An energy based peridynamic state-based failure criterion

Christian Willberg
christian.willberg@dlr.de
Lasse Wiedemann
Martin Rädel

10th European Solid Mechanics Conference
ECSM 2018 05/07/2018

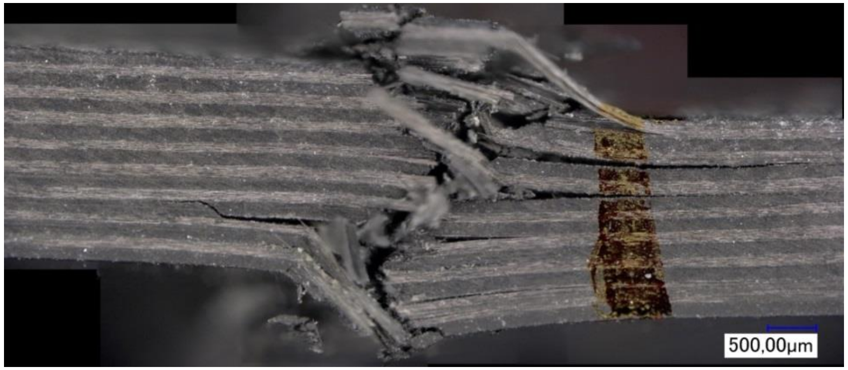
DLR

Simulate of patterns is easy
Simulate of real behavior is complicated

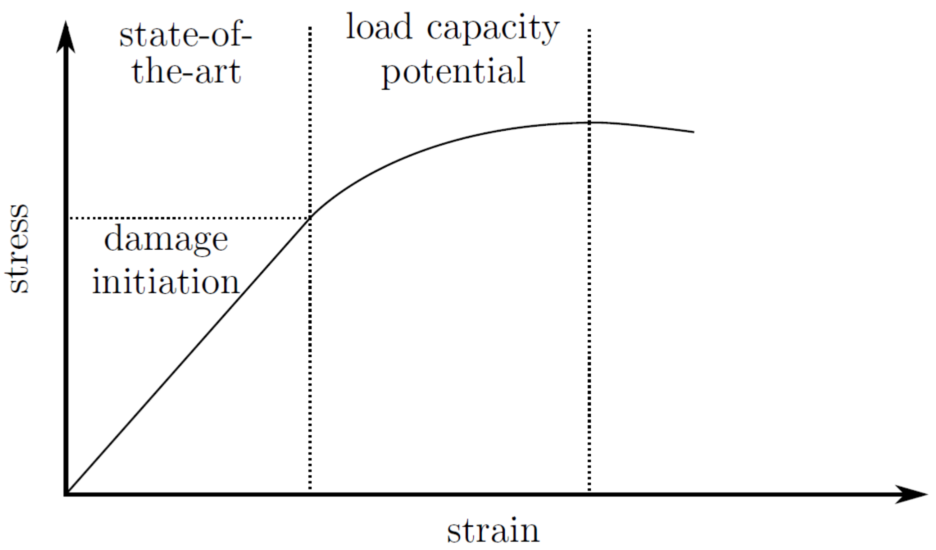
DLR

Motivation

- Challenges:
  - Exploitation of fiber reinforced plastics (FRP) lightweight potential limited
  - Missing reliability of failure predictions



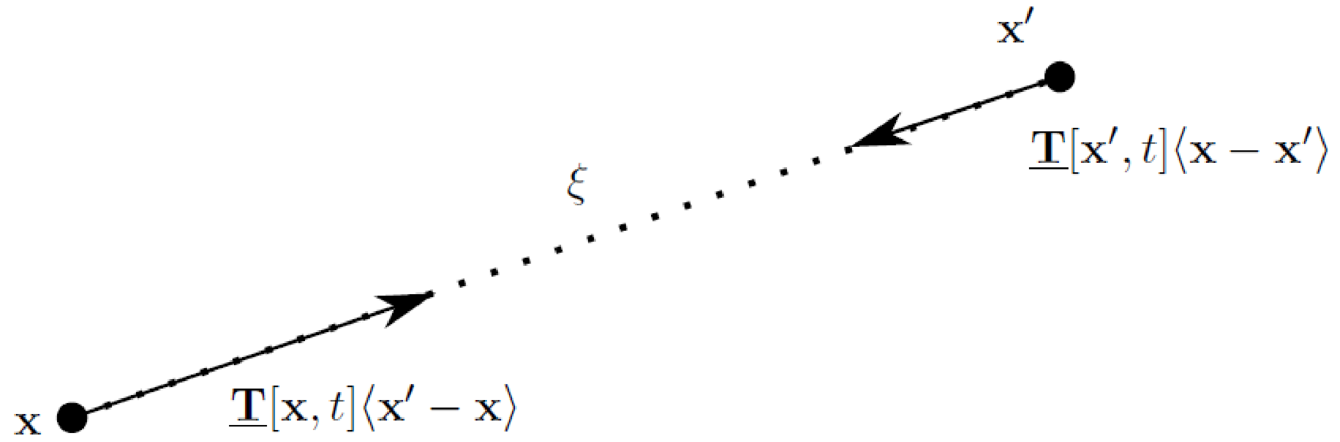
- Goals:
  - Increase understanding of failure mechanisms
  - Reduce number of experiments
  - Derive improved failure criteria for design process of structures



Outline

1. Peridynamic energy based state-based failure criterion
2. Verification of the energy based state-based failure criterion
3. Comparison to critical stretch model

Peridynamics – ordinary state based formulation



$$\rho\left(\mathbf{x}\right)\ddot{\mathbf{u}}\left(\mathbf{x},t\right)$$
$$= \int_{\mathcal{H}} \left(\underline{\mathbf{T}}\left[\mathbf{x},t\right]\left\langle \mathbf{x}'-\mathbf{x}\right\rangle - \underline{\mathbf{T}}\left[\mathbf{x}',t\right]\left\langle \mathbf{x}-\mathbf{x}'\right\rangle\right)\mathrm{d}V + \mathbf{b}\left(\mathbf{x},t\right)$$

Peridynamics – ordinary state based formulation

$$W_{CM} = \frac{1}{2} K \left[\epsilon_{kk}\right]^2 \delta_{ij} + 2G \left[\epsilon_{ij}^d\right]^2 \stackrel{!}{=} W_{PD}$$

$$\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle = \mathbf{F}\boldsymbol{\xi} = \mathbf{F}\langle\mathbf{x}' - \mathbf{x}\rangle \quad \forall\boldsymbol{\xi} \in \mathcal{H}$$

- For small deformations and isotropic material

$$\underline{x} = |\underline{\mathbf{X}}\langle\boldsymbol{\xi}\rangle| \qquad \underline{y} = |\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle| \qquad \underline{e}\langle\boldsymbol{\xi}\rangle = \underline{y} - \underline{x}$$

$$\underline{e}\langle\boldsymbol{\xi}\rangle = |\mathbf{F}\boldsymbol{\xi}| - |\boldsymbol{\xi}| = \epsilon_{ij}\xi_i \frac{\xi_j}{|\boldsymbol{\xi}|}$$

$$\underline{e}^d\langle\boldsymbol{\xi}\rangle = \epsilon_{ij}^d \xi_i \frac{\xi_j}{|\boldsymbol{\xi}|} \qquad \underline{e}^i\langle\boldsymbol{\xi}\rangle = \epsilon_{ii}\xi_i \frac{\xi_i}{|\boldsymbol{\xi}|}$$

$$W_{PD} = \frac{A}{2} \int_{\mathcal{H}} \underline{\omega}\langle\boldsymbol{\xi}\rangle \left[\epsilon_{ij}^d \xi_i \frac{\xi_j}{|\boldsymbol{\xi}|}\right]^2 \mathrm{d}V_{\boldsymbol{\xi}} + \frac{B}{2} \int_{\mathcal{H}} \underline{\omega}\langle\boldsymbol{\xi}\rangle \left[\epsilon_{ii}\xi_i \frac{\xi_i}{|\boldsymbol{\xi}|}\right]^2 \mathrm{d}V_{\boldsymbol{\xi}}$$

Peridynamics – ordinary state based formulation

$$A = \frac{3K}{m_V} \quad \text{and} \quad B = \frac{15G}{m_V}$$

$$m_V = \int_{\mathcal{H}(\mathbf{x})} \underline{\omega}\langle\boldsymbol{\xi}\rangle \underline{x}\underline{x}\, \mathrm{d}V_{\boldsymbol{\xi}} \qquad \theta = \frac{3}{m_V}\int_{\mathcal{H}(\mathbf{x})} \underline{\omega}\langle\boldsymbol{\xi}\rangle \underline{x}\underline{e}\langle\boldsymbol{\xi}\rangle\, \mathrm{d}V_{\boldsymbol{\xi}}$$

$$\underline{t}\langle\boldsymbol{\xi}, t\rangle = \frac{\underline{\omega}\langle\boldsymbol{\xi}\rangle}{m_V}\left[3K\theta\underline{x} + 15G\underline{e}^d\right]$$

$$\underline{\mathbf{T}} = \underline{t}\frac{\underline{\mathbf{Y}}}{|\underline{\mathbf{Y}}|}$$

Damage models

- Could be included via the influence function
- For programming reasons the history dependend scalar value representing the damage function is split from the the influence function

$$\chi\left(\boldsymbol{\xi},t\right)=\begin{cases}1 & \text{no failure}\\ 0 & \text{failure}\end{cases}$$

- Critical energy model by Foster et al.

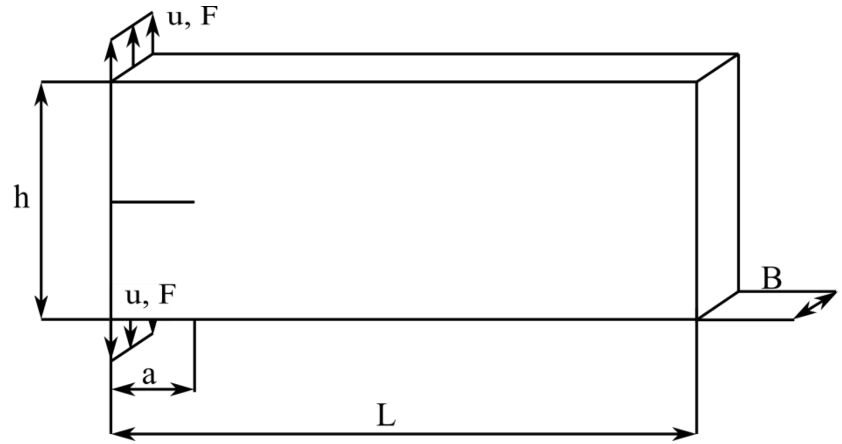
$$W_C=\frac{4G_{0C}}{\pi\delta^4}$$

$$W_{\text{bond}}=0.25\chi(\underline{e}\langle\boldsymbol{\xi}\rangle,t)\left\{\underline{t}\left[\mathbf{x},t\right]-\underline{t}\left[\mathbf{x}',t\right]\right\}\underline{e}>W_C$$

- Critical stretch model

$$s_C=\sqrt{\frac{G_{0C}}{\left[3G+\left(\frac{3}{4}\right)^4\left(K-\frac{5G}{3}\right)\right]\delta}}$$

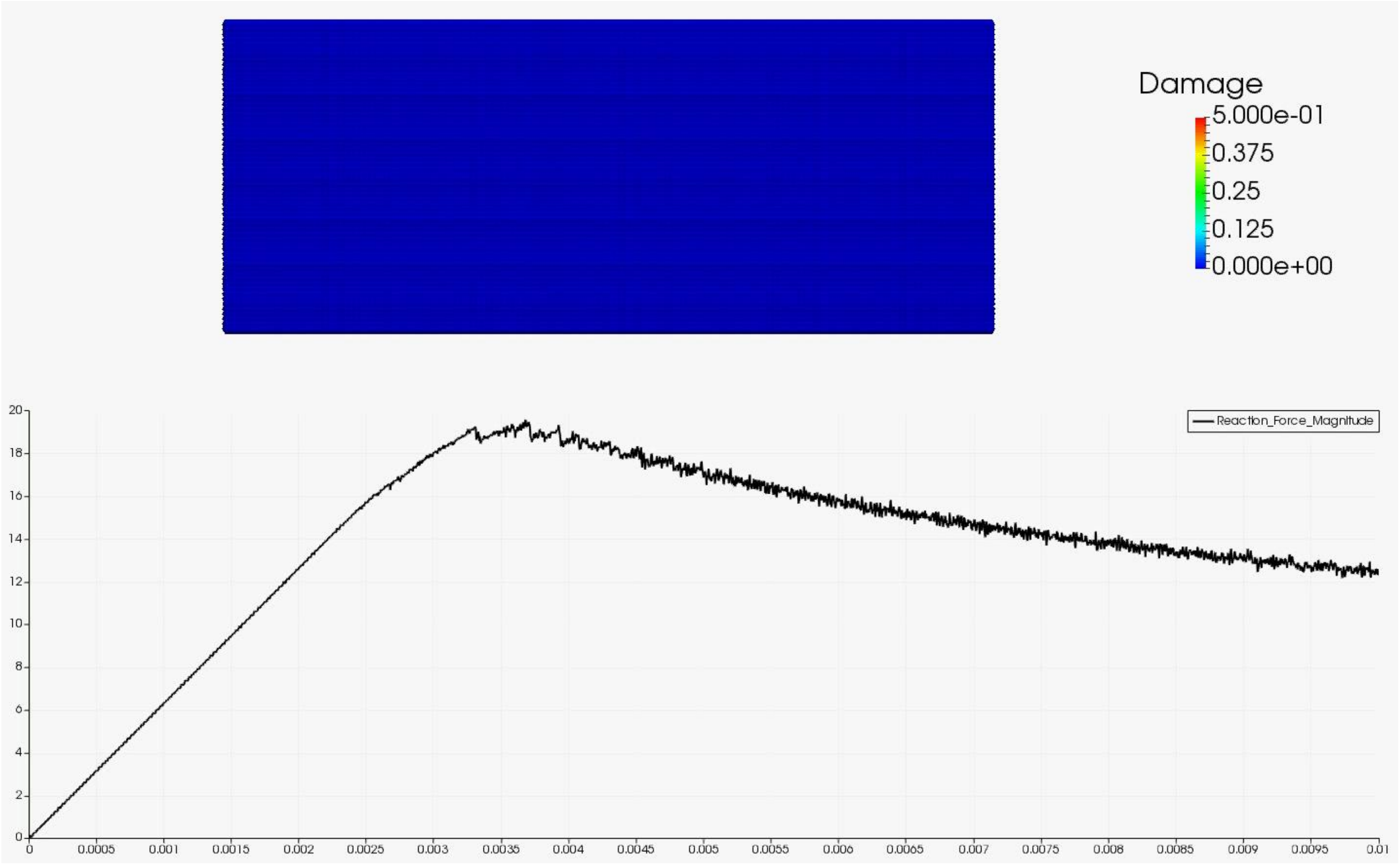
Verification



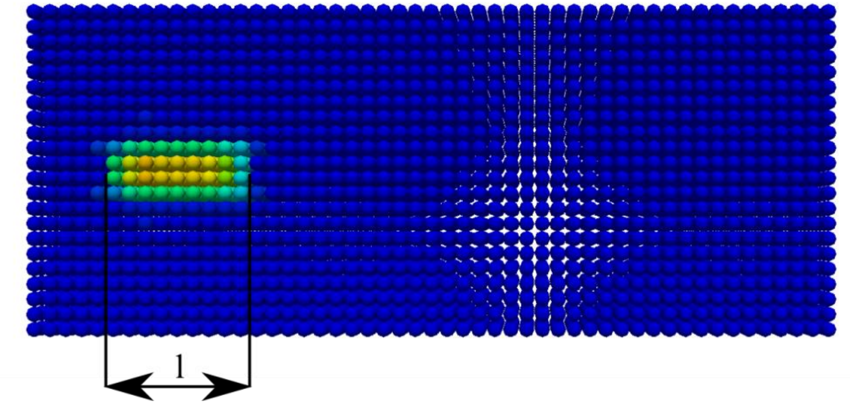
Geometry	a	h	L	B
	0.005m	0.02m	0.05m	0.006m
Material	**Bulk Modulus**	**Shear Modulus**	**Density**	**G_0**
	1.75E+09 Nm^{-2}	8.08E+08 Nm^{-2}	2000 kgm^{-3}	12 Nm^{-1}

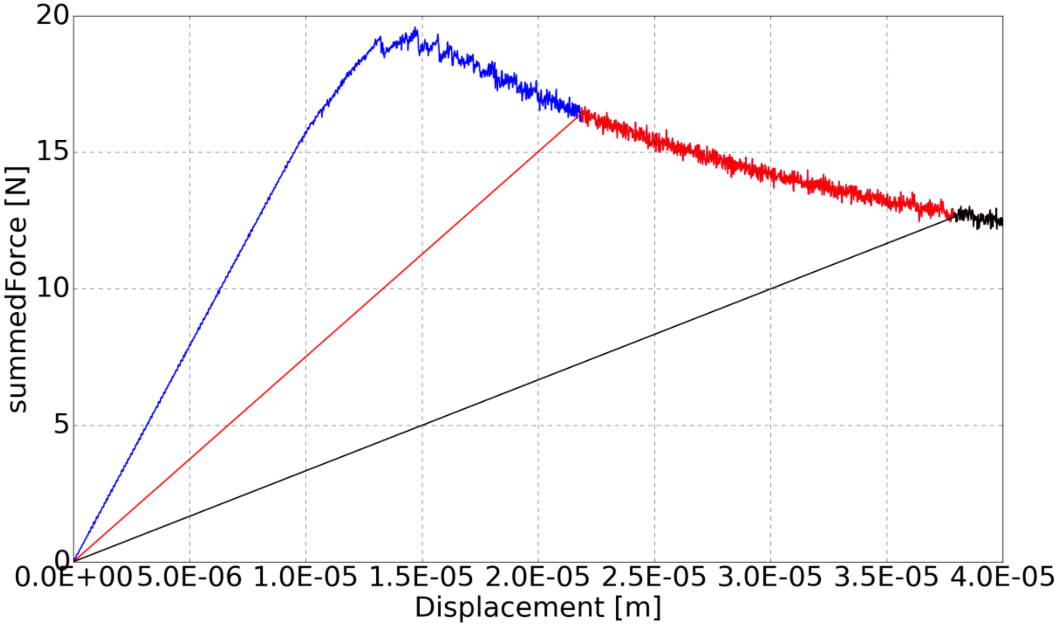
Mesh	2.01dx	3.01dx	4.01dx	5.01dx
0.0005	0.001005	0.001505	0.002005	0.002505
0.00033	0.000663	0.000993	0.001323	0.001653
0.00025	0.000503	0.000753	0.001003	0.001253
0.000125	0.000251		0.000501	

Verification: Double Cantilever Beam (DCB)



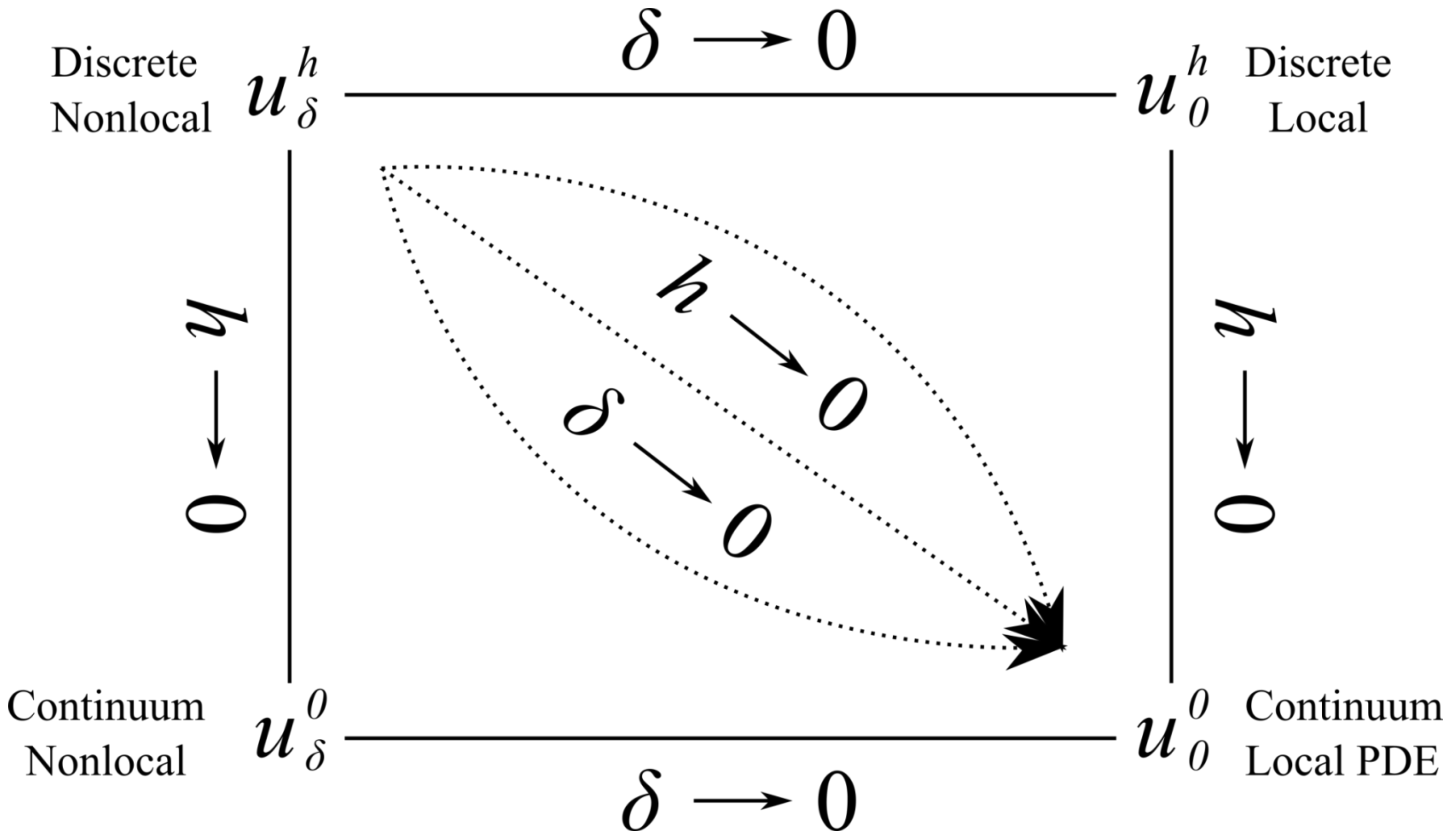
Verification



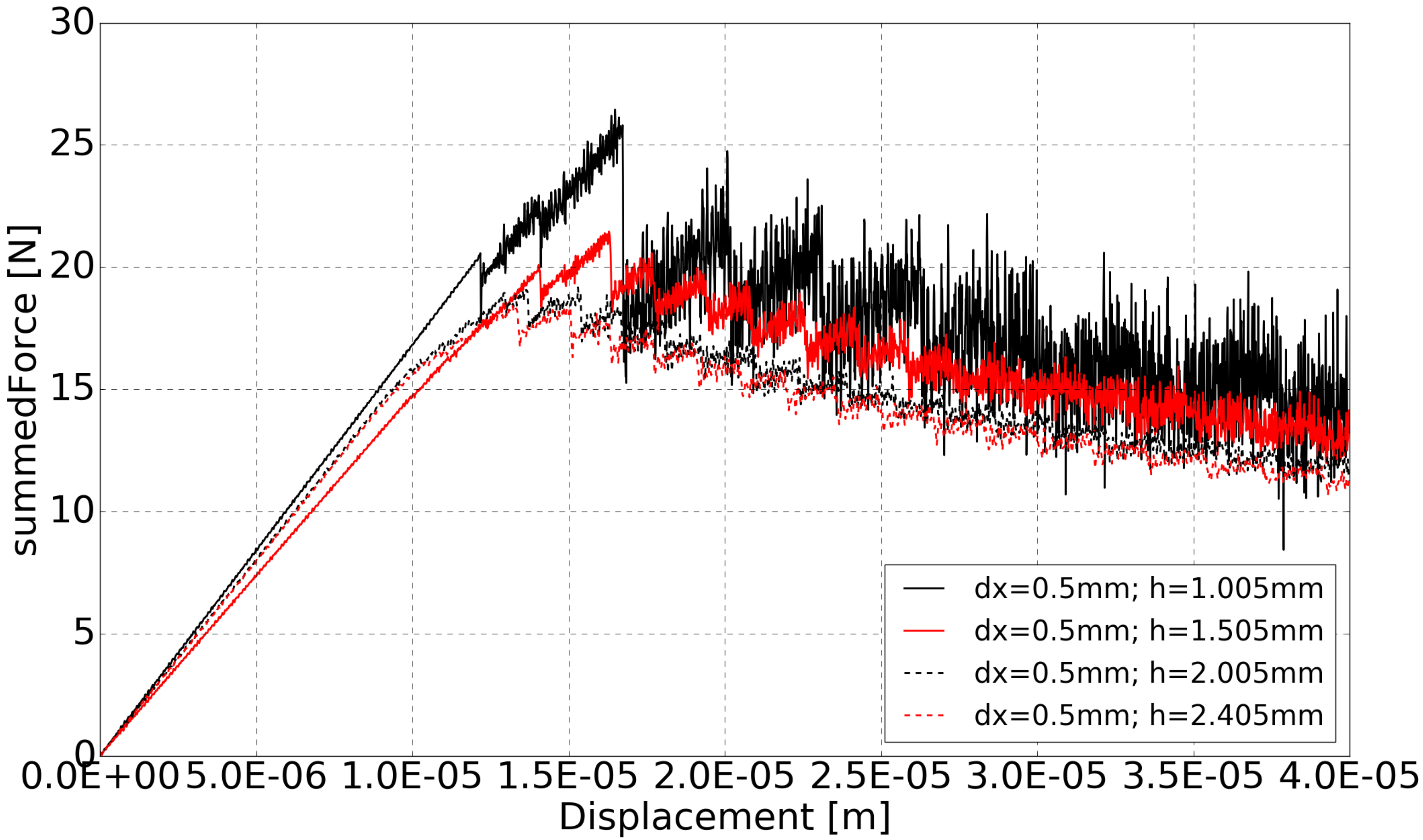


	Line 1	Line 2
δ	G_0	G_0
[m]	[N/m]	[N/m]
$2.015 \cdot 10^{-3}$	12.8	11.4
$3.015 \cdot 10^{-3}$	13.1	12.9
$4.015 \cdot 10^{-3}$	11.1	11.3
$5.015 \cdot 10^{-3}$	11.2	11.9

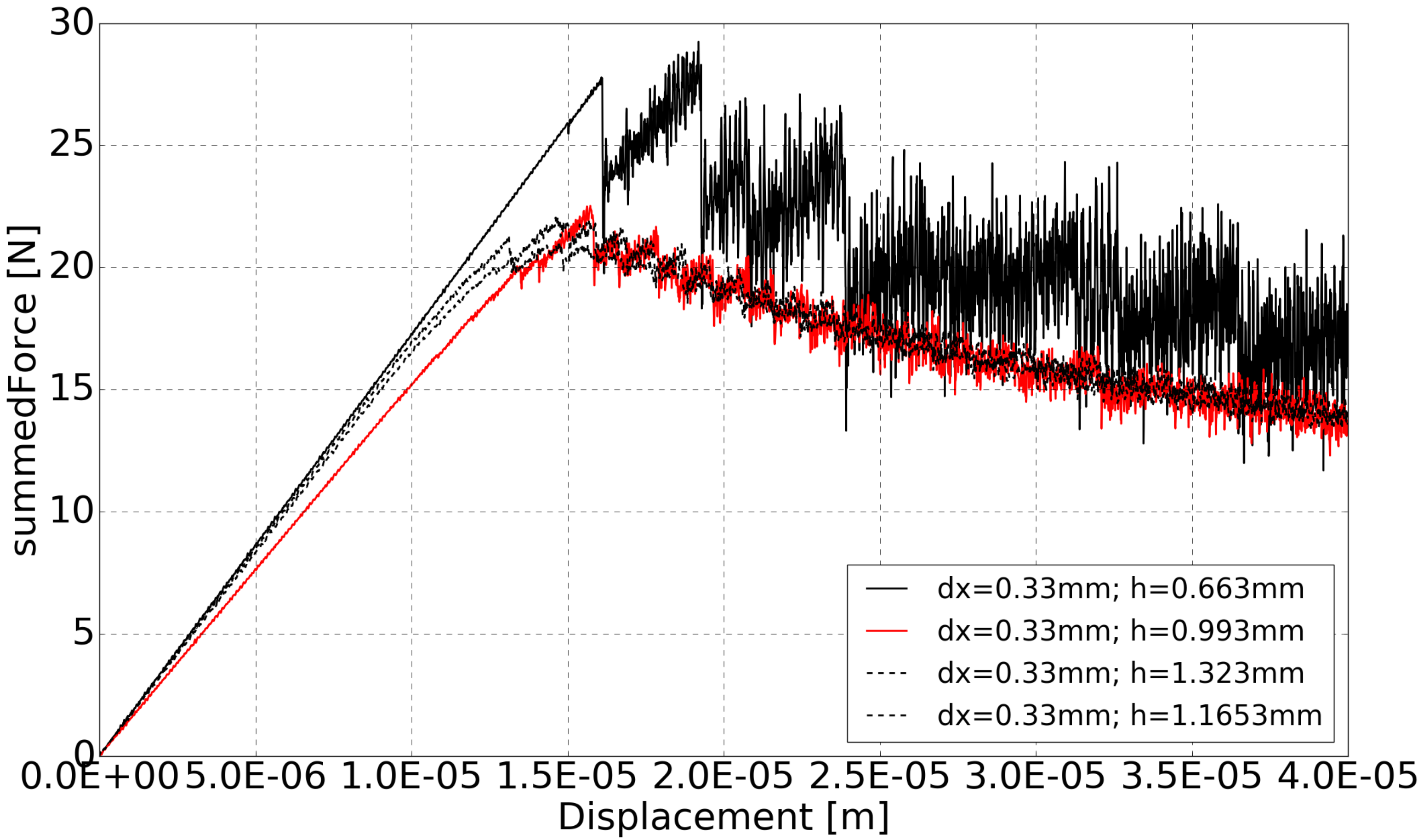
Verification: Convergence



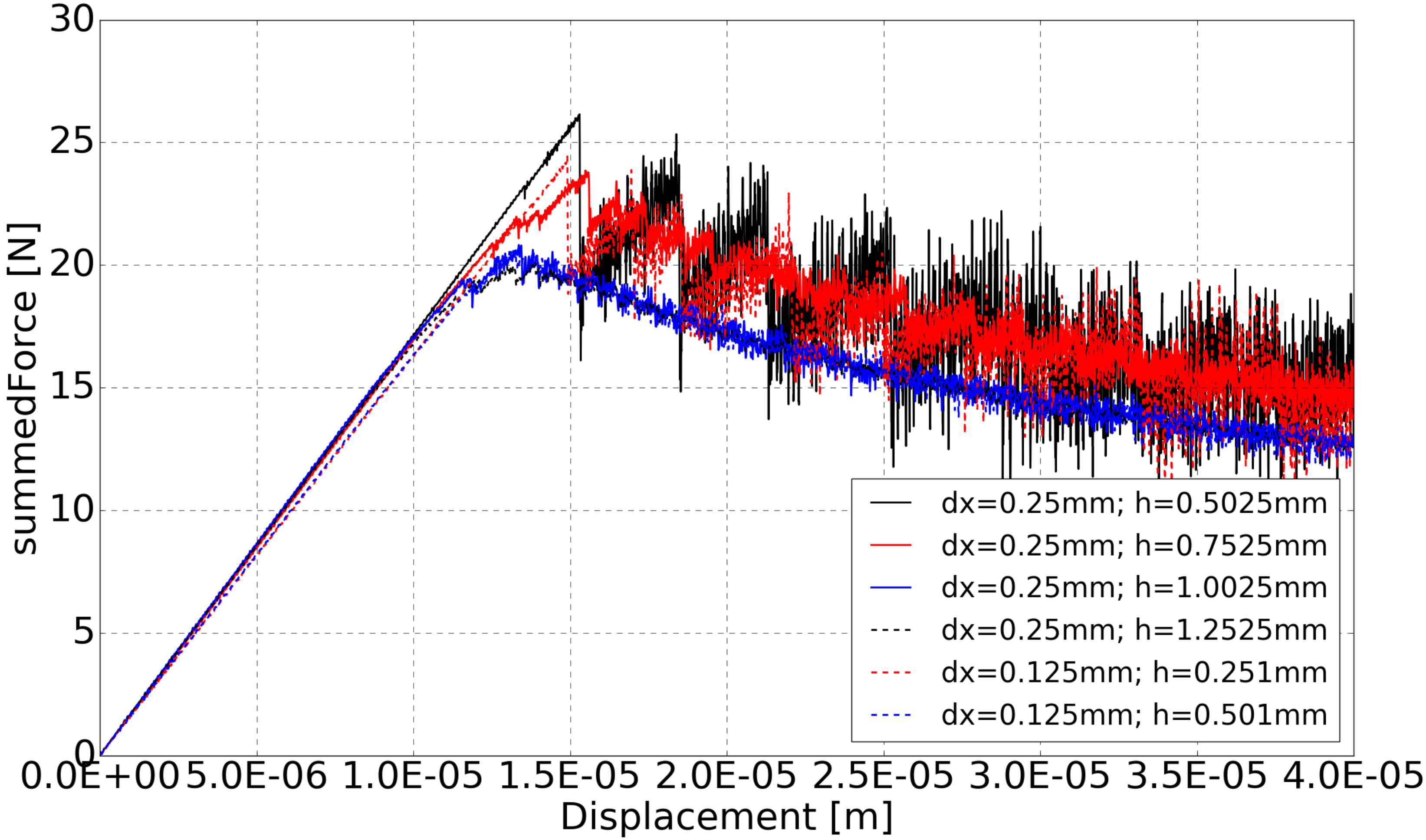
Verification: Convergence



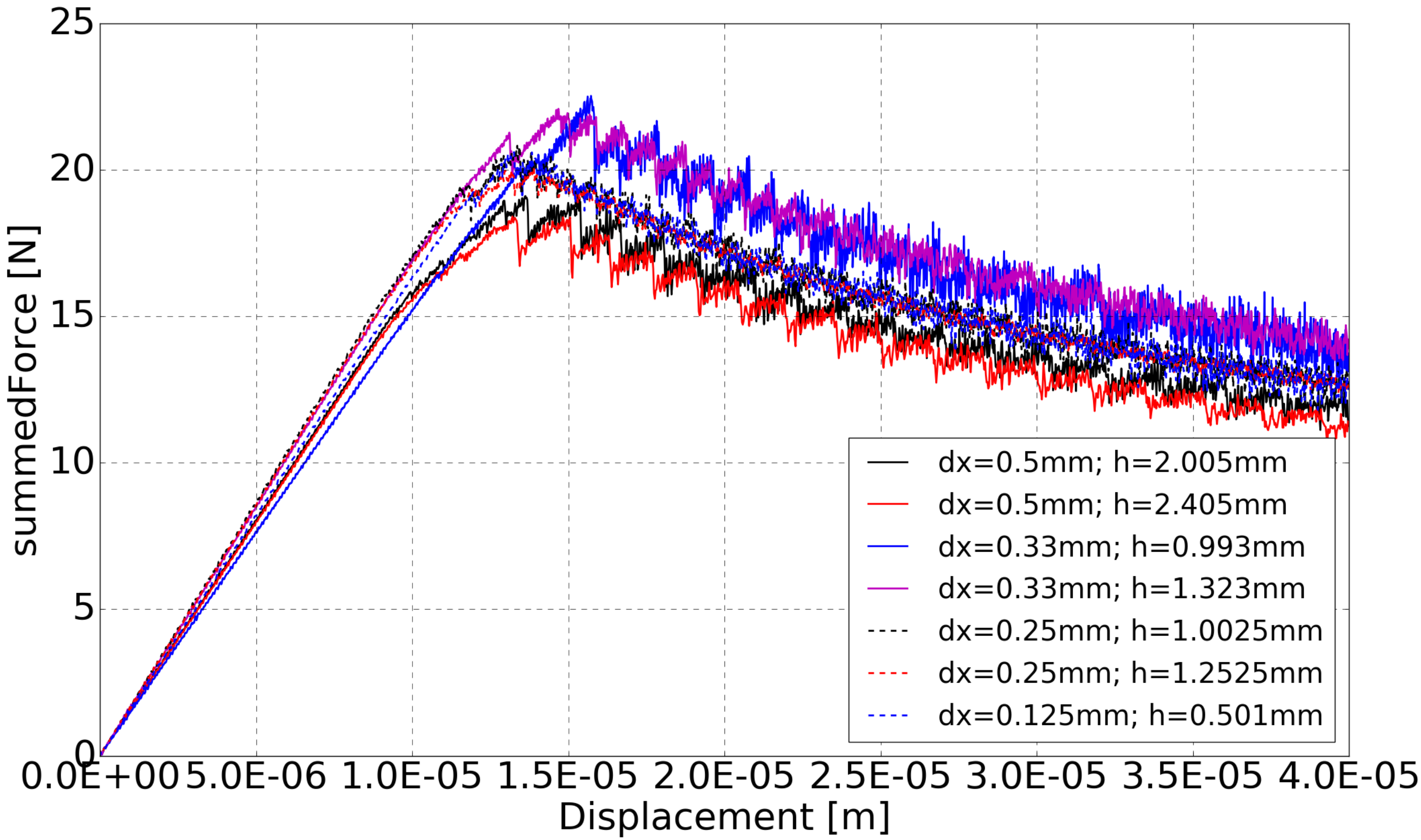
Verification: Convergence



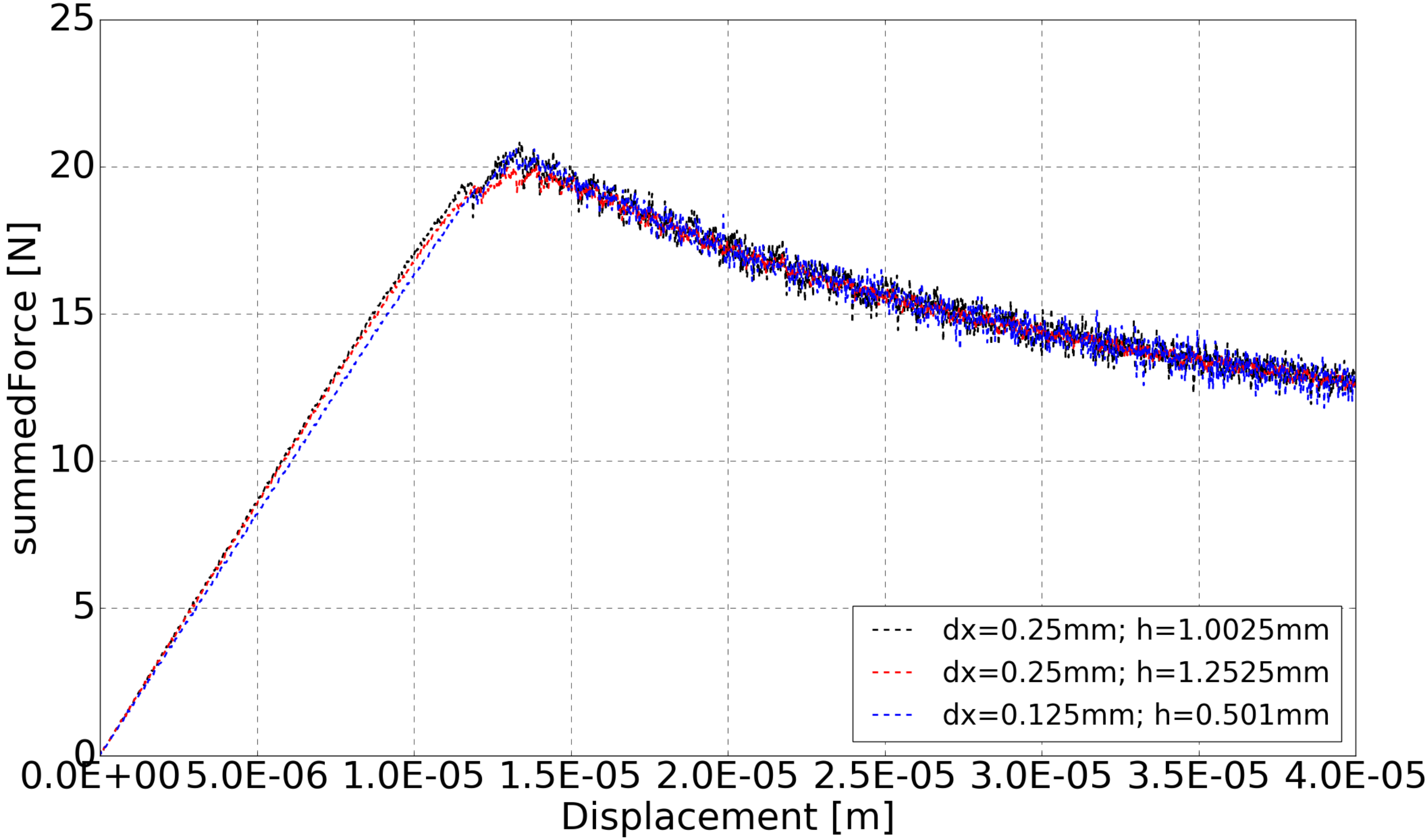
Verification: Convergence



Verification: Convergence

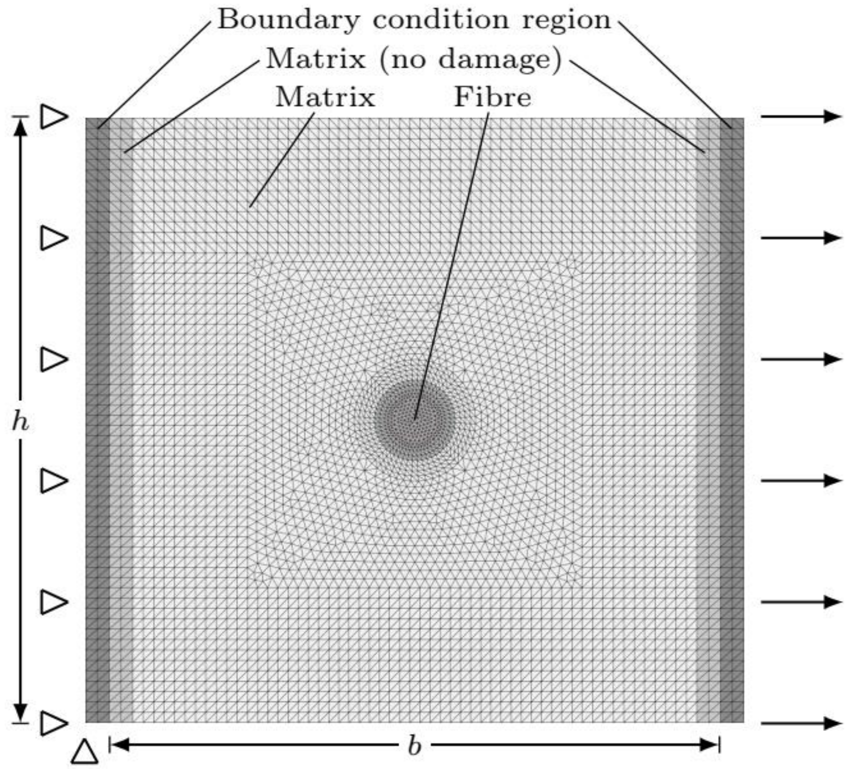


Verification: Convergence

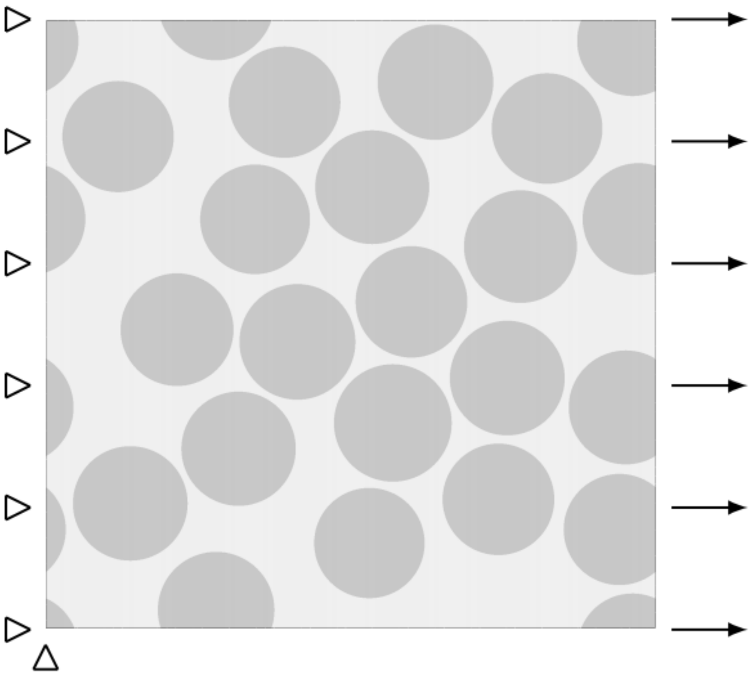


DLR

Comparison: Fibre-matrix models

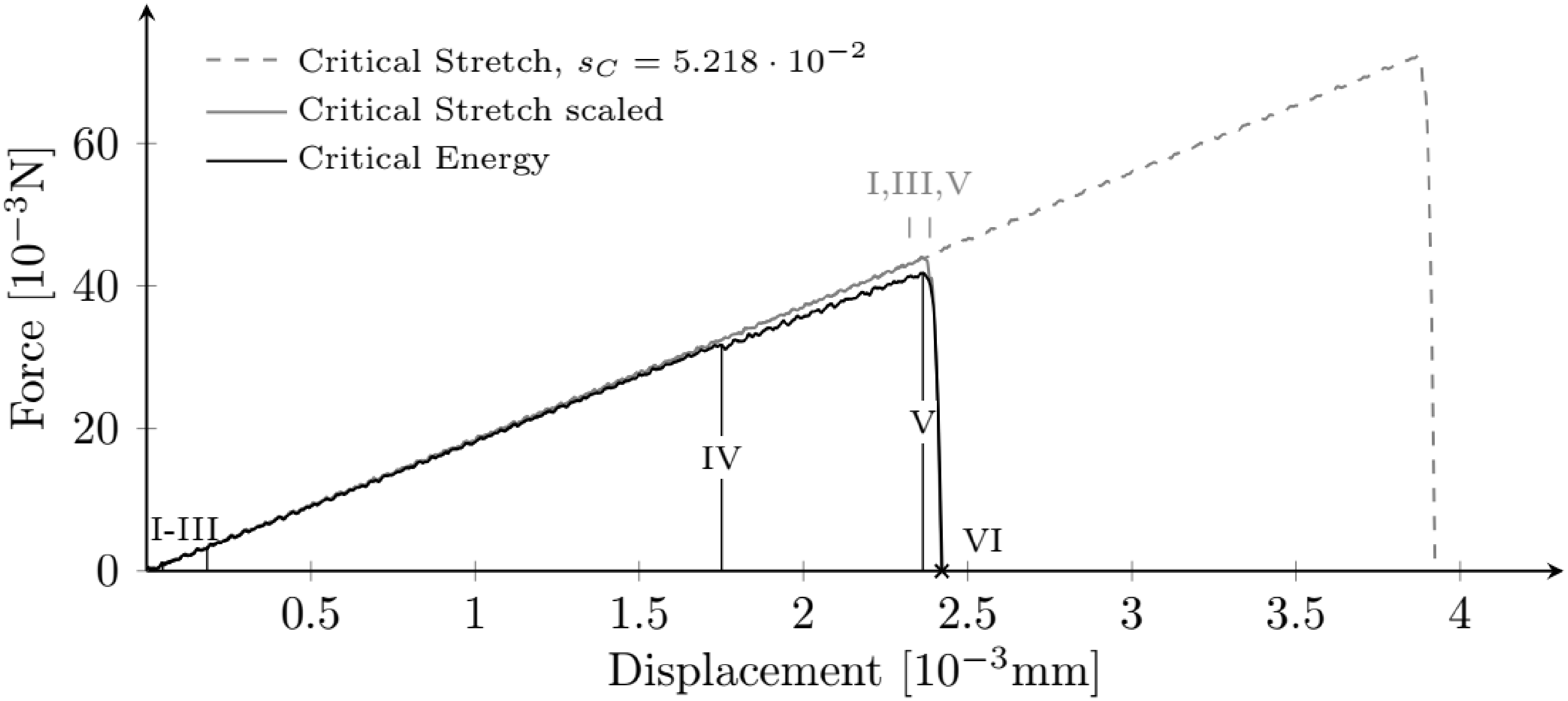


Single fibre

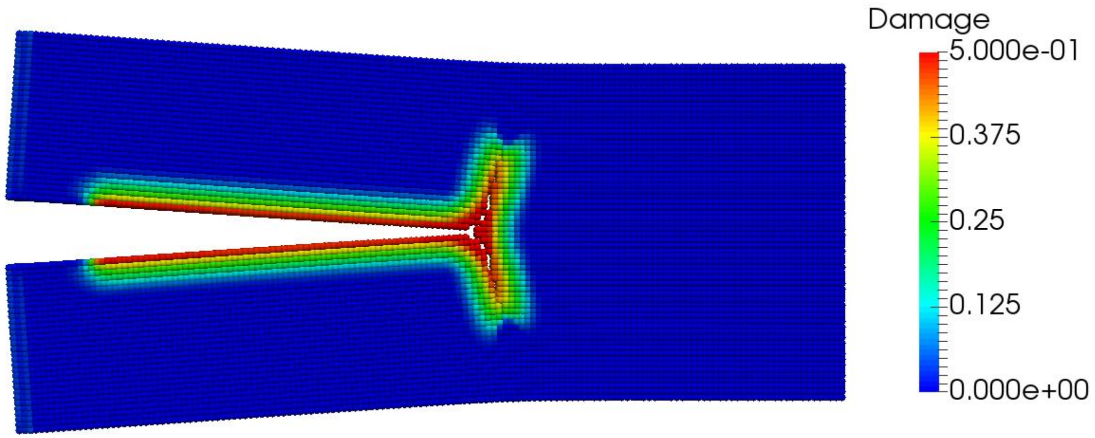


RVE

Comparison: Single fibre – Force-Displacement

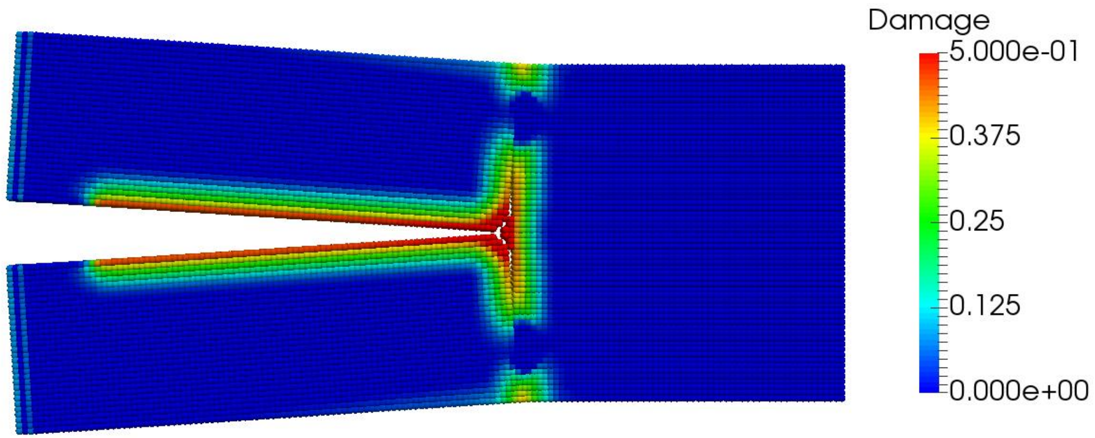


Comparison: DCB



Critical Stretch

$s_c = 0.000433593$
$K = 1.75E09\ N/m^2$
$G = 8.08E8\ N/m^2$
$\delta = 0.002505\ m$
<u>$G_0 = 12\ N/m$</u> → Input

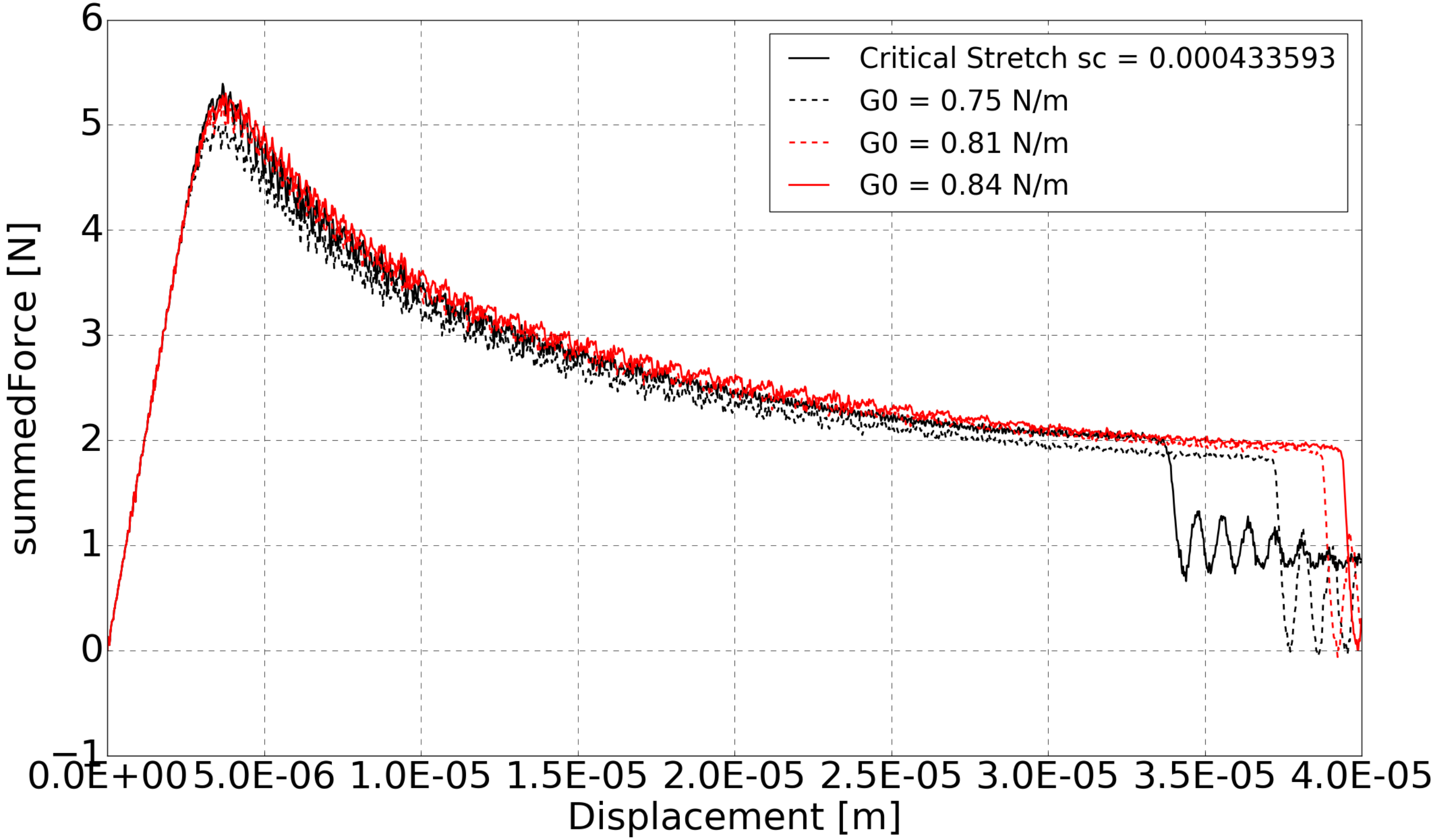


Critical Energy

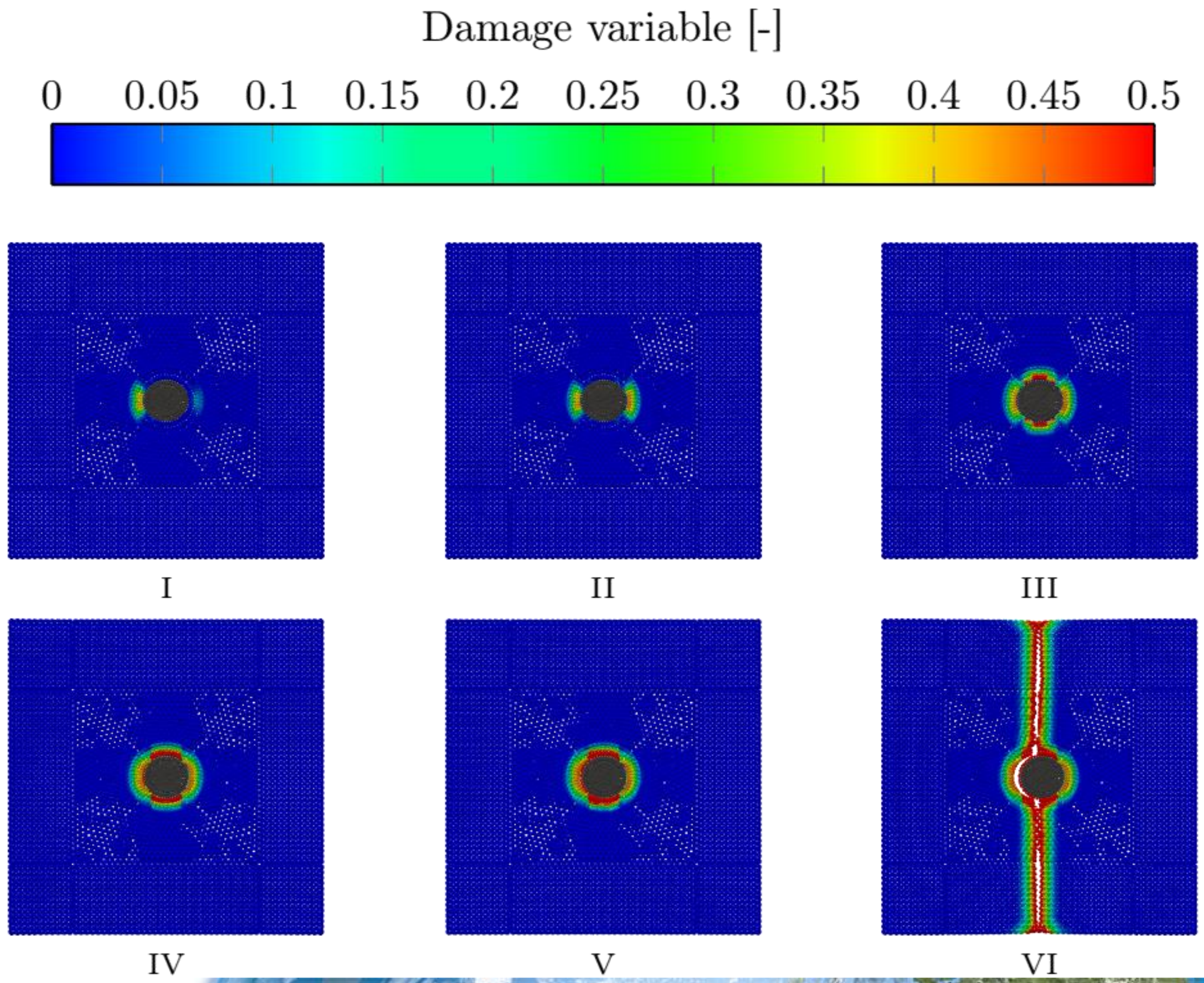
$G_0 = 0.75\text{-}0.84\ N/m$
→ Output and Input Energy Criterion

DLR

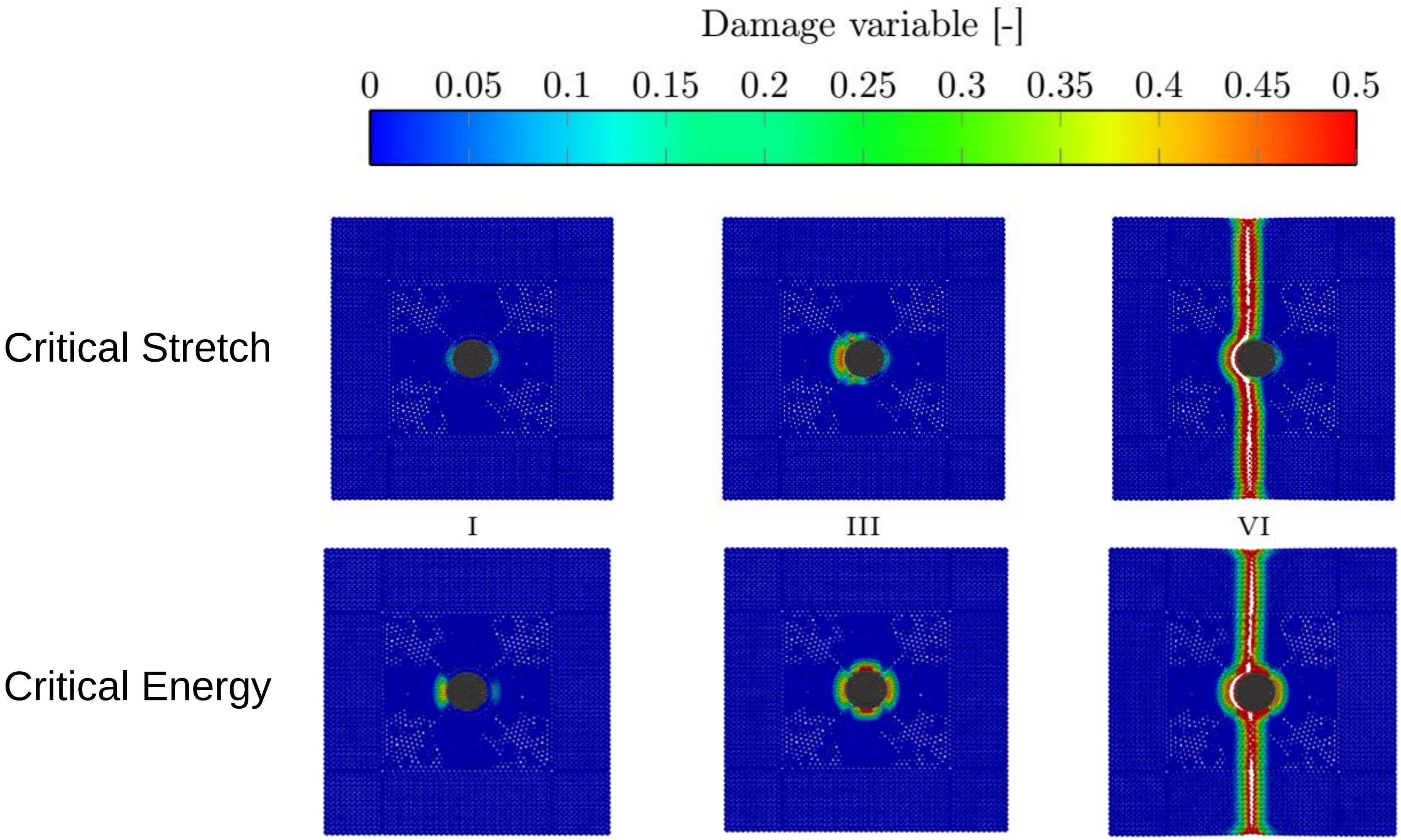
Comparison: Critical stretch fitting



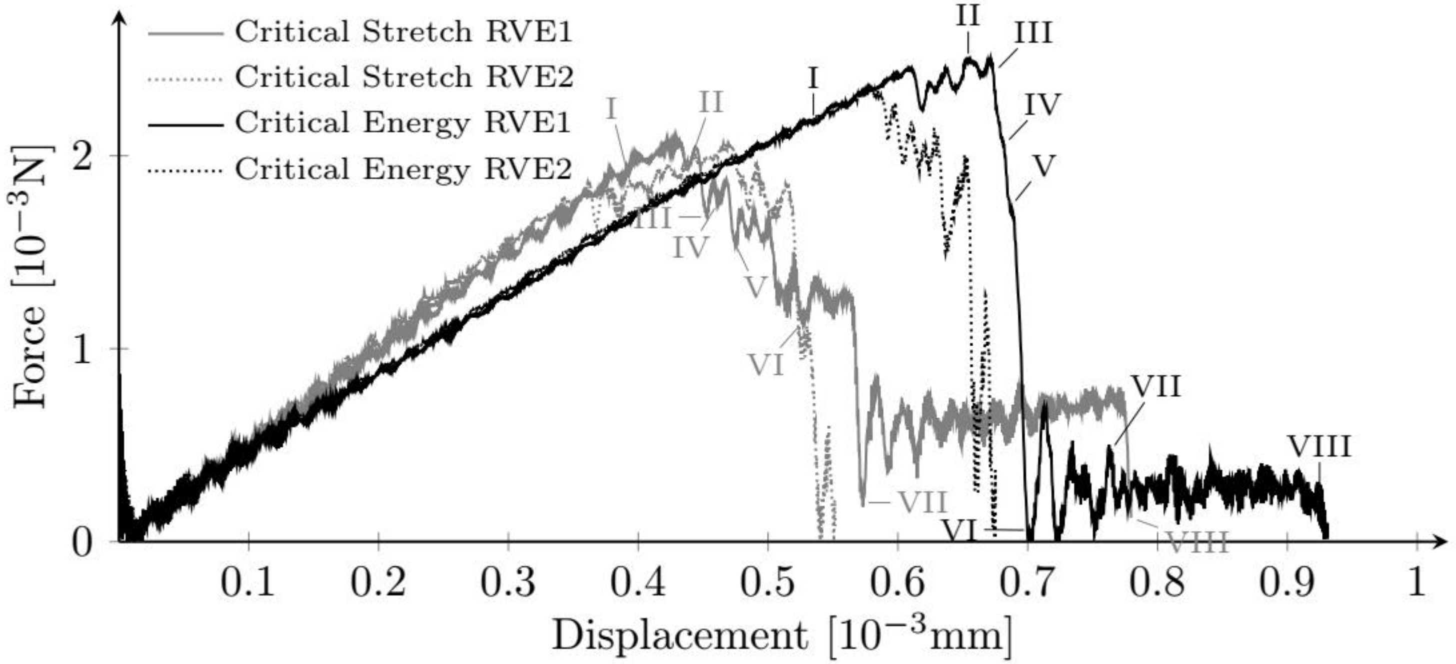
Comparison: Single Fibre – Energy Criterion



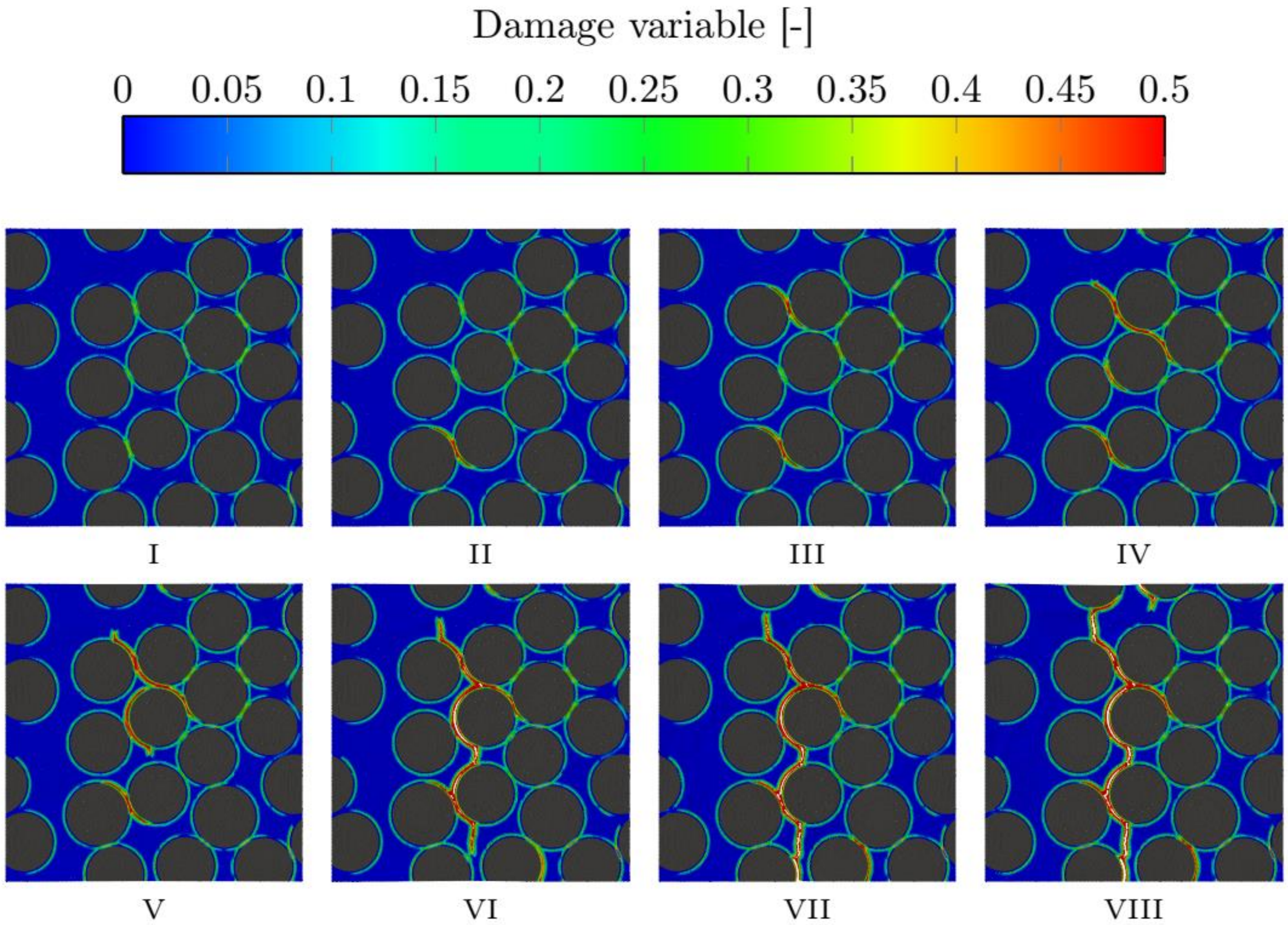
Comparison: Single fibre



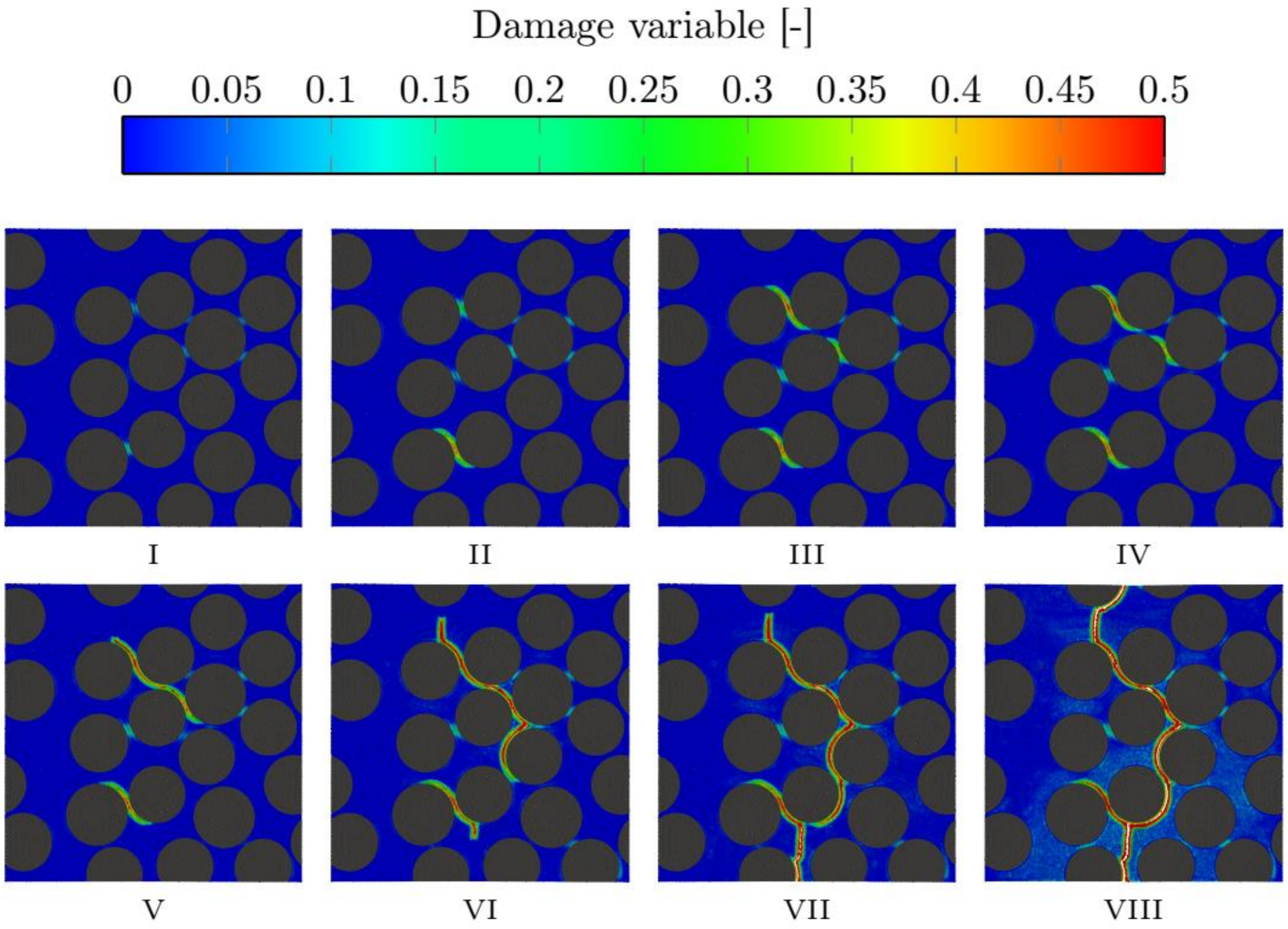
Comparison: RVE-Force-Displacement



Comparison: RVE – Energy Criterion



Comparison: RVE - Critical Stretch



Conclusion

- The energy criterion from Foster et al. has been adapted, implemented and tested
- The criterion is able to represent the energy release rate
- 2dx meshes of any discretization lead to overestimation of the crack initiation load
- 4-5dx shows the best results + converge; <2% difference in results
- Difference between the standard method (critical stretch) and critical energy has been shown
- Fitting of critical stretch model in simple models could not be transferred to complex models

Thank you!

Dr.-Ing. Christian Willberg

DLR
Institute of Composite Structures
and Adaptive Systems

Lilienthalplatz 7
38108 Braunschweig
Germany

Phone: +49 531 295 - 2232
Email: christian.willberg@dlr.de

All presented models and source code can be found here
Rädel, R. & Willberg, C. PeriDoX Repository
https://github.com/PeriDoX/PeriDoX

References

Silling, S. A.; Epton, M.; Weckner, O.; Xu, J. & Askari, E. Peridynamic States and Constitutive Modeling in *Journal of Elasticity,* **2007***, 88*, 151-184

Bobaru, F.; Foster, J. T.; Geubelle, P. H. & Silling, S. A. Handbook of peridynamic Modeling *CRC Press,* **2016**

Foster, J. T.; Silling, S. A. & Chen, W.
An Energy based Failure Criterion for use with Peridynamic States in *International Journal for Multiscale Computational Engineering,* **2011***, 9*, 675-688

Rädel, M.; Bednarek, A.-J. & Willberg, C. Influence of probabilistic material distribution in peridynamics to the crack initiation *6th ECCOMAS Thematic Conference on the Mechanical Response of Composites: COMPOSITES 2017,* **2017**

Willberg, C.; Rädel, M. & Wiedemann, L. “A mode-dependent energy-based damage model for peridynamics and its implementation” in *Journal of Mechanics of Materials and Structures,* **2018**, in review

Energy based state-based failure criterion

$$W_{\text{bond}} = 0.25\chi(\underline{e}\langle\boldsymbol{\xi}\rangle, t)\left\{\underline{t}\left[\mathbf{x}, t\right] - \underline{t}\left[\mathbf{x}', t\right]\right\}\underline{e} < W_C$$

$$\underline{t}\left[\mathbf{x}, t\right] = \chi(\underline{e}\langle\boldsymbol{\xi}\rangle, t)\left(\frac{3K\left[\mathbf{x}, t\right]\theta\left[\mathbf{x}, t\right]}{m_V\left[\mathbf{x}, t\right]}\underline{\omega x} + \frac{15G\left[\mathbf{x}, t\right]}{m_V\left[\mathbf{x}, t\right]}\underline{\omega e}^d\left[\mathbf{x}, t\right]\right)$$

$$\underline{t}\left[\mathbf{x}', t\right] = \chi(\underline{e}\langle\boldsymbol{\xi}\rangle, t)\left(\frac{3K\left[\mathbf{x}', t\right]\theta\left[\mathbf{x}', t\right]}{m_V\left[\mathbf{x}', t\right]}\underline{\omega x} + \frac{15G\left[\mathbf{x}', t\right]}{m_V\left[\mathbf{x}', t\right]}\underline{\omega e}^d\left[\mathbf{x}', t\right]\right)$$

$$\theta\left[\mathbf{x}, t\right] = \frac{3}{m_V\left[\mathbf{x}, t\right]}\int_{\mathcal{H}(\mathbf{x})}\underline{\omega x e}\,\mathrm{d}V_{\boldsymbol{\xi}} \qquad \underline{e}^d\left[\mathbf{x}, t\right] = \underline{e} - \frac{\theta\left[\mathbf{x}, t\right]\underline{x}}{3}$$

$$\theta\left[\mathbf{x}', t\right] = \frac{3}{m_V\left[\mathbf{x}', t\right]}\int_{\mathcal{H}(\mathbf{x}')}\underline{\omega x e}\,\mathrm{d}V_{\boldsymbol{\xi}} \qquad \underline{e}^d\left[\mathbf{x}', t\right] = \underline{e} - \frac{\theta\left[\mathbf{x}', t\right]\underline{x}}{3}$$

DLR