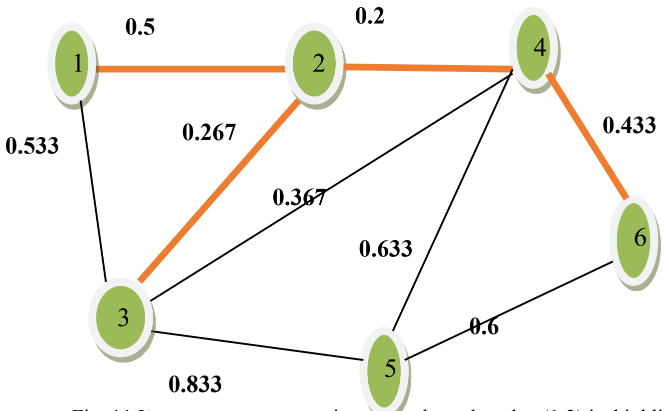

Fig. 11 Undirected neutrosophic graph where the edge (1,2) is highlighted

By referring to the figure 12. The next minimum non zero element 0.533 is now marked. But it produces the cycle so we delete it and mark it as 0 in the place of 0.533.

$$S = \begin{bmatrix} 0 & .5 & \cancel{0.533}\ 0 & 0 & 0 & 0 \\ .5 & 0 & 0.267 & 0.2 & 0 & 0 \\ 0.533 & 0.267 & 0 & 0.367 & 0.833 & 0 \\ 0 & 0.2 & 0.367 & 0 & 0.633 & 0.433 \\ 0 & 0 & 0.833 & 0.633 & 0 & 0.6 \\ 0 & 0 & 0 & 0.433 & 0.6 & 0 \end{bmatrix}$$

Fig .12

The next non zero minimum entries 0.6 is marked it is shown in the figure 13. The corresponding marked edge is portrayed in figure 14.

$$S = \begin{bmatrix} 0 & .5 & \cancel{0.533}\ 0 & 0 & 0 & 0 \\ .5 & 0 & 0.267 & 0.2 & 0 & 0 \\ 0.533 & 0.267 & 0 & 0.367 & 0.833 & 0 \\ 0 & 0.2 & 0.367 & 0 & 0.633 & 0.433 \\ 0 & 0 & 0.833 & 0.633 & 0 & 0.6 \\ 0 & 0 & 0 & 0.433 & 0.6 & 0 \end{bmatrix}$$

Fig .13

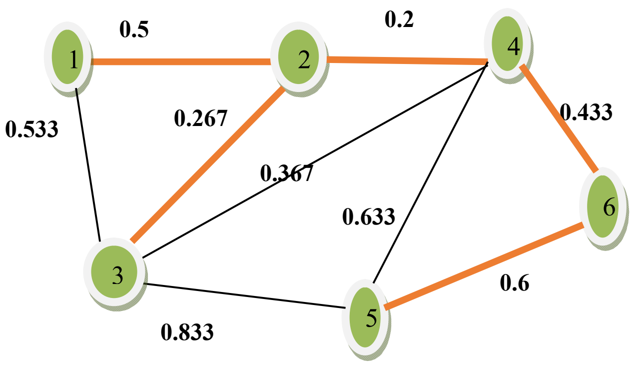

Fig .14 Undirected neutrosophic graph where the edge (5, 6) is highlighted

By referring to the figure 15. The next minimum non zero element 0.633 is marked. But this edge produces a cycle. So, we delete and mark it as 0 in the place of 0.633

S =

0	.5	~~0.533~~ 0	0	0	0
.5	0	0.267	0.2	0	0
0.533	0.267	0	0.367	0.833	0
0	0.2	0.367	0	~~0.633~~ 0	0.433
0	0	0.833	0.633	0	0.6
0	0	0	0.433	0.6	0

Fig .15

By referring to the figure 16. The next minimum non zero element 0.833 is marked. But while drawing the edges it produces the cycle so we delete and mark it as 0 instead of 0.833

S =

0	.5	~~0.533~~ 0	0	0	0
.5	0	0.267	0.2	0	0
0.533	0.267	0	0.367	0.833	0
0	0.2	0.367	0	~~0.633~~ 0	0.433
0	0	~~0.833~~ 0	0.633	0	0.6
0	0	0	0.433	0.6	0

Fig .16

After the above steps, the final path of MST of G is portrayed in figure 17.

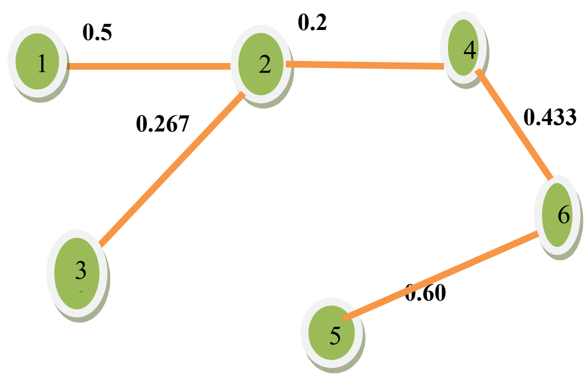

Fig .17. Final path of minimum cost of spanning tree of neutrosophic graph.

According to the procedure of matrix approach presented in section 4. Thus, the crisp minimum cost spanning tree is 2 and the final MST is{1, 2},{2, 3},{2, 4},{4, 6},{6, 5}

6. COMPARATIVE STUDY

In this section, the proposed method presented in section 4 is compared with other existing methods including the algorithm proposed by Mullai et al [27] as follow

Iteration 1: Let $C_1 = \{1\}$ and $\overline{C}_1 = \{2, 3, 4, 5\}$

Iteration 2: Let $C_2 = \{1, 4\}$ and $\overline{C}_2 = \{2, 3, 5\}$

Iteration 3: Let $C_3 = \{1, 4, 3\}$ and $\overline{C}_3 = \{2, 5\}$

Iteration 4: Let $C_4 = \{1,3, 4, 5\}$ and $\overline{C}_4 = \{2\}$

Finally, the single valued neutrosophic minimal spanning tree is

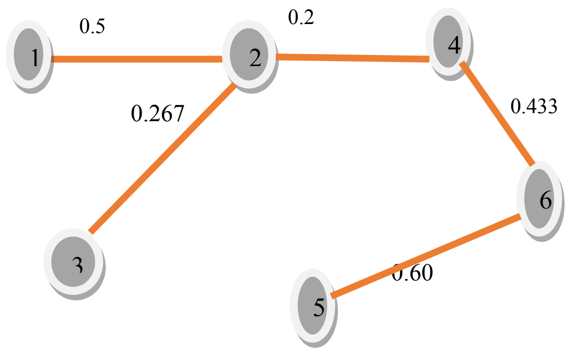

Fig .18 . Single valued neutrosophic minimal spanning tree obtained by Mullai's algorithm.

So, using the score function (4), the SVN MST {1, 2},{2, 3},{2, 4},{4, 6},{6, 5} obtained by Mullai's algorithm is the same as the path obtained by the proposed algorithm.

The difference between the proposed algorithm and Mullai's algorithm is that the proposed approach is based on matrix approach, which can be easily implemented in Matlab, whereas the Mullai's algorithm is based on the comparison of edges in each iteration of the algorithm and this leads to high computation.

7. Conclusion

This paper deals with a MST problem under the neutrosophic environment. The edges of graph are represented by SVNSs. Numerical examples are used to describe the proposed algorithm. The main contribution of this study is to describe an algorithmic approach for MST in uncertain environment using neutrosophic set as edge weights. The proposed algorithm for MST is simple enough and efficient for real world problems. This work can be extended to the case of directed neutrosophic graphs and other structure of graphs including bipolar neutrosophic graphs, interval valued neutrosophic graphs, interval valued bipolar neutrosophic graphs.

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